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User Selection for Multi-beam Satellite Channels: A Stochastic Geometry Perspective

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***Abstract*—In this paper, we analyse the performance of multi-beam satellite communication network where each beam is paired with a single user. Each beam is controlled by a gateway (GW) connected to the satellite via a return channel. We consider terrestrial interference from cellular base stations (BSs) also communicating with the users in the system. Base station (BS) locations are modelled as an independent Poisson point process (PPP). We employ stochastic geometry tools to characterize the terrestrial interference. We also analyse the system based on the random user and the best user selection criteria. Finally, we analyse the coverage probability of the network and validate them with simulation results. Our results show the impact of terrestrial interference on satellite communication and that the best user selection can increase the coverage probability of such systems.**

***Index Terms*—Satellite, multi-beam, interference, Poisson point processes, wireless networks, stochastic geometry.**

I. Introduction

The key goals of current generation wireless communication systems include data rates in the range of Gbps, billions of connected devices, lower latency, improved coverage and reliability and low-cost, energy efficient and environment-friendly operation. Satellite mobile communication is a promising technology in this direction that provides ample coverage with low complexity infrastructure [1]. Multi-beam structure in modern satellite mobile communication has gained massive attention since they can provide high coverage area and larger capacity as multiple isolated spot beams can reuse the frequency.

In contrast to conventional cellular networks, satellite communication systems have long round trip delays between the gateway (GW) and user terminal. It is worth noticing that the variations in the phase component of the channel are rapid due to phase noise contribution from the payload and the time dependent channel variations [2]. On one hand, this time varying nature of the channel coupled with a high round trip delays result in outdated channel phase [2]. On the other hand, there is a limit on reuse level of frequency and leads to inter-beam interference.

Similar to the inter-cell interference in terrestrial cellular networks (3GPP LTE), is this inter-beam interference problem, and it is even more severe with dense reuse. This is because all beams are generated by the satellite array antenna and suppression of out of desired beam coverage strength is only by radiation pattern of beam antenna [3]. In contrast to terrestrial cellular networks where cells are served by geographically separated base stations (BSs) and interference is more effectively suppressed by propagation distance.

In order to meet the increasing demands of wireless traffic, spectrum management is another key technique in wireless communication. Therefore, cognitive satellite communications is shown to be a promising way to improve spectrum efficiency of broadband satellite systems. Hence the terrestrial systems can exploit under-utilized spectrum [4]. Also, gaining interest in research [5] is the co-existance between satellite and terrestrial systems. However, the incumbent BS microwave links cause the terrestrial interference to the cognitive downlink Geostationary (GEO) satellite service terminals. In addition, precoding is being explored in multi-beam satellite systems as a means to mitigate inter-beam interference. The work in [6] shows that spectral efficiency is improved by about 50 percent with the use of linear precoding.

With the incorporation of BSs, terrestrial interference is another key parameter in characterising the performance of downlink mulit-beam satellite system. Stochastic geometry is one of the tools that can be used to characterize the statistical behaviour of aggregate interference. A convenient way to do so is via the Laplace Transform (LT) of interference or its characteristic function (CF). To cite a few examples from the perspective of cellular networks, in [7] the characteristic function of the aggregate interference in a AWGN channel (no fading) was derived, leading to infinite series expressions for the probability density function of interference. Some time later, the approach was revisited and the results generalized to the Rayleigh fading case [8].

The most common user selection approach is random, where users are simply selected randomly. It implies, it is possible that some users with less interference, i.e both intra-beam interference and terrestrial interference, are grouped with some users affected by more interference. This issue will affect on the feasibility of the system. In this paper, we propose to select the users based on the level of the interruption they receive from the beams of other BSs as well as intra-beam interference. In other words, we group the users with highest SINR together and it is the same for users with high interference. In a multi-beam satellite system, the level of the interference is highly dependent on the location of the users.

