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Reconfigurable quantum-optical circuits in a complex medium

Suraj Goel[a], Saroch Leedumrongwatthanakun[a], Natalia Herrera Valencia[a], Will McCutcheon[a],
Armin Tavakoli[b], Claudio Conti[c], Pepijn W. H. Pinkse[d], and Mehul Malik[a]

[a]Institute of Photonics and Quantum Sciences, Heriot-Watt University, Edinburgh, UK
[b]Physics Department, Lund University, Box 118, 22100 Lund, Sweden
[c]Department of Physics, University Sapienza, Piazzale Aldo Moro 2, 00185 Rome, Italy
[d] MESA+ Institute for Nanotechnology, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands

ABSTRACT

Programmable optical circuits form a key part of quantum technologies today. As the size of such circuits is increased, maintaining precise control over every individual component becomes challenging. Here we show how embedding an optical circuit in the higher-dimensional space of a large mode-mixer allows us to forgo control over individual elements, while retaining a high degree of programmability over the circuit. Using this approach, we implement high-dimensional linear optical circuits within a commercial multi-mode fibre placed between controllable phase planes. We employ these circuits to manipulate high-dimensional entanglement in up to 7 dimensions, demonstrating their application as fully programmable quantum gates. Furthermore, we show how these circuits turn the multi-mode fibre itself into a generalised multi-outcome measurement device, allowing us to both transport and certify entanglement. Finally, we show how a high circuit fidelity can be achieved with a low circuit depth by harnessing the resource of a high-dimensional mode-mixer. Our work serves as an alternative yet powerful approach for realising precise control over high-dimensional quantum states of light.

Keywords: complex media, optical circuits, quantum gates, high-dimensional entanglement, quantum optics, inverse design

1. INTRODUCTION

High-dimensional quantum states of light have seen an explosion of interest in recent years for enhancing quantum technologies. While techniques for the generation and measurement of such states have been the focus of intense research,[1,2] methods for their precise manipulation are still severely lacking. The bottleneck in this regard stems from the lack of reconfigurable devices for performing general high-dimensional unitary operations. In such a landscape, complex scattering media that constitute a closed system, such as multi-mode fibers (MMFs), are natural contenders for the transport and manipulation of high-dimensional states of light.[3] Conventionally scalable and re-programmable circuits that are constructed from a sophisticated mesh of interferometers in bulk or with integrated optics, herein referred to as “bottom-up” design,[4] suffer from imperfections in fabrication and alignment.

As an alternative, here we propose a “top-down” approach where a d-dimensional circuit is embedded within large, complex mode-mixers as shown in Fig.1a. The mode-mixers are placed between reprogrammable phase planes, which are programmed using an inverse-design approach.[5] This allows us to encode any high-dimensional unitary by harnessing the large dimensionality of the mode-mixers as an additional resource. In this work, we implement this architecture by placing a multi-mode fiber (MMF) between a pair of spatial light modulators (SLMs) to program a general unitary transformation. We then use inverse design techniques to encode a desired optical circuit inside the MMF. We put this programmable circuit to use as quantum gates for manipulating high-dimensional entangled states of light and characterize them using ancilla-assisted quantum process tomography

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Further author information: (Send correspondence to S.G. and M.M.)
S.G. gs74@hw.ac.uk,
M.M.: m.malik@hw.ac.uk

