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# A distributed algorithm for wide-band radio-interferometry

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**Abstract**—We propose a scalable, randomised algorithm to solve the inverse imaging problem in wide-band radio-interferometry. In the big-data context of the next-generation radio-telescopes, the scalability is paramount due to the large-scale of the problem to be solved. The proposed method distributes the data measured at each frequency and processes it in parallel. We showcase the algorithm capabilities through realistic simulations.

## I. INTRODUCTION

In wide-band radio-interferometry (RI), the electromagnetic signal coming from the sky is probed by an array of antennas, at multiple frequencies  $\nu_i$ , and is correlated at each antenna pair, producing radio measurements  $\mathbf{y}_i \in \mathbb{C}^M$  for each band  $\nu_i$ . To recover a hyper-spectral image of the sky, an ill-posed problem has to be solved, which under simplifying assumptions, can be modelled as  $\mathbf{Y} = \Phi(\mathbf{X}) + \mathbf{N}$ , where  $\mathbf{Y} = (\mathbf{y}_1, \dots, \mathbf{y}_b) \in \mathbb{C}^{M \times b}$  denotes the wide-band measured data at  $b$  bands, corrupted by additive white Gaussian noise  $\mathbf{N} = (\mathbf{n}_1, \dots, \mathbf{n}_b) \in \mathbb{C}^{M \times b}$  and  $\mathbf{X} = (\mathbf{x}_1, \dots, \mathbf{x}_b) \in \mathbb{R}_+^{N \times b}$  is the unknown hyper-spectral image. The linear operator  $\Phi(\mathbf{X}) = ([\Phi_i \mathbf{x}_i]_{i=1:b})$  models the acquisition process, that is an incomplete Fourier sampling.

## II. CONVEX MINIMISATION PROBLEM

We assume a linear mixture model for the image cube and solve a convex minimisation problem imposing low-rankness, joint-sparsity and positivity of the image cube  $\mathbf{X}$  [1]. We introduce multiple data fidelity terms defined for each frequency band, to achieve a high degree of parallelism. The minimisation problem can be defined as

$$\min_{\mathbf{X}} f(\mathbf{X}) + \mu g_1(\Psi^\dagger \mathbf{X}) + g_2(\mathbf{X}) + \sum_{i=1}^b h_i(\bar{\Phi}_i(\mathbf{X})), \quad (1)$$

with the functions involved:  $f = \iota_{\mathcal{D}}, \mathcal{D} = \mathbb{R}_+^{N \times b}$  accounting for the positivity constraint;  $g_1(\mathbf{Z}) = \|\mathbf{Z}\|_{\ell_{2,1}}$  imposing joint-sparsity in a concatenation of wavelet basis  $\Psi$ ;  $g_2(\mathbf{Z}) = \|\mathbf{Z}\|_*$  imposing low-rankness onto the desired solution;  $h_i = \iota_{\mathcal{B}_i}, \mathcal{B}_i = \{\mathbf{Z} \in \mathbb{C}^{M \times b} : \|\mathbf{Z} - \mathbf{Y}_i\|_F \leq \epsilon_i\}$  enforcing data fidelity by constraining the solution to belong to the  $\epsilon_i$ -balls defined by the known noise statistics. We denote with  $\mathbf{Y}_i = (\alpha_1 \mathbf{y}_1, \dots, \alpha_b \mathbf{y}_b) \in \mathbb{C}^{M \times b}$  the measurement matrix active only at the band  $\nu_i$  such that  $\alpha_j = 0, \forall j \neq i$ . The associated linear operator is  $\bar{\Phi}_i(\mathbf{X}) = ([\alpha_j \Phi_i \mathbf{x}_i]_{i=1:b})$  with  $\alpha_j = 0, \forall j \neq i$ .

To solve (1), we use a randomised primal-dual algorithm [2] that relies on forward-backward (FB) iterations to manage the non-smooth functions. The algorithmic structure has been employed for distributed, single-band imaging [3] and for non-distributed wide-band imaging [1]. The operations are detailed in Algorithm 1. All the proximal FB steps have closed-form solutions. The proximity operator for the joint-sparsity prior is a row-wise soft-thresholding operation, for row  $k$  defined as  $(\mathcal{S}_\alpha^{\ell_{2,1}}(\mathbf{Z}))_{k,:} = \frac{\bar{z}(\|\bar{z}\|_{\ell_2} - \alpha)}{\|\bar{z}\|_{\ell_2}}$  if  $\|\bar{z}\|_{\ell_2} > \alpha$  and  $(\mathcal{S}_\alpha^{\ell_{2,1}}(\mathbf{Z}))_{k,:} = 0$  otherwise. The nuclear norm produces the soft-thresholding of the eigenvalues of  $\mathbf{Z}$ ,  $\mathcal{S}_\alpha^*(\mathbf{Z}) = \mathbf{H}_1 \mathcal{S}_\alpha^{\ell_1}(\Sigma) \mathbf{H}_2^\dagger$ . Data fidelity is enforced by the projections  $\mathcal{P}_{\mathcal{B}_i}$  onto the  $\epsilon_i$  sized  $\ell_2$  balls, for

each band and positivity is imposed via the projection  $\mathcal{P}_{\mathcal{D}}$  onto the positive orthant  $\mathcal{D}$ .