The contributions of the paper can be summarized as follows:

- We provide an analytical framework for the analysis of multi-beam satellite with terrestrial BSs. In particular, we characterize the terrestrial interference using stochastic geometric approaches.
- *Performance analysis:* To analyze the performance of our model, we use the coverage probability.
- Via numerical results, we show the feasibility of two user selection criteria with respect to the mentioned performance metrics.

II. System Model

The multi-beam satellite system architecture is considered where the coverage area is served by K spot beams. It is assumed that the satellite downlink operates in the same band with incumbent microwave BSs as shown in Fig. 1. Without loss of generality, a single feed per beam is assumed. Hence, each beam is considered to be paired with a single user terminal at a given time constant. A single GW which manages a cluster of K adjacent beams on ground by K antenna feeds is considered in the system. It is assumed that perfect CSI is obtainable at GW. Thus, each user's signal is precoded at the GW and transmitted across all beams. We consider a zero-forcing (ZF) precoder and time division multiplexing (TDM) transmission, and assume an ideal link between the GW and the satellite.

The terrestrial BSs cause interference to the user terminals. To decrease the impact of terrain interference, we employ the directional transmissions at microwave links, which are generally deployed for backhauling point-to-point application [9]. Note that the downlink interference from the cognitive satellite to the terrestrial receivers is negligible due to the limitation in the maximum Effective Isotropic Radiated Power (EIRP) density of the current Ka band satellite system [10].

A. Network model

In this section, we illustrate our system model of a multi-beam satellite network consisting of multiple users with terrestrial BSs. The BSs in the network are modeled as points in $\mathbb{R}^2$ which are distributed as a homogeneous Poisson point process (PPP) Φ_{BS} with intensity λ_{BS}. We assume that the transmissions are simultaneous and use a universal frequency reuse scenario where all users can use the same channel. We consider a typical user which receives information from a multi-beam satellite.

B. Satellite system model

1) Free space loss: The free space loss is not identical for each multi-beam due to the earth's curvature. Therefore, in order to mitigate this effect, the free space loss coefficient of the kth beam multi-beam can be given as [11]

$$L_{\max} = \left(\frac{\lambda}{4\pi}\right)^2 \frac{1}{d_0^2 + d_k^2}, \tag{1}$$

where $d_0 \approx 35786\,\mathrm{km}$, λ is wavelength and d_k is the distance of the kth beam center from the center of the central beam.

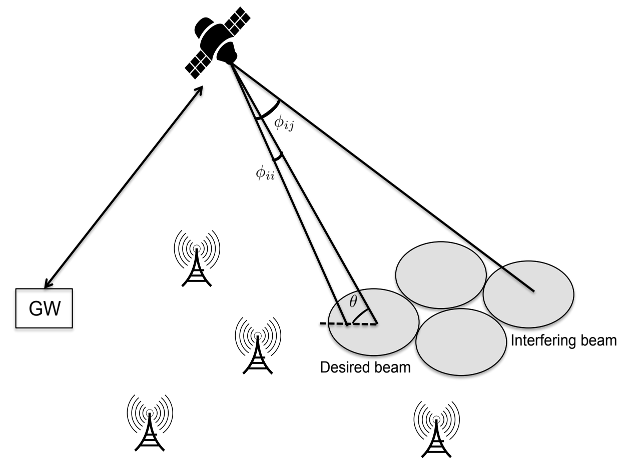

Fig. 1: An illustration of network set-up.

2) Fading model: We assume that the forward link contains both the line-of-sight (LOS) component and the scatter component. Hence, consider ω be the average receive power of LOS term, b_0 as half of the average power of scattered component, and m as the Nakagami fading coefficient by definition. Leveraging the results from [11], the Shadowed-Rician (SR) fading model can be considered to model both the LOS and scatter components. Therefore the probability density function can be written as

$$f_{|h|^2}(x) = \left(\frac{2mb_0}{2mb_0+\Omega}\right)^m \frac{1}{2b_0}\exp\left(-\frac{x}{2b_0}\right) \times {}_1F_1\left(m, 1, \frac{\Omega\, x}{2b_0(2mb_0+\Omega)}\right), \tag{2}$$