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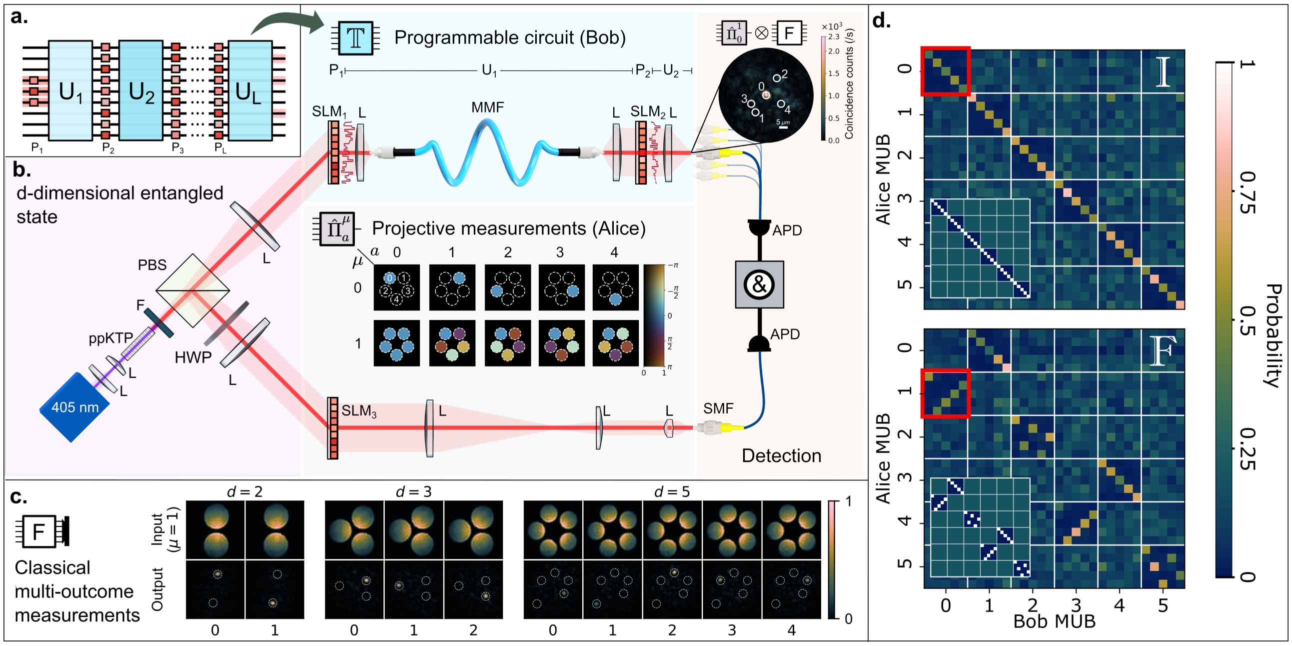

Figure 1. (a) Schematic of the top-down design where large mode mixers are separated by reconfigurable phase-planes. (b) Experimental apparatus for manipulation of high-dimensional entangled states using a two-plane programmable circuit using a multi-mode fiber (MMF) and two spatial light modulators (SLMs). (c) CCD images of multi-outcome measurements performed in the Fourier macro-pixel basis with classical light. (d) Normalized two-photon coincidence counts in all mutually unbiased bases for the characterization of identity and Fourier gates.

(AA-QPT). Further, we use some of these gates to perform measurements in mutually unbiased bases (MUBs) allowing us to certify high-dimensional entanglement despite the presence of systematic errors. We finally assess the programmability and scalability of the top-down approach by performing numerical simulations.

2. INVERSE DESIGN FOR MANIPULATING ENTANGLEMENT

We use the top-down approach to describe the linear optical circuit as a transformation built from a cascade of optical mode mixers $\{U_j\}$ separated by phase shifters $\{P_j\}$. In this approach, we embed a d-dimensional circuit $\mathbb{T}$ within these large complex mode-mixers U_j with dimension $n \gg d$.[5] A programmable circuit with depth L (the number of layers) is represented as:

$$\mathbf{T} \approx \prod_{j=1}^{L \leqslant \mathcal{O}(\mathbf{d})} U_j P_j, \tag{1}$$

Experimentally, we implement this architecture by placing a multi-mode fibre (MMF) between a pair of spatial light modulators (SLMs) to program general unitary circuits as shown in Fig 1b. After recovering U_1 by employing multi-plane neural networks (MPNN),[6] we perform the wavefront matching (WFM) algorithm ,[7] which is an inverse design technique to encode a desired circuit $\mathbb{T}$ in our experiment. We implement a variety of different target circuits including the identity-$\mathbb{I}$, high-dimensional analogs of Pauli-$\mathbb{Z}$ and $\mathbb{X}$, Fourier-$\mathbb{F}$, and random unitaries-$\mathbb{R}$ in dimensions $d = \{2, 3, 5, 7\}$ manipulating states in the macro-pixel and the orbital-angular-momentum basis.