## Algorithm 1 Randomised PD for distributed WB RI.

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given  $\mathbf{X}^{(0)}, \bar{\mathbf{X}}^{(0)}, \mathbf{V}_1^{(0)}, \mathbf{V}_2^{(0)}, \mathbf{U}_1^{(0)}, \dots, \mathbf{U}_b^{(0)}, \mu, \tau, \sigma_1, \sigma_2, \sigma_3$ 
repeat for  $t = 1, \dots$ 
  generate active set  $\mathcal{A} \subset \{1, \dots, b\}$ 
  do in parallel
     $\mathbf{V}_1^{(t)} = \mathbf{V}_1^{(t-1)} + \Psi^\dagger \bar{\mathbf{X}}^{(t-1)} - \mathcal{S}_{\mu/\sigma_1}^{\ell_{2,1}}(\mathbf{V}_1^{(t-1)} + \Psi^\dagger \bar{\mathbf{X}}^{(t-1)})$ 
     $\mathbf{V}_2^{(t)} = \mathbf{V}_2^{(t-1)} + \bar{\mathbf{X}}^{(t-1)} - \mathcal{S}_{1/\sigma_2}^*(\mathbf{V}_2^{(t-1)} + \bar{\mathbf{X}}^{(t-1)})$ 
     $\forall i \in \mathcal{A}$  do in parallel
       $\mathbf{U}_i^{(t)} = \mathbf{U}_i^{(t-1)} + \bar{\Phi}_i(\bar{\mathbf{X}}^{(t-1)}) - \mathcal{P}_{\mathcal{B}_i}(\mathbf{U}_i^{(t-1)} + \bar{\Phi}_i(\bar{\mathbf{X}}^{(t-1)}))$ 
    end
  end
   $\mathbf{X}^{(t)} = \mathcal{P}_{\mathcal{D}}\left(\mathbf{X}^{(t-1)} - \tau\left(\sigma_1 \Psi \mathbf{V}_1^{(t)} + \sigma_2 \mathbf{V}_2^{(t)} + \sigma_3 \sum_{i=1}^b \bar{\Phi}_i^\dagger(\mathbf{U}_i^{(t)})\right)\right)$ 
   $\bar{\mathbf{X}}^{(t)} = 2\mathbf{X}^{(t)} - \mathbf{X}^{(t-1)}$ 
until convergence

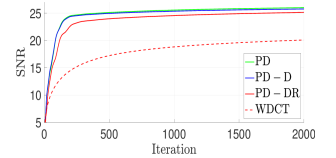
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## III. SIMULATIONS AND RESULTS

We simulate a wide-band image cube following the spectral curvature model  $\mathbf{x}_i = \mathbf{x}_0(\nu_i/\nu_0)^{-\gamma + \beta \log(\nu_i/\nu_0)}$ , where  $\mathbf{x}_0$  is a  $256 \times 256$  sized image of a radio region in the M31 galaxy;  $\gamma$  and  $\beta$  are the spectral index maps of size  $N$  and modelled as correlated Gaussian random fields. The wide-band cube is generated for  $b = 16$  bands in the range  $[1.4, 2.8]$  GHz. The wide-band data are simulated using realistic  $uv$ -coverages from the VLA array-configuration with  $M = 33120$  measurements at each band and are corrupted with zero-mean Gaussian noise with an input signal-to-noise ratio (SNR) of 30 dB.

The figure reveals the SNR evolution for the different algorithms. We can see that the non-distributed primal-dual algorithm denoted by PD [1] and the distributed version PD-D exhibit comparable behaviour, reaching a SNR = 26 dB. For the proposed distributed randomised algorithm PD-DR, we fix the probability of selecting an active subset  $\mathcal{A}$  from the full data  $\mathbf{Y}$  to 0.5. This has the advantage of lower infrastructure and memory requirements, at the expense of an increased number of iterations to achieve convergence. Also, when compared to the approach proposed in [4] and denoted by WDCT, our proposed algorithm presents superior performance; PD-DR reaches a SNR = 25 dB that is 5 dB higher than WDCT.



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