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Financial Fairness and Conditional Indexation

Torsten Kleinow and Hans Schumacher

Department of Actuarial Mathematics and Statistics,
the Maxwell Institute for Mathematical Sciences and
Netspar - Network for Studies on Pensions, Aging and Retirement,
School of Mathematical and Computer Sciences
Heriot-Watt University, Edinburgh, EH14 4AS, United Kingdom
t.kleinow@hw.ac.uk

Department of Econometrics and Operations Research and Netspar
Tilburg University, PO Box 90153, 5000 LE Tilburg, the Netherlands
j.m.schumacher@uvt.nl

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Collective pension contracts can generate advantages for their participants by implementing forms of risk sharing. To ensure the continuity of a collective scheme, it has to be monitored whether the contracts offered to participants are financially fair in terms of their market value. When risk sharing is implemented by means of optionalities such as conditional indexation, the analysis of financial fairness is not straightforward. In this paper, we use a stylised overlapping generations model to study financial fairness for a conditional indexation scheme. We find that financial fairness for all participants at all times is not feasible within a scheme of this type. However, financial fairness for incoming generations at the moment of entry can be realised. We show how to compute the fair contribution rate as a function of the current nominal asset/liability ratio for a given level of nominal entitlements. At low levels of the ratio, the fair contribution for incoming generations is also relatively low; nevertheless, the joining of a new generation still has a positive effect on the asset/liability ratio.

Keywords: collective pensions, conditional indexation, intergenerational risk sharing, overlapping generations models.

1 Introduction

Improved mortality and increasing uncertainty about investment returns have led to the closure of many traditional defined benefit (DB) pension schemes. As an alternative, defined ambition schemes (DA) have been suggested, for example, in the Netherlands. While in traditional DB schemes certain annuity levels were guaranteed subject to the pension plan or the employer staying solvent, the annuity levels of DA schemes represent a target, or ambition, rather than a guarantee. For a more detailed discussion of the DA scheme in the Netherlands, see for example, Bovenberg et al. [2015].

A characteristic of DA schemes is the presence of more or less complex rules for adapting benefit payments and accrual of pension rights to changing circumstances, arising for instance from fluctuations in investment returns or from revisions of mortality forecasts. In this way, DA schemes implement forms of intergenerational risk sharing. Discussions of various schemes in the literature include Gollier [2008], Dai and Schumacher [2009], Cui et al. [2011], Kleinow [2011], and Bovenberg and Mehlkopf [2014].

The sustainability of collective pension schemes hinges on the willingness of all involved stakeholders to continue participation. As pointed out by Bovenberg et al. [2007], it may not be attractive for new generations to enter an existing fund when the fund is running a deficit. Of course, whether or not incoming generations do have a motive for nonparticipation depends on the rules of the pension scheme. In this paper, we analyse the participation constraint in the context of a particular type of schemes known as conditional indexation schemes.

In contrast to Beetsma et al. [2012], our definition of the participation constraint is in terms of market values rather than in terms of utilities. In other words, we consider the open market as an alternative to participation in the collective. A generation that enters into a collective scheme with specified rules effectively acquires a contingent claim. The only contingencies that can affect the payoffs relate to events that occur between the time at which the generation enters the system and the time of its death; indeed, events that occur before are not contingencies, and events that occur later cannot be taken into account. Financial contracts that depend on events that take place during the generation's lifetime are also offered in the open market. If a generation does not get a "fair deal" on a market value basis in the collective scheme (that is, if the market-consistent value of the contingent claim that the generation receives from the scheme is less than the market-consistent value of the contributions that are charged), then, under sufficient market completeness, it would be able to achieve higher utility by making use of suitable financial instruments that are traded in the open market.

In debates on pension reform, one can hear different opinions on whether such market completeness is indeed present. Market frictions reduce the threat of deviations from market consistency to the continuity of schemes that are well designed in terms of utility improvement. Deviations that are too large, however, would endanger sustainability. A certain "tolerance band" might be identified, but in this paper we will keep to the point of view of strict financial fairness in market value terms. We will work under the tacit assumption that, from a utility point of view, participation in the scheme that we consider is sufficiently attractive to successive generations. This assumption allows us to avoid explicit modelling of preferences.

In the design of risk sharing systems over time, the constraints imposed by incoming participants are not the only ones that should be taken into account. There is a rich literature on the conditions for continued risk sharing in overlapping generations models applied to financial intermediation, starting from Diamond and Dybvig [1983]. Here we consider the interests of three parties: incoming generations, incumbent generations, and an infinitely-lived sponsor. We derive the consequences of financial fairness to each of these stakeholders.

In order to analyse financial fairness, one must specify the rules of the scheme that is considered. In this paper we focus attention on conditional indexation (CI) schemes. In such schemes, pension benefits are subject to an indexation function which refers to an exogenous index such as inflation or a salary index, but which also depends on the solvency of the fund: if the asset/liability ratio is low, there will be no or little indexation, or in some cases, promised pensions might be reduced.

In the CI scheme we assume, pension benefits are regularly adjusted by a factor consisting of an exogenous index multiplied by a CI function which depends on the nominal asset/liability ratio of the pension fund. The nominal asset/liability ratio, or the nominal funding ratio as we shall call it, is defined as the ratio of assets to *nominal* liabilities, that is, liabilities without taking uncertain future indexation into account. This is in contrast to what might be called the *market-consistent* funding ratio, which is the ratio of assets to the market value of liabilities, given the rules of the indexation scheme. For the nominal funding ratio only the accrued benefits for existing members are relevant, while potential increases of those benefits in the future are ignored.

Using an overlapping generations model, we find that the value of benefits promised to a new generation depends on the contribution that this generation makes to the assets of the fund, but it is also necessary to consider the unknown contributions that future generations will make. Therefore, the valuation of promised benefits in a CI pension scheme turns out to be a non-standard option pricing problem with two major complications: the premium itself has an impact on the payoff, and the premium paid by future generations affects the payoff as well. In this respect, our approach is different from the approach used by de Jong [2008], who does not consider the feedback effect between the level of new contributions and the indexation of benefits. Instead, the focus of his research is on optimal portfolio choice. In contrast, we assume that the asset allocation is fixed, which we believe is a good approximation of common practice, and which allows us to concentrate on the effect of current and future contributions on the value of current pension promises.

The paper makes two main contributions to the existing literature. Firstly, we consider the financial position of the three stakeholders in the proposed CI pension scheme, namely: existing members, new members and the sponsor, for example, the employer. We then show that it is not possible to treat all three stakeholders fairly at all points in time.

