# Pseudomodes as an effective description of memory: Non-Markovian dynamics of two-state systems in structured reservoirs

# Pseudomodes as an effective description of memory: Non-Markovian dynamics of two-state systems in structured reservoirs

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We investigate the non-Markovian dynamics of two-state systems in structured reservoirs. We establish a connection between two theoretical quantum approaches, the pseudomodes [B. M. Garraway, Phys. Rev. A **55**, 2290 (1997)] and the recently developed non-Markovian quantum jump method [J. Piilo *et al.*, Phys. Rev. Lett. **100**, 180402 (2008)]. This connection provides a clear physical picture of how the structured reservoir affects the system dynamics, suggesting the role of the pseudomodes as effective description of environmental memory.

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## I. INTRODUCTION

Recent advances in experimental techniques for coherent control of small quantum systems have paved the way to a series of spectacular experiments aimed at both testing fundamental features of quantum theory and implementing logic gates for quantum information processing [1,2]. Quantum properties, however, are very fragile. Any interaction between quantum systems and their surroundings gives rise to decoherence and dissipation phenomena, destroying the quantumness of the state of the system. For this reason during the last decade several theoretical and experimental studies have been devoted to the investigation of the dynamics of open quantum systems [3–5].

One of the approaches to the description of open quantum systems consists in separating the total system into two parts, the quantum system of interest and the surrounding environment. The environment is often modeled as an infinite collection of quantum harmonic oscillators in thermal equilibrium [3]. This type of reservoir describes, e.g., the quantized electromagnetic field. One of the key quantities characterizing the reservoir, and therefore determining the open system dynamics, is the spectral distribution or structure function. This quantity describes the frequency-dependent coupling between the system and the continuum of harmonic oscillators forming the environment.

In many physical situations the system-reservoir coupling strength does not depend strongly on the frequency of the reservoir oscillators. In this case the reservoir spectral density can be conveniently approximated by a flat spectrum. One typically refers to such systems as Markovian open quantum systems. A typical feature of the dynamics of Markovian open quantum system is the irreversible flow of energy and/or information from the system to the environment.

In certain physical contexts, however, the quantum system of interest interacts with "structured" reservoirs, whose spectral density strongly varies with frequency. We refer to such systems as non-Markovian open quantum systems. Non-Markovian dynamics is characterized by the existence of a memory time scale during which some energy/information that has been transferred from the system to the environment feeds back into the system.

Many solid-state systems such as Josephson junctions, display strongly non-Markovian dynamics [5]. Moreover, often the e.m. field surrounding a quantum system, e.g., an atom, can be conveniently engineered to prevent or inhibit the occurrence of decoherent processes such as atomic spontaneous emission. Atoms placed inside cavities or photonic band-gap materials are examples of quantum systems interacting with such engineered structured reservoirs [6]. The formalism we develop here for a two-level atom can be applied to any two-level system in bosonic structured reservoir, e.g., N-$V$ vacancy in diamond embedded in photonic band gap [7].

The theoretical description of the dynamics of non-Markovian quantum system is usually very complicated, only a few simple systems are amenable to an exact solution [3]. A number of methods have been formulated for treating non-Markovian dynamics [8–10] but the connection between these methods has so far remained unexplored. Most importantly, due to the mathematical difficulties in dealing with non-Markovian systems, a simple intuitive physical picture of the memory of a non-Markovian reservoir and of how such memory allows to partly restore some of the coherence lost to the environment is highly desirable. Such a simple effective description is our main result. By connecting two non-Markovian approaches, the pseudomodes [8] and the non-Markovian quantum jumps (NMQJ) methods [9], we are able to identify, for simple exemplary cases, where the memory of a non-Markovian reservoir resides.

Our results provide insight into the problem of the existence of pure state trajectories and the physical meaning of the master equation unraveling for non-Markovian systems. This issue has been recently considered in Refs. [11,12]. Both the pseudomodes and the NMQJ approaches are consistent with the interpretation given in Ref. [12].