We put this programmable circuit to use for manipulating high-dimensional entangled states of light produced using spontaneous parametric down-conversion (SPDC) shared between Alice and Bob, as shown in Fig. 1b. While Bob's photon is locally transformed using the transformation $\mathbb{T}$, Alice's photon is measured using sequential single-outcome projective measurements. We characterize the programmed circuits using ancilla-assisted

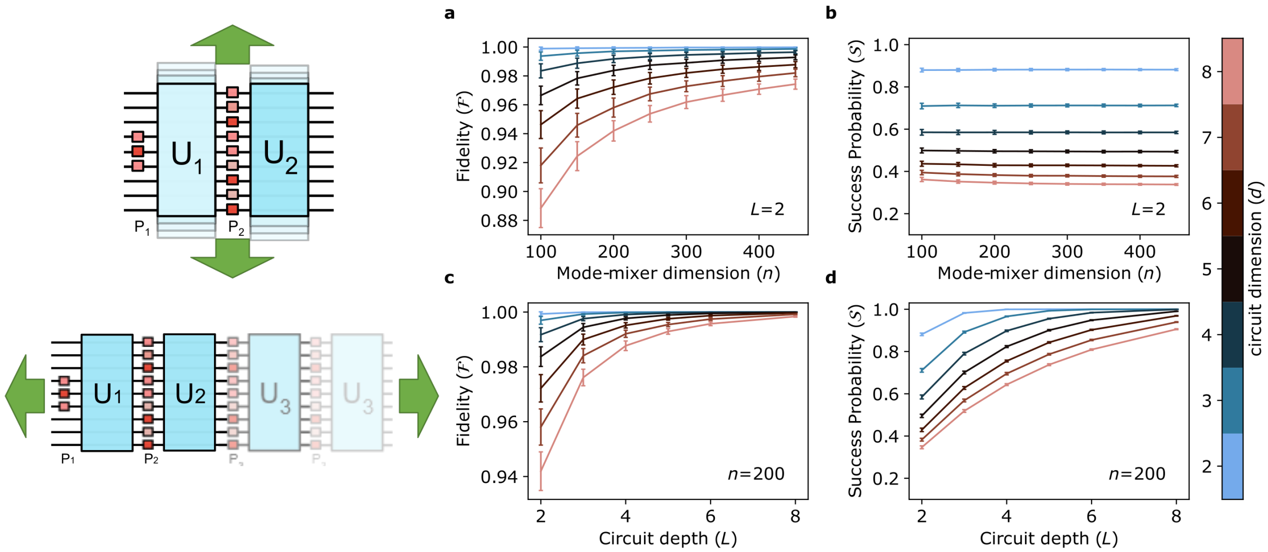

Figure 2. (a) Fidelity and (b) success probability of a d-dimensional quantum optical circuit as a function of the dimension of mode mixers (n) for a circuit depth of $L = 2$, and (c) and (d) as a function of L for $n = 200$

quantum process tomography (AA-QPT) which involves the characterization of both the initial and final state using quantum state tomography (QST) to recover the Choi state of the process. The fidelity of the processes, quantified through the fidelity of the experimental Choi states to the ideal Choi states, is reported in Table 1 for all dimensions in the macro-pixel basis.