Secondly, we then consider a scheme in which only the young generation (new members) will be treated fairly at the time they join the scheme, and in which any generation is obliged to allow any future generation to join the scheme subject to the future generation paying a fair premium. We calculate the fair value of the pension contract at the time a generation joins the scheme. We find that the fair level of contribution to be paid by the incoming generation is driven mainly by the current nominal funding ratio; when the funding ratio is low, the young generation should pay less than when the funding ratio is high. Our calculations indicate that, even when the contribution of incoming generation is reduced when the funding ratio is low, the older generations still benefit from the joining of the new generation.

By treating the young generation fairly, we ensure that they are happy to join the pension scheme rather than being forced to join a pension scheme, which they might perceive as unfair, given the information they have at the time of joining. In such a scheme each generation is driven only by self-interest. However, we show that even when decisions are driven by self-interest only, risk sharing between generations takes place.

The remainder of the paper is organised as follows. The model that we use is developed in Section 2 below. In Section 3, we ask whether it would be possible, within the CI schemes that we consider, to guarantee financial fairness for all involved parties at all times, and we arrive at the conclusion that this is not feasible. Instead, we continue under the assumption that entry implies commitment to future participation. In Section 4 we analyse the effect of incoming generations on older generations under this assumption, and we obtain a formula for the fair level of contributions for generations entering an infinitely-lived scheme. The formula justifies an approximation that is based on a finite horizon; we use this in Section 5 to obtain quantitative results. The intergenerational risk sharing effects are described in Section 6, and conclusions follow in Section 7.

2 Model and Notation

We consider an overlapping generations model in which each generation works for two periods and then retires at the end of the second period. Such a model was first introduced by Samuelson [1958] and has been applied widely in the economic literature; see for example Weil [2008]. In our model, members of each generation will pay into a pension fund and receive a lump sum payment at retirement. We will call the generation that enters at time $t = 1, 2, \dots$ generation t. We will assume that each generation t makes a single contribution, C_t, at time t (start of working life). In return generation t is promised to receive a nominal benefit to be paid as a lump sum at retirement at time $t + 2$. In addition, the initial nominal benefits are adjusted at the end of each period, that is, at times $t + 1$ and $t + 2$. The nominal benefits promised at time s to generation t are denoted by N_s^t, so that N_t^t is the initial promised benefit, N_{t+1}^t is the nominal promise at time $t + 1$ after adjustments at the end of the first period, and

N_{t+2}^{t} is the actual pension payment received by generation t when that generation retires at time $t+2$. We take the sizes of generations to be equal, so that the quantity N_{t+2}^{t} can also be viewed as the pension benefit per member of generation t.

Let A_t denote the value of the pension fund's assets at time t after the payment N_t^{t-2} has been made to the generation that retires at t, but before new contributions are received from generation t. It follows that the asset value after new contributions have been received is A_t+C_t, and the value of those assets at $t+1$ before pension payments N_{t+1}^{t-1} is $(A_t+C_t)\exp(R_{t+1})$ where R_{t+1} denotes the return during the period $[t, t+1)$. After the payment of N_{t+1}^{t-1} to generation $t-1$ has been made at $t+1$ the value of the assets reduces to

$$A_{t+1} = (A_t + C_t)\exp(R_{t+1}) - N_{t+1}^{t-1} \tag{1}$$

and the cycle starts again from that new asset value.

The adjustment of nominal benefits is called indexation in this paper. The focus of our study is on the calculation of fair contributions C_t to be made by generation t for a given initial nominal benefit N_t^t, and conditionally on an agreed indexation procedure. Pension schemes differ in that respect. In a pure defined contribution (DC) scheme there are no guarantees, and therefore no promises are made at time t. The payment N_{t+2}^{t} at retirement will be decided at that time depending on the available assets. On the other extreme end of the scale we have defined benefit schemes, where an initial promise is made, and that promise is adjusted throughout working life independently of the available assets until the promised payment is made at retirement. We ignore here adjustments during retirement since we assume a lump sum payment, N_{t+2}^{t} at the end of the second period. Although we study the risk for the pension fund sponsor in this paper, we also assume that the pension fund cannot default on its obligations but that the sponsor will always fund outstanding pension benefits.

Defined contribution (DC) and defined benefit (DB) schemes have been widely discussed in the literature and the risk sharing between pension fund sponsors (employer, state) and pension fund members (employees, citizens) is well understood; see for example Exley et al. [1997], Josa-Fombellida and Rincón-Zapatero [2004], and Vigna and Haberman [2001].

In contrast, the focus of this paper is on pension schemes where indexation is conditional on the value of the assets available to the pension fund. The basic argument is that if asset values are low then indexation is low, while high asset values lead to higher indexation. To be precise, we assume for the nominal promises made to generations $t-1$ and t that

$$N_{t+1}^{t-1} = I_{t+1} N_t^{t-1},\ \ N_{t+1}^{t} = I_{t+1} N_t^{t} \text{ with } I_{t+1} = g\left(\frac{A_t + C_t}{N_t^{t-1} + N_t^{t}}\exp(R_{t+1})\right) \tag{2}$$

where I_{t+1} is the indexation factor and g is the indexation function which depends on the ratio of available assets $(A_t+C_t)\exp(R_{t+1})$ at $t+1$ before payments and before new contributions, and the total liabilities $N_t^{t-1} + N_t^{t}$ of the scheme which consist of the promises made earlier to the two generations $t-1$ and t. The function g is often called the indexation ladder. In more realistic situations the indexation factor I_t would be influenced by other factors as well, such as realised inflation; for simplicity we ignore such additional factors here. Note that the same indexation factor I_{t+1} in (2) is applied to the promised nominal benefits of both generations t and $t-1$ which are in the scheme at the time of indexation.

The effect of conditional indexation is such that a given increase in value of the assets does not lead to a proportional increase of the market-consistent funding ratio, because the market value of liabilities is increased as well due to better perspectives for indexation. If the policy ladder g in (2) is too steep, then it may even happen that an increase of asset value causes a decline of the market-consistent funding ratio. For this reason we include an upper bound on the steepness of the ladder function in the standing assumption below.

(A1): The ladder function g is continuous, piecewise differentiable, and $0 \leq g' \leq 1$.