where ${}_1F_1$ is the hypergeometric function and the parameters b_0, m and Ω are connected with the elevation angle θ as illustrated in Fig. 1. The detailed characterization of the parameters b_0, m and Ω in terms of θ can be found in [11]. Hence, we omit the corresponding expressions of the parameters in this paper. Although the SR fading model is widely used in literature, the probability density function (PDF) and cumulative density function (CDF) are too complex to work with SINR expressions. Therefore, we approximate the squared SR model with Gamma random variable. Thus, the parameters of Gamma random variable are given by [11]

$$\alpha = \frac{m(2b_0+\Omega)^2}{4mb_0^2+4mb_0\Omega+\Omega^2},\ \nu = \frac{4mb_0^2+4mb_0\Omega+\Omega^2}{m(2b_0+\Omega)}. \tag{3}$$

3) Beamforming gain at user terminal: It is worth noticing that the average SINRs are highly dependent on both satellite beam pattern and user position. Therefore, the beam gain can be approximated as [12]:

$$G_{ii} = L_{\max} G_{s,i} G_{r,i} \left(\frac{J_1(x)}{2x} + 36\frac{J_3(x)}{x^3}\right)^2, \tag{4}$$

where $x = 2.07123 \sin(\phi_{ii})/\sin(\phi_{3dB})$, J_1 and J_3 are the first-kind Bessel functions of order 1 and 3. $G_{s,i}$ is the satellite transmit antenna gain for the ith beam and $G_{r,i}$ is the satellite user's receive gain. Note that ϕ_{ii} is denoted as the off-axis angle of the ith desired beam, ϕ_{ij} means for the off-axis angle from the ith desired beam to center of the jth intrefering beam.

Therefore, G_{ii} can be calculated from (4) with ϕ_{ii}. Similarly, G_{ij} is also calculated by (4) in term of ϕ_{ij}, is observed antenna gain between the jth interfering beam and the ith user.

C. Terrestrial system model

1) Fading model: In cellular networks, small scale fading impact on the transmitted signals is higher when compared to satellite systems. It is extensively mentioned in [13], [14] that in cellular analysis, the Nakagami fading model can capture a generalised propagation environment. Hence, we consider Nakagami-m channel model, and the channel power is distributed according to

$$H \sim f_H(x;m) \triangleq \frac{m^m x^{m-1} e^{-mx}}{\Gamma(m)}, \tag{5}$$

where $\Gamma(m)$ is the gamma function.

2) Directional beamforming modeling: In order to reduce the terrestrial interference impact on the satellite user terminals, we employ directional beamforming at BSs. Accordingly, antenna arrays are deployed at the transmitters. It is worth noticing that the receiver, i.e satellite user terminals, are also equipped with the directional antennas. Hence, we consider that all the antennas at transmit and receiver pairs are directional antennas with sectorized gain pattern. Let $\bar{\theta}$ be the beamwidth of the main lobe. Then the antenna gain pattern for a BS or satellite user at some angle ϕ is given as [15]:

$$G_q(\bar{\theta}) = \left\{ \begin{array}{ll} G_q^{\mathrm{M}} & \mathrm{if}|\bar{\phi}| \leq \bar{\theta} \\ G_q^{\mathrm{m}} & \mathrm{if}|\bar{\phi}| \geq \bar{\theta} \end{array} \right\}, \tag{6}$$

where $q \in \mathrm{BS}, \mathrm{U}$, $\bar{\phi} \in [0, 2\pi)$ is the angle of boresight direction, G_q^{M} and G_q^{m} are the array gains of main and side lobes, respectively.