## II. PSEUDOMODE AND NMQJ METHODS

### A. Time-local master equation

Consider a two-level atom interacting with a structured electromagnetic reservoir in the vacuum state. The Hamiltonian of such a system in the rotating wave approximation is

 

$$H = \hbar\omega_0\sigma_+\sigma_- + \sum_\lambda \hbar\omega_\lambda a_\lambda^\dagger a_\lambda + \sum_\lambda (\hbar g_\lambda^* a_\lambda \sigma_+ + \text{H.c.}), \quad (1)$$

where $\sigma_\pm$ are the Pauli raising and lowering operators for the two-level system, $\omega_0$ is the atomic transition frequency, $a_\lambda$ and $a_\lambda^\dagger$ are the annihilation and creation operators for the mode $\lambda$ of the field having frequency $\omega_\lambda$ and coupling constant $g_\lambda$. We assume that initially only one excitation is present in the system. The state of the total system at time $t$ takes the form

$$|\psi(t)\rangle = c_0|g,0_\lambda\rangle + c_1(t)|e,0_\lambda\rangle + \sum_\lambda c_\lambda(t)|g,1_\lambda\rangle, \quad (2)$$

with $|g,0_\lambda\rangle$, $|e,0_\lambda\rangle$, and $|g,1_\lambda\rangle$ the states containing zero excitations, one atomic excitation and one excitation in the $\lambda$ mode of the e.m. field, respectively.

The problem of a two-level system interacting with a zero-temperature reservoir is in principle exactly solvable using Laplace transforms. The exact non-Markovian master equation describing the dynamics of the atomic system takes the form [3]

$$\frac{d\rho_A}{dt} = \frac{S(t)}{2i}[\sigma_+\sigma_-,\rho_A] + \gamma(t)\left[\sigma_-\rho_A\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-,\rho_A\}\right], \quad (3)$$

where $\rho_A$ is the atomic density operator. The time-dependent Lamb shift $S(t)$ and the time dependent decay rate $\gamma(t)$ are given by

$$S(t) = -2\,\text{Im}\left\{\frac{\dot{c}_1(t)}{c_1(t)}\right\}, \quad \gamma(t) = -2\,\text{Re}\left\{\frac{\dot{c}_1(t)}{c_1(t)}\right\}. \quad (4)$$

The master equation (3) can be simulated by means of the non-Markovian quantum jump method [9] which extends the Monte Carlo wave-function approach [13] to non-Markovian systems with negative decay rates.

### B. Pseudomode method

Alternatively, one can investigate the dynamics using the pseudomode theory [8,14]. This method relies on the strong connection between the atom dynamics and the shape of the reservoir spectral distribution. More precisely, the key quantities influencing the time evolution of the atom are the poles of the spectral distribution in the lower half complex $\omega_\lambda$ plane. By introducing some auxiliary variables, called pseudomodes, defined in terms of the position and of the residue of the poles of the spectral distribution, one can derive a Markovian master equation in the Lindblad form for the extended system comprising the atom and the pseudomodes. This exact master equation describes the coherent interaction between the atom and the pseudomodes in presence of decay of the pseudomodes due to the interaction with a Markovian reservoir.

For a Lorentzian spectral distribution the pseudomode approach leads to the following master equation

$$\frac{d\rho}{dt} = -i[H_0,\rho] - \frac{\Gamma}{2}[a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a], \quad (5)$$