For simplicity of notation, it will be assumed as well that all amounts are expressed in terms of a suitable numéraire, so that interest rates can be taken to be zero. We assume furthermore that there is a common valuation operator that is applied by all stakeholders to assign a value at time t to uncertain payoffs that will be realised at time s. Under our assumption that amounts are stated in terms of a numéraire, the valuation operator takes the form $\mathrm{E}_t[X_s]$ where X_s is an uncertain payoff realised at time s, expectation is taken with respect to the pricing measure that corresponds to the chosen numéraire, and the subscript t indicates conditioning with respect to information that is available at time t. In the context of valuation, the term "expectation" when used below will always refer to expectation under the pricing measure.

3 Financial Fairness for Everyone

Before turning to the calculation of the fair contribution we will investigate whether the considered CI pension scheme is consistent in itself, in the sense that it is actually possible to treat all involved parties fairly at every point in time.

We consider a scheme at time t, after indexation has been applied and payments to generation $t-2$ have been made, but before generation t has joined. The pension fund has assets with a value of A_t and a nominal liability of $N_t^{t-1} = N_{t-1}^{t-1} I_t$ since only generation $t-1$ is still a member of the fund.

There are three stakeholders that should be treated financially fairly when new members make their contribution and are given nominal promises. They are: the new members, the existing members and the pension fund sponsor (employer/state for example). These three parties now need to negotiate how much the new generation should contribute in return for a nominal promise of N_t^t which will be adjusted according to a given indexation function g. In a CI pension scheme where all three stakeholders are treated fairly at any point in time, the new generation might not want to join if new contributions are perceived as too high, or the new generation might not be allowed to join by the sponsor or the old generation if they disagree on the new contribution C_t to be paid by the new generation.

The existing members of the fund, generation $t-1$, will argue that the value at time t of the random payment that they will receive at $t+1$ should not be reduced by the fact that generation t joins. Specifically, in the context of a conditional indexation scheme, the payment that generation $t-1$ will receive when they retire at time $t+1$ is

$$N_{t+1}^{t-1} = N_t^{t-1} g\left(\frac{A_t + C_t}{N_t^{t-1} + N_t^t}\exp(R_{t+1})\right) \tag{3}$$

which depends on the contribution C_t and the nominal benefits N_t^t. We will use the notation V_s^t to indicate the market value at time s of the pension promise held by generation t. The value of the pension promise to generation $t-1$ at time t depends on the contribution C_t made by the incoming generation at time t as well as on the nominal promise N_t^t that this generation receives; therefore we write $V_t^{t-1} = V_t^{t-1}(C_t, N_t^t)$. The older generation will only be treated fairly if any new contributions and promises do not reduce the fair value of their pension deal, that is:

$$V_t^{t-1}(C_t, N_t^t) \geq V_t^{t-1}(0,0) \tag{4}$$

where the RHS in (4) is the market value of benefits if no generation joins at time t.

The sponsor, for example the employer, will have to deal with all shortfalls in the pension scheme. The recent closure of many DB schemes has shown that the sponsor will not always agree to new members joining a pension scheme. The sponsor will consider the surplus (or deficit), denoted by S_t, of the pension fund using the same valuation operator as the other two stakeholders. The surplus in the pension fund after new members joined the fund at time t depends again on the contribution that those new members make:

$$S_t(C_t, N_t^t) = A_t + C_t - \left[V_t^{t-1}(C_t, N_t^t) + V_t^t(C_t, N_t^t)\right] \tag{5}$$

where $A_t + C_t$ is the value of the available assets at time t after generation t has joined, and $V_t^{t-1}(C_t, N_t^t) + V_t^t(C_t, N_t^t)$ is the market value (at time t) of the combined pension promises made to generations $t-1$ and t.

The sponsor will compare this surplus with the surplus of the fund without generation t joining, that is,

$$S_t(0,0) = A_t - V_t^{t-1}(0,0). \tag{6}$$

Again, we argue that financial fairness for the sponsor means that

$$S_t(C_t, N_t^t) \geq S_t(0,0) \tag{7}$$

and that the sponsor will close the fund for new members if that condition is not fulfilled. Combining equations (5), (6) and (7) and solving for $V_t^{t-1}(0,0)$ we obtain

$$V_t^{t-1}(C_t, N_t^t) + V_t^t(C_t, N_t^t) - C_t \leq V_t^{t-1}(0,0). \tag{8}$$

Finally, new members will also demand to be treated fairly or decide not to join the pension scheme. They require that their contributions do not exceed the value of the nominal promises they receive. The restriction imposed by the incoming generation is

$$C_t \leq V_t^t(C_t, N_t^t). \tag{9}$$

The constraints imposed by the three stakeholders are (4), (8), and (9). The last two inequalities imply

$$V_t^{t-1}(C_t, N_t^t) \leq V_t^{t-1}(0, 0). \tag{10}$$

Combining this with (4) leads to

$$V_t^{t-1}(C_t, N_t^t) = V_t^{t-1}(0, 0). \tag{11}$$

This equation in combination with (8) leads to the inequality

$$C_t \geq V_t^t(C_t, N_t^t) \tag{12}$$

which together with (9) implies that

$$C_t = V_t^t(C_t, N_t^t). \tag{13}$$

We find that the three inequalities (4), (8), and (9) can only be satisfied if all inequalities are in fact equalities. This happens when the two conditions (11) and (13) are both satisfied. On the one hand, the condition in equation (13) says that new members will pay a contribution which is equal to the value of the promises they receive. On the other hand, the condition in equation (11) says that any new contributions and promises should not change the value of promises made to existing members.

Given a particular choice of the policy ladder in a conditional indexation scheme, equation (13) establishes a relationship between the contribution C_t made by the incoming generation at time t and the promise N_t^t that they receive. We will show below how to compute this relation. The question remains whether it is possible to design a conditional indexation scheme in such a way that the equality (11) is satisfied. Note that equality should hold under all circumstances that might occur at time t; in our stylised model, these circumstances are specified by the current asset value and the nominal pension promise to the older generation. If either C_t or N_t^t is prescribed, then only the ladder function g is available to make (11) hold. Since this is an equality in two parameters (current asset value and promise to older generation) while the policy ladder is a function of only one variable, it is unlikely that a solution exists.