For any intended link[1] q, $G_q = G_{\mathrm{BS}}^{\mathrm{M}} G_{\mathrm{BS}}^{\mathrm{M}}$, for any other link t, G_t will depend on the directivity gains of the main (i.e., G^{M}) and side (i.e., G^{m}) lobes of the antenna beam pattern. Accordingly, the effective antenna gain for an interferer seen by the typical receiver is given by:

$$G_t = \left\{ \begin{array}{ll} G_{\mathrm{BS}}^{\mathrm{M}} G_{\mathrm{U}}^{\mathrm{M}}, & \mathrm{p}_{\mathrm{MM}} = (\frac{\bar{\theta}}{2\pi})^2 \\ G_{\mathrm{BS}}^{\mathrm{M}} G_{\mathrm{U}}^{\mathrm{m}}, & \mathrm{p}_{\mathrm{Mm}} = \frac{\bar{\theta}(2\pi-\bar{\theta})}{(2\pi)^2} \\ G_{\mathrm{BS}}^{\mathrm{m}} G_{\mathrm{U}}^{\mathrm{M}}, & \mathrm{p}_{\mathrm{mM}} = \frac{\bar{\theta}(2\pi-\bar{\theta})}{(2\pi)^2} \\ G_{\mathrm{BS}}^{\mathrm{m}} G_{\mathrm{U}}^{\mathrm{m}}, & \mathrm{p}_{\mathrm{mm}} = (\frac{2\pi-\bar{\theta}}{2\pi})^2 \end{array} \right\}, \tag{7}$$

where p_{tk}, with $t, k \in \{\mathrm{M}, \mathrm{m}\}$ denotes the probability that the antenna gain $G^t G^k$ is seen by the receiver. Here, the effective gain can be considered as a random variable, which can take any of the above mentioned values. Herein after, we denote G_r^M is the satellite user's main lobe gain[2].

D. Signal model

The overall channel gain between the jth beam and ith user can be given as

$$h_{ij} = h_j \, G_{ij}(\phi_{ij})^{1/2}, i, j = 1, \cdots, K, \tag{8}$$

[1]The transmission between BS and another BS is regarded as intended link.
[2]We omit the subscript i in $G_{r,i}$.

Consider P_i is the transmit power of ith beam, and x_i is the transmitted information symbol from beam i. Hence, the received signal at ith beam user can be formulated as

$$y_i = \sqrt{P_i} h_{ii} x_i + \sum_{j=1, j\neq i}^{K} \sqrt{P_j} h_{ij} x_j + I_{\mathrm{BS}} + \omega_i, \tag{9}$$

where ω_i is the noise power at beam i, and I_{BS} is the terrestrial interference.

E. SINR modeling

The SINR for the intended link i at the i^{th} user can then be formulated as

$$\gamma_i \triangleq \frac{P_i G_{ii} |h_i|^2}{\sigma^2 + \sum\limits_{j=1, j\neq i}^{K} P_j G_{ij} |h_j|^2 + I_{\mathrm{BS}}}, \tag{10}$$

where h_{ii} is the fading gain at the user of interest, σ^2 is the noise power, and h_{ij} denotes each interference fading gain.

The second term of the denominator is in (10) is zero due to successful ZF precoding. Hence, the SINR for the intended link i at any particular user considering terrestrial interference can be re-written as

$$\hat{\gamma}_i \triangleq \frac{P_i G_{ii} |h_i|^2}{\sigma^2 + \sum\limits_{l \in \Phi_{\mathrm{BS}}} P_t G_t |h_l|^2 r_l^{-\alpha}}, \tag{11}$$

where P_t is transmit power of BS, r_l is the distance between BS and satellite user, and α is the path loss exponent.

III. Coverage Probability

In this section, we analyze the proposed system model from the perspective of coverage probability. In particular, we characterize the coverage probability with random user selection and best user selection criteria. The best user can be considered to be the one that receives the maximum received path gain with less interference.

In the following proposition, we give coverage probability of a random user subjected to terrestrial interference.