where

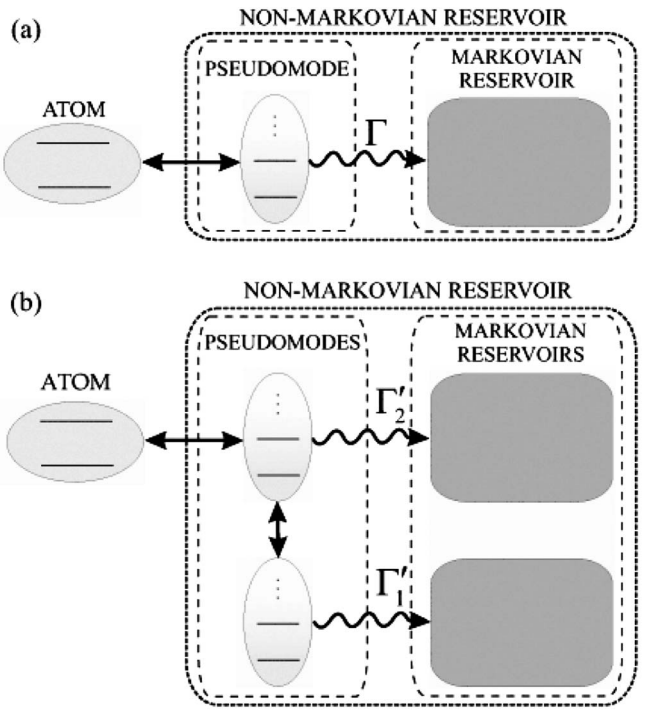


FIG. 1. Diagrammatic representation of the atom-pseudomode dynamics. (a) Atom interacting with a Lorentzian structured reservoir: the atom interacts with a single pseudomode which leaks into a Markovian reservoir. (b) Atom in a "simple" photonic band gap: we see a more complex memory architecture, the second pseudomode acts as a memory for the atom, while the first pseudomode acts as a memory for the first one.

$$H_0 = \omega_0\sigma_+\sigma_- + \omega_c a^\dagger a + \Omega_0[a^\dagger\sigma_- + a\sigma_+] \quad (6)$$

and $\rho$ is the density operator for the atom and the pseudomode. Since a Lorentzian function has only one pole in the lower half complex plane, the atom interacts with one pseudomode only as displayed in Fig. 1(a). The constants $\omega_c$ and $\Gamma$ are, respectively, the oscillation frequency and the decay rate of the pseudomode and they depend on the position of the pole $z_1 \equiv \omega_c - i\Gamma/2$ while $\Omega_0$ is the pseudomode coupling constant.

Both master equations (3) and (5) are exact. Hence we expect to obtain an equation of motion for the atom of the form of Eq. (3) by tracing out the pseudomode in Eq. (5). Additionally it is interesting to see the expressions of the coefficients $\gamma(t)$ and $S(t)$ as functions of the pseudomode amplitude. Indeed the equation obtained from Eq. (5) has the form

$$\frac{d\rho_A}{dt} = \frac{A(t)}{2i}[\sigma_+\sigma_-,\rho_A] + B(t)\left[\sigma_-\rho_A\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-,\rho_A\}\right], \quad (7)$$

where

$$A(t) = 2\left[\omega_0 + \Omega_0\frac{\text{Re}\{c_1(t)b_1^*(t)\}}{|c_1(t)|^2}\right], \quad (8)$$

and

$$B(t) = 2\Omega_0 \frac{\mathrm{Im}\{c_1(t)b_1^*(t)\}}{|c_1(t)|^2}, \tag{9}$$

where $b_1(t)$ is the pseudomode amplitude. By using the differential equations governing the atom-pseudomode dynamics [8]

$$i\frac{d}{dt}c_1 = \omega_0 c_1 + \Omega_0 b_1, \quad i\frac{d}{dt}b_1 = z_1 b_1 + \Omega_0 c_1, \tag{10}$$

it is easy to prove that $A(t)=S(t)$ and $B(t)=\gamma(t)$.