The invariance condition (11) can be given a more concrete form if we make more specific assumptions. We assume that the returns R_t over the period $[t-1, t)$ form a stochastic process defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Using the representation of the valuation operator in terms of an expectation under a pricing measure $\mathbb{Q}$ defined on $(\Omega, \mathcal{F})$, we can write the market value at time t of the pension promise to generation $t-1$ as

$$\begin{aligned} V_t^{t-1}(C_t, N_t^t) &= \mathrm{E}_t\left[N_{t+1}^{t-1} \,|\, N_t^{t-1}, A_t\right] \\ &= \mathrm{E}_t\left[N_t^{t-1} g\left(\frac{A_t + C_t}{N_t^{t-1} + N_t^t}\exp(R_{t+1})\right) \,\middle|\, N_t^{t-1}, A_t\right]. \end{aligned} \tag{14}$$

Furthermore, we assume that the indexation function g is non-decreasing and not constant, see assumption (A1) in Section 2, and that R_{t+1} can take any value in $\mathbb{R}$. We then obtain that (11) is only fulfilled, if

$$\frac{A_t + C_t}{N_t^{t-1} + N_t^t} = \frac{A_t}{N_t^{t-1}}. \tag{15}$$

Therefore, to ensure financial fairness for all three involved stakeholders, the nominal funding ratio should not change when generation t joins and makes new contributions. This holds if and only if

$$C_t = \frac{A_t}{N_t^{t-1}} N_t^t. \tag{16}$$

Financial fairness for all stakeholders therefore requires that the relationship between C_t and N_t^t which is specified implicitly in (11) actually leads to the quite simple form (16). In other words:

$$\frac{A_t}{N_t^{t-1}} N_t^t = V_t^t(C_t, N_t) = \mathrm{E}_t[I_{t+2} I_{t+1}] N_t^t.$$

Using the expressions for the asset value at time $t+1$ as given in (1) and for the indexation factor as given in (2), and also using the relation in (15) both at time t and at time $t+1$, the above can be rewritten as

$$\begin{aligned} \frac{A_t}{N_t^{t-1}} = \mathrm{E}_t\Bigg[g\Bigg(\Bigg[\left(1 + \frac{N_t^{t-1}}{N_t^t}\right)\frac{(A_t/N_t^{t-1})\exp(R_{t+1})}{g((A_t/N_t^{t-1})\exp(R_{t+1}))} - \frac{N_t^{t-1}}{N_t^t}\Bigg]\exp(R_{t+2})\Bigg) \\ \cdot\, g((A_t/N_t^{t-1})\exp(R_{t+1}))\Bigg]. \end{aligned}$$

Here it is seen explicitly that the variables A_t and N_t^{t-1} appear as independent variables. Any particular structure that would allow matching the left hand side and the right hand side for all values of these variables by a suitable choice of the scalar function g does not appear to be present.

We conclude that the relation (11) cannot be expected to be satisfied. Consequently, the joining of a new generation may either increase or decrease the market value of the claims held by the older generation. We will discuss below under which conditions either of these possibilities is realised.

4 Financial Fairness for New Members Only

Since it is not possible to treat all three stakeholders fairly at all times, we will now consider a pension scheme in which new members are always allowed to join at time t as long as this generation t pays its fair contribution C_t which is derived as a solution of $C_t = V_t^t(C_t, N_t^t)$ for a given promised benefit N_t^t and an agreed valuation operator V_t^s. We argue that such an arrangement is reasonable for both generations since the old generation $t-1$ enjoyed the same privilege when they were young and joined the scheme. Their continued participation in the scheme has been priced into the contribution that they paid.

Before we turn to the calculation of C_t under this assumption, we discuss the effect of the new generation on the other two stakeholders (existing members and sponsor). Under the assumption of market completeness, both the sponsor and the old generation are able to hedge completely against the effect of the new generation entering, since all incoming generations pay a market-consistent price for the rights they receive. The implementation of a (complete or partial) strategy may be more feasible for the sponsor than for the participants, and we focus below in particular on the consequences of the joining of the new generation for the old generation.

4.1 Effect on Sponsor and Old Generation

We consider a scheme in which new generations pay a contribution that is equal to the market value of the pension promise they receive, and in which they are always allowed to join regardless of the effect on the nominal funding ratio which determines indexation. In such a scheme, the joining of any generation t will almost surely have an effect on the financial positions of generation $t-1$ and the sponsor.

4.1.1 Increased Nominal Funding Ratio

We first discuss the situation in which the nominal funding ratio increases due to contributions made by generation t, that is,

$$\frac{A_t + C_t}{N_t^{t-1} + N_t^t} > \frac{A_t}{N_t^{t-1}}\,.$$

In this situation, the value $V_t^{t-1}(C_t, N_t^t)$ of the pension promised to generation $t-1$ will be larger than the value $V_t^{t-1}(0,0)$ of those promises without generation t. The reason is that g is a non-decreasing function, and therefore, for any realised return R_{t+1} the pension payment N_{t+1}^{t-1} will be larger than without the new generation joining.

On the other hand, the surplus in the pension fund will be reduced: $S_t(C_t, N_t^t) < S_t(0,0)$. This follows from the inequality $V_t^{t-1}(C_t, N_t) > V_t^{t-1}(0,0)$ and the equality $C_t = V_t^t(C_t, N_t)$, since

$$\begin{aligned}S_t(C_t, N_t) &= A_t + C_t - (V_t^{t-1}(C_t, N_t) + V_t^t(C_t, N_t))\\ &= A_t - V_t^{t-1}(C_t, N_t) < A_t - V_t^{t-1}(0,0) = S_t(0,0).\end{aligned}$$

This seems to be counter-intuitive, since it might be argued that any increase in the funding ratio should improve the solvency of the fund. However, in the considered pension scheme, an increased funding ratio also increases the liabilities of the fund, and therefore, the surplus is reduced. The reduction is to the detriment of the sponsor, who is the owner of the surplus.

Instead of the difference $S_t(C_t, N_t^t) - S_t(0,0) < 0$, which measures the absolute change in the surplus caused by generation t, we may also consider a funding ratio based on the market (risk-neutral) value $V_t^{t-1}(C_t, N_t^t) + V_t^t(C_t, N_t^t)$ of the total liabilities. In case the current surplus is positive ($A_t > V_t(0,0)$), we find that

$$\frac{A_t + C_t}{V_t^{t-1}(C_t, N_t^t) + V_t^t(C_t, N_t)} = \frac{A_t + C_t}{V_t^{t-1}(C_t, N_t^t) + C_t} < \frac{A_t + C_t}{V_t^{t-1}(0,0) + C_t} < \frac{A_t}{V_t^{t-1}(0,0)} \tag{17}$$

which means that the market-consistent funding ratio will be reduced.