A. Random user selection

Proposition 1. *The CCDF of a typical user SINR considering the terrestrial interference can be given as*

$$F_{\hat{\gamma}_i}(T) \approx \sum_{l=1}^{\alpha} \binom{\alpha}{l} (-1)^{l+1} \exp\left(\frac{-A\, l\, T \sigma^2}{P_i G_{ii}}\right) \tag{12}$$

$$\times \prod_{t,k \in \{\mathrm{M},\mathrm{m}\}} \exp\left[-2\pi p_{tk} \lambda_{\mathrm{BS}} r \left(1 - \left(\frac{1}{1 + \frac{A\, l\, T\, P_j G_t^{tk}}{P_i G_{ii} m r^{\alpha}}}\right)^m\right) \mathrm{dr}\right].$$

Proof. Now, the CCDF of SINR distribution using (11) can be given as

$$F_{\hat{\gamma}_i} = \mathbb{P}\left[\frac{P_i G_{ii} |h_i|^2}{\sigma^2 + I_{\mathrm{BS}}} > T\right] = \mathbb{P}\left[|h_i|^2 > \frac{T}{P_i G_{ii}} \left(\sigma^2 + I_{\mathrm{BS}}\right)\right]. \tag{13}$$

Leveraging the tight upper bound of a Gamma random variable of parameters α and ν as $\mathbb{P}[|h_i|^2 < \gamma] < (1 - e^{-A\nu\gamma})^{\alpha}$

with $A = \alpha(\alpha!)^{\frac{-1}{\alpha}}$, and by applying Binomial theorem we approximate (13) as

$$F_{\hat{\gamma}_i}(T) \approx \sum_{l=1}^{\alpha} \binom{\alpha}{l} (-1)^{l+1} e^{\frac{-A\, l\, T \sigma^2}{P_i G_{ii}}} \mathcal{L}\{I_{\Phi_{\mathrm{BS}}}\}(s). \quad (14)$$

However, the terrestrial interference due to BSs needs to be characterized before proceeding to next section. Given that the interference from BSs could be either from main lobe or side lobe, therefore leveraging the notion of mark stochastic geometry, we have

$$\mathbf{I}_{\Phi_{\mathrm{BS}}} = \mathbf{I}_{\Phi_{\mathrm{BS}}}^{MM} + \mathbf{I}_{\Phi_{\mathrm{BS}}}^{Mm} + \mathbf{I}_{\Phi_{\mathrm{BS}}}^{mM} + \mathbf{I}_{\Phi_{\mathrm{BS}}}^{mm}. \quad (15)$$

By definition of the Laplace transform, we have

$$\mathcal{L}\{I_{\Phi_{\mathrm{BS}}}\} = \mathcal{L}\{I_{\Phi_{\mathrm{BS}}}^{MM}\}\mathcal{L}\{I_{\Phi_{\mathrm{BS}}}^{Mm}\}\mathcal{L}\{I_{\Phi_{\mathrm{BS}}}^{mM}\}\mathcal{L}\{I_{\Phi_{\mathrm{BS}}}^{mm}\}. \quad (16)$$

Hence the conditional Laplace transform of $\mathbf{I}_{\Phi_{\mathrm{BS}}}^{MM}$ for a given G_t can be written as

$$\begin{aligned} \mathcal{L}\{I_{\Phi_{\mathrm{BS}}}^{MM}\}(s) &= \mathbb{E}[\exp(-s I_{\Phi_{\mathrm{BS}}}^{MM})], \quad (17) \\ &= \mathbb{E}_{\Phi_{\mathrm{BS}}, h_t, G_t}\left[\exp\left(-s\, P_t G_t |h_t|^2 r^{-\alpha}\right)\right], \\ &\overset{(a)}{=} \mathbb{E}_{\Phi_{\mathrm{BS}}, G_t}\left\{\prod_{i \in \Phi_{\mathrm{BS}}} \left(\frac{1}{1 + \frac{s\, P_t G_t r^{-\alpha}}{m}}\right)^m\right\}, \\ &\overset{(b)}{=} \mathbb{E}_{\Phi_{\mathrm{BS}}, G_t}\left\{\exp\left[-2\pi\lambda_{\mathrm{BS}} r \left(1 - \left(\frac{1}{1 + \frac{s\, P_j G_t}{m r^{\alpha}}}\right)^m\right) \mathrm{dr}\right]\right\}, \end{aligned}$$

where (a) follows from the use of the moment generating function of Gamma random variable with Nakagami fading parameter m, (b) follows due to the use of probability generating functionals of PPPs.