The programmable circuit allows us to perform generalized basis transformations to spatially separated localized pixels, enabling multi-outcome measurements for any given input as shown in Fig. 1c. These multi-outcome measurements can be used for efficiently certifying high-dimensional entanglement using measurements in mutually unbiased bases (MUBs). Fig. 1d shows examples of normalized two-photon coincidence count data in all MUBs for five-dimensional $\mathbb{I}$ and $\mathbb{F}$ gates programmed for the macro-pixel basis. We use measurements in two MUBs (red boxes in Fig. 1d) and employ a computational approach using semidefinite programming to show that our data can only be reproduced by a quantum model that relies on five-dimensional entanglement.

3. PROGRAMMABILITY AND SCALABILITY

While we have successfully demonstrated the ability to perform various optical circuits in multiple spatial-mode bases in two, three, five and seven dimensions, it is imperative to question the limits of this approach. We address these limits by numerically studying the programmability and scalability of our approach. We simulate circuits based on Eq.1 and vary the three principle design parameters - the dimension of the mode mixers n, the dimension of the target gate d and the depth of the circuit L.

Table 1. Quantum process fidelities of inverse-designed experimental gates to the ideal gates in the macro-pixel basis

Gate	$d = 2$	$d = 3$	$d = 5$	$d = 7$
$\mathbb{I}$	$96.7 \pm 0.9\%$	$97.4 \pm 0.7\%$	$88.0 \pm 0.7\%$	$71.9 \pm 1.1\%$
$\mathbb{Z}$	$97.7 \pm 0.9\%$	$96.1 \pm 0.5\%$	$80.6 \pm 1.0\%$	$65.2 \pm 1.0\%$
$\mathbb{X}$	$97.6 \pm 0.8\%$	$95.0 \pm 0.7\%$	$79.2 \pm 1.0\%$	$60.1 \pm 1.0\%$
$\mathbb{F}$	$95.7 \pm 0.9\%$	$89.3 \pm 0.8\%$	$76.9 \pm 1.1\%$	$58.9 \pm 0.7\%$
$\mathbb{R}$	$96.8 \pm 0.7\%$	$91.8 \pm 0.8\%$	$80.1 \pm 1.1\%$	$63.5 \pm 0.7\%$

*Errors are reported to one standard deviation

Fig. 2a-d shows the fidelity $\mathcal{F}$ and success probability $\mathcal{S}$ of these simulated gates plotted against various parameters. At first, we keep the number of layers constant at $L = 2$ and increase the mode mixer size n for various dimensions of circuits d. We observe that by merely increasing the size of the mode mixers one can achieve near unity fideleties for all circuit dimensions. The success probability in this regime where $d/n < 0.1$ stays approximately the same for all sizes of mode mixers. This implies that we can improve our experimental results by simply using a larger mode mixer without increasing the amount of losses in our system.

Next, increase the number of layers L and notice that both success probability and fidelity drastically improve. This implies that full scalability can be achieved by adding more layers in the design. We notice that the success probability converges to unity when the number of layers approach twice the dimensionality of the circuit $L \approx 2d$. We also observe that in the regime where $d/n < 0.1$, the fidelity converges to unity when the total number of reconfigurable elements nL exceeds the number of parameters needed to define a d-dimensional unitary transform, thereby showing a high level of programmability with our design.

4. CONCLUSION

To summarize, we have introduced a new top-down approach to design programmable optical circuits in the transverse spatial domain using scattering media. We used these circuits as high-dimensional quantum gates to manipulate and measure entangled states of light. We characterised these circuits using process tomography with high-fideleties in up-to seven dimensions in multiple bases. We further used the identity and the Fourier gates to certify entanglement five dimensions. Finally, we demonstrate the programmability and scalability of our approach with the aid of numerical simulations and observe that high-fidelity circuits can be achieved with relatively low circuit depth by employing a large dimensional mode-mixer. Our work paves a way for practical implementations of programmable optical circuits with clear applications in next-generation quantum communication and computing technologies.

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