However, if the current surplus is negative ($A_t < V_t(0,0)$), the market-consistent funding ratio is improved by the incoming new generation if the condition

$$V_t^{t-1}(C_t, N_t^t) - V_t^{t-1}(0,0) < \left(\frac{V_t^{t-1}(0,0)}{A_t} - 1 \right) C_t \tag{18}$$

is satisfied, that is, when the increase of the market value of the rights of the existing generation is relatively small compared to the contribution made by the incoming generation. In this scenario, the deficit (negative surplus) would still increase, but the market-consistent funding ratio would improve. The new funding ratio (LHS in (17)) will still be less than 100%, but it will be closer to 100% than the funding ratio without the new generation (RHS in (17)).

4.1.2 Decreased Nominal Funding Ratio

Similar arguments hold if the new generation reduces the nominal asset/liability ratio. In that case, the surplus of the fund would increase, the value of the promises made to generation $t-1$ would decrease, and the funding ratio based on market values would increase if the fund runs a deficit, but it might decrease if the fund runs a surplus depending on a condition similar to (18).

4.2 Valuation

After having investigated the effect of new generations on the financial positions of the sponsor and the old generation, we are now interested in quantifying the fair contribution required by any new generation for a fixed initial nominal promise. To this end we will first study in more detail how the value $V_t^{t-1}(C_t, N_t^t)$ of the pension contract of existing members is affected by the contribution C_t of new members and the nominal promises N_t^t made to them. In a second step we will then consider the conditional expectation of $V_t^{t-1}(C_t, N_t^t)$ at time $t-1$ to find the fair contribution required from generation $t-1$.

Using risk-neutral valuation we obtain for the value at time t of actual pension payments N_{t+1}^{t-1} made at time $t+1$:

$$V_t^{t-1}(C_t, N_t^t) = \mathrm{E}_t[N_{t+1}^{t-1}] = \mathrm{E}_t \left[N_t^{t-1} g \left(\frac{A_t + C_t}{N_t^{t-1} + N_t^t} \exp(R_{t+1}) \right) \right]. \tag{19}$$

We know that

$$V_t^{t-1}(C_t, N_t^t) = V_t^{t-1}(0,0) \iff C_t = \frac{A_t}{N_t^{t-1}} N_t^t$$

that is, the funding ratio is not changed by new contributions and new rights. We now use a Taylor expansion of $V_t^{t-1}(c,n)$ in c for a given value of $n = N_t^t$:

$$V_t^{t-1}(C_t, N_t^t) = V_t^{t-1}\left(\frac{A_t}{N_t^{t-1}} N_t^t, N_t^t\right) + \left[C_t - \frac{A_t}{N_t^{t-1}} N_t^t\right] \frac{\partial}{\partial c} V_t^{t-1}\left(c, N_t^t\right)\Big|_{c=\frac{A_t}{N_t^{t-1}} N_t^t} + O\left(\left[C_t - \frac{A_t}{N_t^{t-1}} N_t^t\right]^2\right).$$

We introduce the notation

$$h_t^{t-1}(n) = \frac{\partial}{\partial c} V_t^{t-1}\left(c, n\right)\Big|_{c=\frac{A_t}{N_t^{t-1}} n}. \tag{20}$$

The value at time $t-1$ of this pension contract is then, to first-order approximation,

$$\begin{aligned} V_{t-1}^{t-1} &= \mathrm{E}_{t-1}\left[V_t^{t-1}(C_t, N_t^t)\right] \\ &= \mathrm{E}_{t-1}\left[V_t^{t-1}\left(\frac{A_t}{N_t^{t-1}} N_t^t, N_t^t\right)\right] - \mathrm{E}_{t-1}\left[\frac{A_t}{N_t^{t-1}} N_t^t h_t^{t-1}(N_t^t)\right] + \mathrm{E}_{t-1}\left[C_t h_t^{t-1}(N_t^t)\right]. \end{aligned} \tag{21}$$

The first two terms in (21) can be calculated since they are independent of contributions made by future generations. The third term contains C_t and can therefore not be computed immediately. However, treating new members fairly at any time means that $V_{t-1}^{t-1} = C_{t-1}$. We therefore establish an approximate relationship between contributions at successive times:

$$C_{t-1} = \mathrm{E}_{t-1}\left[V_t^{t-1}\left(\frac{A_t}{N_t^{t-1}} N_t^t, N_t^t\right)\right] - \mathrm{E}_{t-1}\left[\frac{A_t}{N_t^{t-1}} N_t^t h_t^{t-1}(N_t^t)\right] + \mathrm{E}_{t-1}\left[C_t h_t^{t-1}(N_t^t)\right]. \tag{22}$$

Using this formula recursively we obtain

$$\begin{aligned} C_{t-1} &= \mathrm{E}_{t-1}\left[V_t^{t-1}\left(\frac{A_t}{N_t^{t-1}} N_t^t, N_t^t\right)\right] - \mathrm{E}_{t-1}\left[\frac{A_t}{N_t^{t-1}} N_t^t h_t^{t-1}(N_t^t)\right] \\ &\quad + \mathrm{E}_{t-1}\left[\mathrm{E}_t\left[V_{t+1}^t\left(\frac{A_{t+1}}{N_{t+1}^t} N_{t+1}^{t+1}, N_{t+1}^{t+1}\right) h_t^{t-1}(N_t^t)\right]\right] \\ &\quad - \mathrm{E}_{t-1}\left[\mathrm{E}_t\left[\frac{A_{t+1}}{N_{t+1}^t} N_{t+1}^{t+1} h_{t+1}^t(N_t^t)\right] h_t^{t-1}(N_t^t)\right] \\ &\quad + \mathrm{E}_{t-1}\left[\mathrm{E}_t\left[C_{t+1} h_{t+1}^t(N_{t+1}^{t+1})\right] h_t^{t-1}(N_t^t)\right] \\ &\ \ \vdots \\ &= \mathrm{E}_{t-1}\left[V_t^{t-1}\left(\frac{A_t}{N_t^{t-1}} N_t^t, N_t^t\right)\right] \end{aligned}$$