Finally, using equation (16), the Laplace transform of $\mathbf{I}_{\Phi_{\mathrm{BS}}}$ is given as

$$\begin{aligned} \mathcal{L}\{I_{\Phi_{\mathrm{BS}}}\}(s) &= \mathbb{E}[\exp(-s I_{\Phi_{\mathrm{BS}}})], \quad (18) \\ &= \prod_{t,k \in \{\mathrm{M,m}\}} \exp\left[-2\pi p_{tk} \lambda_{\mathrm{BS}} r \left(1 - \left(\frac{1}{1 + \frac{s\, P_j G_t^{tk}}{m r^{\alpha}}}\right)^m\right) \mathrm{dr}\right], \end{aligned}$$

This proof concludes by substituting (18) into (14). □

B. Best user selection

The distribution of interest, $i.e.$, the path gain distribution of the best satellite user, can be given in following lemma with the help of *order statistics*.

Lemma 1. *Let $\bar{\gamma} = \max\{\hat{\gamma}_i\}$. Then the probability distribution of the user which exhibits the maximum received SINR can be given as*

$$F_{\bar{\gamma}} = \prod_{i=1}^{K} (1 - F_{\hat{\gamma}_i}) = (1 - F_{\hat{\gamma}_i})^K, \quad (19)$$

where K is the total number of satellite users.

Proof. This proof follows from [16]. □

IV. Numerical Analysis

In this section, we validate our system model and also verify the accuracy of the results mentioned in the propositions. In general, the computations are done through Monte Carlo simulations, which are then used to validate the analytical results. We consider the carrier frequency of 4 GHz. Unless

TABLE I: Simulation Parameters

Notation	**Parameter**	**Values**
d_0	Orbit	35786 Km
r_d	Beam radius	250 Km
$G_{s,i}$	Satellite antenna gain	30 dBi
$G_{r,i}$	Satellite terminal gain	15 dBi
$3dB$	Angle	0.4^o
ϕ_{ii}	Off-axis angle of desired user	0.6^o
ϕ_{ij}	Off-axis angle of interfering user	0.8^o
λ_{BS}	Density of BSs	1e-06
G^M	BS antenna gain of main lobe	15 dB
α	Path loss exponent	2.5
P_{BS}	Node transmit power	1 Watt
m	Nakagami parameter	1

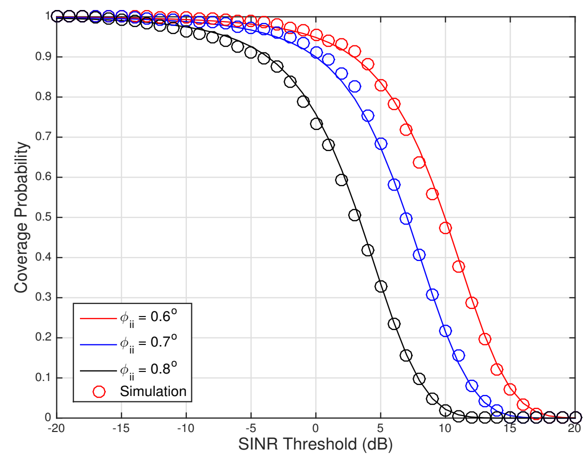

Fig. 2: Coverage probability as a function of SINR threshold under random user selection for different off-axis angles.