$$
\begin{aligned}
&+ \sum_{k=0}^{\infty} \mathrm{E}_{t-1}\left[\left\{V_{t+k+1}^{t+k}\left(\frac{A_{t+k+1}}{N_{t+k+1}^{t+k}} N_{t+k+1}^{t+k+1}, N_{t+k+1}^{t+k+1}\right)\right.\right. \\
&\qquad\qquad \left.\left. - \frac{A_{t+k}}{N_{t+k}^{t+k-1}} N_{t+k}^{t+k}\right\} \prod_{n=0}^{k} h_{t+n}^{t+n-1}(N_{t+n}^{t+n})\right] \\
&= \mathrm{E}_{t-1}\left[V_t^{t-1}\left(\frac{A_t}{N_t^{t-1}} N_t^t, N_t^t\right)\right] \\
&+ \sum_{k=0}^{\infty} \mathrm{E}_{t-1}\left[\left\{\mathrm{E}_{t+k}\left[V_{t+k+1}^{t+k}\left(\frac{A_{t+k+1}}{N_{t+k+1}^{t+k}} N_{t+k+1}^{t+k+1}, N_{t+k+1}^{t+k+1}\right)\right]\right.\right. \\
&\qquad\qquad \left.\left. - \frac{A_{t+k}}{N_{t+k}^{t+k-1}} N_{t+k}^{t+k}\right\} \prod_{n=0}^{k} h_{t+n}^{t+n-1}(N_{t+n}^{t+n})\right] \\
&= \mathrm{E}_{t-1}\left[V_t^{t-1}\left(\frac{A_t}{N_t^{t-1}} N_t^t, N_t^t\right)\right] \\
&+ \sum_{k=0}^{\infty} \mathrm{E}_{t-1}\left[\left\{\mathrm{E}_{t+k}\left[V_{t+k+1}^{t+k}(0,0)\right] - \frac{A_{t+k}}{N_{t+k}^{t+k-1}} N_{t+k}^{t+k}\right\} \prod_{n=0}^{k} h_{t+n}^{t+n-1}(N_{t+n}^{t+n})\right] \qquad (23)
\end{aligned}
$$

where we used the tower rule for conditional expectations and

$$V_{t+1}^t\left(\frac{A_{t+1}}{N_{t+1}^t} N_{t+1}^{t+1}, N_{t+1}^{t+1}\right) = V_{t+1}^t(0,0) \qquad \forall\, t.$$

For the conclusion, we introduce some notation:

$\tilde{C}_t$ is the fair contribution of the generation entering at time t assuming that the next generation will not change the nominal funding ratio at time $t+1$, i.e.

$$\tilde{C}_t = \mathrm{E}_t\left[V_{t+1}^t\left(\frac{A_{t+1}}{N_{t+1}^t} N_{t+1}^{t+1}, N_{t+1}^{t+1}\right)\right] = \mathrm{E}_t\left[V_{t+1}^t(0,0)\right]$$

$\hat{C}_t$ is the contribution of the generation entering at time t such that the nominal funding ratio is not changed at time t, i.e.

$$\hat{C}_t = \frac{A_t}{N_t^{t-1}} N_t^t.$$

We can then rewrite (23) as

$$C_{t-1} = \tilde{C}_{t-1} + \sum_{k=0}^{\infty} \mathrm{E}_{t-1}\left[\left\{\tilde{C}_{t+k} - \hat{C}_{t+k}\right\} \prod_{n=0}^{k} h_{t+n}^{t+n-1}(N_{t+n}^{t+n})\right]. \qquad (24)$$

Our conclusion is that the fair contribution at time $t-1$ given all future nominal promises consists of two parts. The first part is the fair contribution at $t-1$ assuming that the next generation will not change the nominal funding ratio. To that we add the expected value of all changes to the nominal funding ratio made by future generations. However, instead of the actual change in the funding ratio resulting from $C_{t+k}-\hat{C}_{t+k}$ which we cannot calculate without knowing future contributions, we only require $\tilde{C}_{t+k}-\hat{C}_{t+k}$ in (24) for all future generations.

Those differences, $\tilde{C}_{t+k}-\hat{C}_{t+k}$, are then multiplied with $\prod_{n=0}^{k} h_{t+n}^{t+n-1}(N_{t+n}^{t+n})$ in (24). Using (19) and the definition of h in (20) we obtain for those derivatives

$$\begin{aligned}
\frac{\partial}{\partial c} V_t^{t-1}\left(c, N_t^t\right) &= \frac{\partial}{\partial c}\mathrm{E}_t\left[N_t^{t-1} g\left(\frac{A_t+c}{N_t^{t-1}+N_t^t}\exp(R_{t+1})\right)\right] \\
&= \mathrm{E}_t\left[N_t^{t-1}\frac{\partial}{\partial c} g\left(\frac{A_t+c}{N_t^{t-1}+N_t^t}\exp(R_{t+1})\right)\right] \\
&= \mathrm{E}_t\left[N_t^{t-1} g'\left(\frac{A_t+c}{N_t^{t-1}+N_t^t}\exp(R_{t+1})\right)\frac{1}{N_t^{t-1}+N_t^t}\exp(R_{t+1})\right] \\
&= \frac{N_t^{t-1}}{N_t^{t-1}+N_t^t}\mathrm{E}_t\left[g'\left(\frac{A_t+c}{N_t^{t-1}+N_t^t}\exp(R_{t+1})\right)\exp(R_{t+1})\right] \\
&\leq \frac{N_t^{t-1}}{N_t^{t-1}+N_t^t}\max_x g'(x)\mathrm{E}_t\left[\exp(R_{t+1})\right]
\end{aligned}$$

and it follows that $h_t^{t-1}(N_t^t) < 1$ if the policy ladder g is chosen such that

$$\max_x g'(x) < \frac{N_t^{t-1}+N_t^t}{N_t^{t-1}}\frac{1}{\mathrm{E}_t\left[\exp(R_{t+1})\right]}\,. \tag{25}$$

As we are taking expectations under a pricing measure, and prices are taken relative to a numéraire so that the interest rate can be taken to be zero, we have $\mathrm{E}_t[\exp(R_{t+1})] = 1$. Consequently, the inequality (25) is satisfied under our standing assumption (A1) that $g' \leq 1$. Therefore, the product $\prod_{n=0}^{k} h_{t+n}^{t+n-1}(N_{t+n}^{t+n})$ in (24) is a decreasing function of the number of factors, which means that generations which are far in the future have less influence on the fair contribution today. This result justifies to calculate fair contributions under the assumption that there is only a finite number of future generations, since the impact of generations $t+k$ for large k can be ignored.

5 Fair Contributions with a Finite Number of Future Generations

The discussions in the previous section justify an approximation in which only a finite number of future generations are considered as long as (25) is satisfied. In this case we do not need to use the approximation in (24), but we can calculate the fair contributions numerically by using a recursive scheme starting with the last generation. To be specific, we assume throughout that $N_t^t = 1$; in other words, we work with a fixed initial promise and generation size.