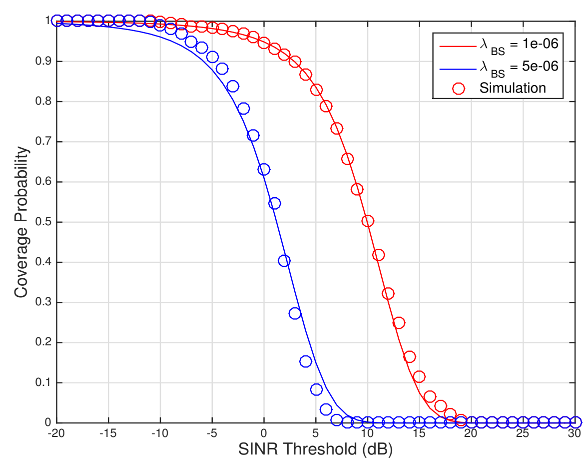

Fig. 3: Coverage probability as a function of SINR threshold under random user selection for different λ_{BS}.

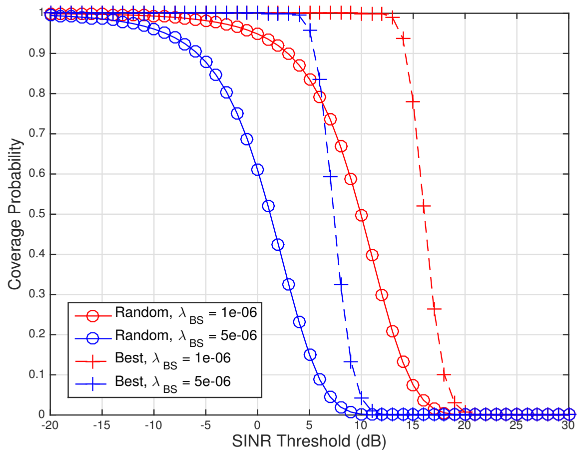

Fig. 4: Coverage probability as a function of SINR threshold at random and best user for different λ_{BS}.

stated otherwise, most of the values of the parameters used are inspired from literature [11], [12]. A few of the parameters and their corresponding values are given in Table I. All other parameters and values will be explicitly mentioned wherever used.

First we compare the coverage probability with different off-axis angles as a function of SINR threshold in Fig. 2. This result validates Proposition 1. It can be seen that the gap between the analytical and simulation results obtained after numerical evaluation is very tight. It can also be seen from the figure that the off-axis angles have considerable impact on the received SINR. As we increase the off-axis angle of desired satellite user, the probability of coverage of a random satellite user for a fixed transmit power reduces.

After establishing the effect of off-axis angles in the previous figure, we now look into the effect terrestrial interference on the coverage probability. Hence, in the next figure we consider terrestrial interference as a function of λ_{BS} under the Rayleigh fading model. Accordingly, in Fig. 3 we compare the coverage probability with different λ_{BS}. This result also validates Proposition 1 as the performance gap between the analytical and simulation results is minimal. It is evident from the figure that the coverage probability decreases as the increase in BS density λ_{BS}.

Similar to previous Fig. 3, we analyze the coverage probability for various λ_{BS} but with respect to the best user in Fig. 3. The settings for this figure are kept similar to Fig. 3. It is shown from the figure that similar to Fig. 3, higher BS density reduces the coverage probability of a user. However, we would like to note that the best user criteria has better coverage probability as compared to random user.

V. Conclusion

This paper deals with the downlink multi-beam satellite communication network with inter-beam interference and terrestrial interference from microwave BSs. The received SINR at satellite user takes into account the path loss caused by shadowed fading. Two robust user selection criteria's are proposed. Namely, random user selection - users are selected randomly, the best user based on received signal-to-noise-interference (SINR) is selected. In particular, stochastic geometric tools are employed while modelling the terrestrial interference and user selection. With the help of analytical and simulation results on coverage probability, we show the effect of varying different network parameters such as BS density, path loss exponent, off-axis angle and SINR threshold. It is found that the terrestrial interference has a major impact on the coverage probability of a satellite user. The simulation results also reveal that the best user criteria considered to be a viable method, which in turn can increase the coverage probability of user network.

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