Our purpose in this section is therefore to compute the fair value of contributions in the conditional indexation scheme described by (1–2), with a fixed initial pension promise, under the assumption that the scheme will be terminated at a given future time T. In essence, this is an option pricing problem: the contribution is considered as the premium to be paid by the incoming generation at time t for the contingent payoff that they will receive at time $t+2$. There are two major complications compared to standard option pricing problems: first, the premium itself has an impact on the payoff, and secondly, the premium paid by the next generation impacts the payoff as well. In order to deal with these complications, we use a computational scheme that employs two auxiliary functions:

- the expected value (under the pricing measure) of the payoff at time $t+2$, as a function of existing liabilities at time $t+1$ and the asset value at time $t+1$ *before* the contribution of the generation that comes in at time $t+1$ has been paid, under the assumption that this contribution will take its fair value (one-step-ahead expectation);
- the expected value (under the pricing measure) of the payoff at time $t+2$, as a function of existing liabilities at time t and the asset value at time t *after* the contribution that comes in at time t has been paid (two-step-ahead expectation).

With some abuse of notation, the first function will be denoted by $E[N_{t+2}^t \mid A_{t+1}, N_{t+1}^t]$ as in (19), and the second by $E[N_{t+2}^t \mid A_t + C_t, N_t^{t-1}]$. As in (13), the fair contribution at time t is determined as a function of asset values at time t, A_t, and existing liabilities at time t, N_t^{t-1}, by the equation

$$C_t = E[N_{t+2}^t \mid A_t + C_t, N_t^{t-1}]. \tag{26}$$

The two auxiliary functions mentioned above can be used to evaluate the right hand side, since

$$E[N_{t+2}^t \mid A_t + C_t, N_t^{t-1}] = E\big[E[N_{t+2}^t \mid A_{t+1}, N_{t+1}^t] \mid A_t + C_t, N_t^{t-1}\big]$$

where A_{t+1} and N_{t+1}^t are expressed in terms of A_t, C_t, and N_t^{t-1} according to the equations of the conditional indexation scheme.

Since the scheme terminates at time T, there is no new generation coming in at time $T-1$. Within the equations above, this can be expressed by $N_{T-1}^{T-1} = 0$ and $C_{T-1} = 0$. The above equations then allow to compute the fair contribution C_{T-2} as a function of A_{T-2} and N_{T-2}^{T-3}. Using this, the function C_{T-3} can be computed, and so on.

Figure 1 shows the results of computations in a particular case. We assume a quite simple policy ladder that applies no indexation if the nominal funding ratio falls below 105%, and full indexation when the funding ratio is above 140%; linear interpolation is applied between these two bounds. Thinking of generations as following each other by 25 years, we set full indexation to 35%, which corresponds to real wage growth on a 25-year horizon if annual real wage growth is equal to 1.2%. Our indexation function is then given by:

$$g(x) = \begin{cases} 1 & \text{for} \quad x < 1.05 \\ x - 0.05 & \text{for} \quad 1.05 \le x \le 1.4 \\ 1.35 & \text{for} \quad 1.4 < x \end{cases}$$

Asset returns are supposed to be lognormally distributed with 25% volatility; on a 25-year horizon, this corresponds to a quite conservative investment policy (5% annual volatility).

The fair contribution is determined largely by the nominal funding ratio, but the structure of liabilities (pension rights for different generations) also plays a role. The figure shows two curves, corresponding to the older generation having obtained no indexation at the previous time step ("low liability") or full indexation ("high liability"). The two curves are seen to cross. A high pension promise to the older generation has a depressing effect on the value of the pension payoff for the younger generation at low funding ratios; however, when the funding ratio is high in spite of a high liability to the older generation, then the level of assets must be quite high, which is good for the indexation perspectives of the younger generation. From some point on (around 125%) the second effect more than compensates for the first. Also drawn in the figure is the level of contribution that would keep the nominal funding ratio constant. It is seen that at levels up to about 160%, the fair contribution helps to improve the funding ratio; at higher levels, the effect of the fair contribution is to reduce it.

The graph shows the situation five steps before termination, because this appears already enough to represent a stationary situation. The fair contribution is computed, for a fixed value of assets A_t and a fixed level of the existing liabilities N_t^{t-1}, by the iterative procedure that is naturally suggested by equation (26). The iteration converges quickly; at a tolerance level of 10^{-5} (compared to the level of the contribution between 1 and 2), convergence typically takes place within three or four iterations steps. Computations were carried out on a grid of 150 steps in the direction of the asset value and 20 steps in the direction of the pension promise to the older generation. Total computation time, using an implementation in Matlab on a standard PC, was in the tens of seconds.

6 Intergenerational Risk Sharing and Sustainability of the Pension Scheme

We have seen in Section 4.1 how any new generation affects the financial positions of existing members and the sponsor of the CI pension scheme. We have considered the two scenarios of reduced or increased nominal funding ratios. In figure 1 we find that for low nominal funding ratios the fair contribution made by generation t exceeds the contribution $C_t = \frac{A_t}{N_t^{t-1}} N_t^t$, which does not change the nominal funding ratio. This is the case both in the low liability and in the high liability scenario in Fig. 1. It follows that low nominal funding ratios are increased by the new contributions made by generation t with the consequences discussed in section 4.1. In particular, the new contributions improve the financial position of existing members. It follows that existing members benefit after a period of low returns from the contributions made by generation t.

On the other hand we find in Fig. 1 that the fair contribution of generation t will be less

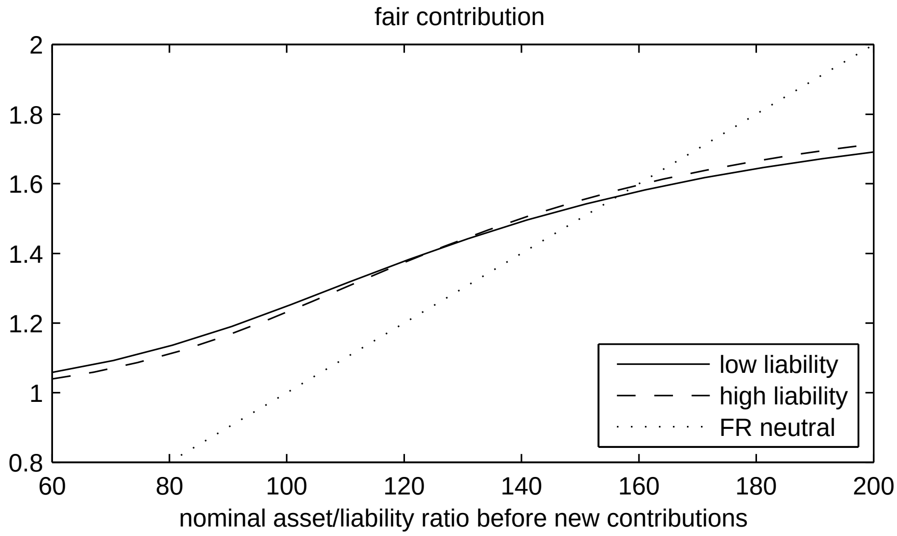

Figure 1: The plot shows the fair contribution of the generation entering at time t as a function of the funding ratio at time t, in two cases: "low liability" means that the promise made to the older generation is at level 1 (no indexation in previous time step), "high liability" means that the promise to the older generation is at level 1.35 (full indexation in previous time step). The dotted line represents contribution levels that leave the funding ratio unchanged.

than $\frac{A_t}{N_t^{t-1}} N_t^t$ if the nominal funding ratio at time t is rather high, that is, after high returns R_t have been enjoyed by the existing members and previous generations.

In that way generation t participates in the high or low returns experienced in previous periods. This risk sharing mechanism is achieved despite each generation only paying the financially fair contribution conditional on the information available at the time they join the scheme. This is in contrast to risk sharing mechanisms which rely on a social planner, and on forcing each new generation to join. We argue that in the CI pension scheme new generations will always be happy to join since they are only required to pay a fair contribution for the benefits they receive.

One could imagine a pension scheme in which new generations are always forced to join and to make a contribution which will restore the funding ratio to some specified level. Although such a pension scheme would have a more stable funding ratio, new generations would not be treated fairly and might not join given the choice. On the other hand, the CI pension scheme combines financial fairness with the risk sharing mechanisms described above. And since new contributions are such that the funding ratio is improved in a scenario with a low existing funding ratio, the funding ratio is indeed stabilised to some extent. Some further analysis from a risk management point of view on the development of the funding ratio over time would be relevant, but this is beyond the scope of the present paper.

7 Conclusions and Recommendations

Our purpose in this paper has been to investigate the consequences of imposing financial fairness on a conditional indexation scheme. We have only considered a very simple form of such a collective pension scheme. Nevertheless, we do believe that some conclusions and policy recommendations can be drawn from our findings.

We have argued that financial fairness at all times for all cohorts cannot be achieved without essentially breaking up the scheme. On the other hand, it is possible to realize financial fairness for incoming generations. This means the collective scheme requires a commitment from participating generations which guarantees that they will continue their participation, and that they will allow new generations to enter. Under some circumstances, older generations must accept that the value of their pension rights is reduced. However, in a scheme that is constructed on the basis of financial fairness for incoming generations, the fact that such circumstances may arise has already been taken into account in the contribution to be paid by these generations, and in that sense the conditional reduction of rights is still fair. Obviously this is a point that needs to be clearly communicated to plan members.

We recommend that, in the design of collective schemes which allow for adjustment of accrued rights (either by conditional indexation or by other rules), calculations be made analogous the ones that have been presented here, in order to ensure at least approximate financial fairness for incoming generations. The sustainability of the system would be at stake if this property is not fulfilled. From a technical point of view, the computations are not straightforward. In Section 5 we have outlined a method that can be used for this particular option pricing problem. It is a recursive method, which makes use of suitable auxiliary functions. Scaling up the computational procedure to determine fair contributions in a more real-life situation would, of course, still require a substantial amount of work. It is encouraging to observe in the example that the fair contribution level is determined largely by the current nominal funding ratio and depends only to a minor extent on the structure of existing liabilities, which in a multi-generation framework can be a complicated object.

Our conclusion is that, in conditional indexation schemes which aim for financial fairness at entry, the contribution to be paid by incoming generations should be reduced when the funding ratio is low. This is fair, since a low funding ratio means that indexation is less likely. We have kept the nominal rights constant for simplicity, but obviously an alternative way to achieve financial fairness would be that incoming generations are granted higher nominal rights in case of a low funding ratio, and lower rights in case of a high funding ratio.

The recommendation to reduce contributions for incoming generations in case of low funding ratios, and to increase contributions in case of high funding ratios, may raise the question whether the funding ratio would not be destabilized in this way. In calculations that we have made in a specific case, however (see Fig. 1), it is shown that such an effect need not occur. At low funding ratios, the reduction of the contribution is limited to such an extent that the joining of the new generation still helps to increase the funding ratio. Conversely, when the funding ratio is sufficiently high, it will be reduced by the arrival of the new generation. Of course, the result depends on the specific assumptions we made in the example, and calculations need to be done anew in other cases. More detailed information about the variability of the funding ratio can be obtained from ALM studies.

In our analysis, we have focused on the effects of financial fairness in the context of a given conditional indexation scheme. Clearly, these effects may depend on the conditional indexation rule that is used as well as on other parameters of the collective scheme, in particular the investment policy. An assumption that the indexation function is not too steep has been used above in providing mathematical support to the intuitive notion that generations in the very distant future should be less important for setting the current level of contributions; see (25) and assumption (A1). It may be surmised that sufficient flatness of the indexation function is also important in relation to the stability of the funding ratio. If this is confirmed in further studies, we obtain in this way a policy recommendation regarding indexation functions. Similarly, policy recommendations regarding the investment policy can be obtained by studying the way in which a given policy influences the impact of the imposition of financial fairness.

In this paper we have assumed the investment mix to be fixed in time, both for simplicity and because this is in line with a practice that is often followed. However, one might envisage that the investment policy could be adapted for instance to the current level of the funding ratio. A situation in which management has complete freedom to change the portfolio at any time has been discussed by Kleinow [2011] for a related pension contract. For an analysis along the lines of the present paper, it would be necessary to assume that the composition of the investment portfolio is determined according to a specified rule, rather than being at the discretion of the trustees. Financial fairness can then in principle again be determined by solving an option pricing problem, although the complexity of this problem would be increased.

The analysis of this paper, while applied in a highly idealized situation, indicates that collective schemes that implement risk sharing through adjustment of accrued rights can be designed in such a way as to ensure financial fairness for incoming generations. Our findings suggest that such schemes offer a route to effective risk sharing between generations driven by self interest only, rather than by enforcement.

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