### C. Connection between pseudomode and NMQJ approaches

Once we have proven the equivalence between the two master equations, we focus on the simple case of a Lorentzian spectral distribution off resonant with the atomic transition frequency (damped Jaynes-Cummings model with detuning). In the strong-coupling regime $\Gamma \ll \Omega_0$ the atomic dynamics is strongly non-Markovian. The excited-state population of the atom oscillates in time indicating that the energy dissipated into the environment flows back into the system as a consequence of the reservoir memory. At the same time the atomic decay rate $\gamma(t)$ attains negative values. The key role of the pseudomode is exposed when we look at the time derivative of the pseudomode population and, using the differential equations in Eq. (10), we obtain

$$\frac{d|b_1(t)|^2}{dt} + \Gamma|b_1(t)|^2 = \gamma(t)|c_1(t)|^2. \tag{11}$$

The equation above shows that the *compensated rate of change* of the pseudomode population, given by the left-hand side of Eq. (11) (where the effect of the pseudomode leakage is removed), is directly related to the atomic decay rate. This means that the information about the dissipative dynamics of the atom into the structured reservoir is all contained in the pseudomode dynamics. Equation (11) and its physical interpretation is one of the main results of the paper.

We conclude this section analyzing the connection between the NMQJ unraveling of the master equation in Eq. (3) and the Monte-Carlo wave function unraveling of the pseudomode master equation in Eqs. (5) and (21). In the NMQJ description the ensemble members living in the Hilbert space of the system are always in a pure state. In particular the density matrix of the ensemble can be written as

$$\rho_A(t) = \frac{N_0(t)}{N}|\psi_0(t)\rangle\langle\psi_0(t)| + \frac{N_1(t)}{N}|\psi_1\rangle\langle\psi_1|, \tag{12}$$

where

$$|\psi_0(t)\rangle = C_g(t)|g\rangle + C_e(t)|e\rangle, \tag{13}$$

$$|\psi_1\rangle = |g\rangle, \tag{14}$$

$N$ is the total number of ensemble members, $N_1(t)$ is the number of ensemble members who have jumped into the ground state, and $N_0(t)$ is the number of members who have not jumped, or which have gone through a jump-reverse-jump cycle due to the negative decay rate [9].

In the pseudomode description the unraveling of the master equation (5) is in the extended Hilbert space containing the pseudomode. The density matrix of such ensemble, expressed in the atom-pseudomode basis, is the following:

$$\rho(t) = \frac{N_0^P(t)}{N^P}|\psi_0^P(t)\rangle\langle\psi_0^P(t)| + \frac{N_1^P(t)}{N^P}|\psi_1^P\rangle\langle\psi_1^P|, \tag{15}$$

where

$$|\psi_0^P(t)\rangle = C_{g0}^P(t)|g,0\rangle + C_{g1}^P(t)|g,1\rangle + C_{e0}^P(t)|e,0\rangle, \tag{16}$$

$$|\psi_1^P\rangle = |g,0\rangle, \tag{17}$$

$N^P$ is the total number of ensemble members, $N_1^P(t)$ is the number of ensemble members who have decayed into the atom-pseudomode ground state via a pseudomode jump, and $N_0^P(t)$ is the number of members who have not jumped. If we want to look at the time evolution of the ensemble members in the atomic Hilbert space only, we have to trace out the pseudomode auxiliary degree of freedom. This leads to the following reduced atomic density matrix:

$$\begin{aligned}\rho_A(t) = {} & \frac{N_0^P(t)}{N^P}[(|C_{g0}^P(t)|^2 + |C_{g1}^P(t)|^2)|g\rangle\langle g| + |C_{e0}^P(t)|^2|e\rangle\langle e| \\ & + C_{g0}^{P*}(t)C_{e0}^P(t)|e\rangle\langle g| + C_{g0}^P(t)C_{e0}^{P*}(t)|g\rangle\langle e|] \\ & + \frac{N_1^P(t)}{N^P}|g\rangle\langle g|\end{aligned} \tag{18}$$

in which the ensemble members are clearly in a mixed state. A comparison between Eqs. (12) and (18) illustrates the connection between the two unravelings. In particular it is illustrative to consider the ground-state population,

$$\begin{aligned}\langle g|\rho_A(t)|g\rangle &= \frac{N_1(t)}{N} + \frac{N_0(t)}{N}|C_g(t)|^2 \\ &= \frac{N_1^P(t)}{N^P} + \frac{N_0^P(t)}{N^P}(|C_{g0}^P(t)|^2 + |C_{g1}^P(t)|^2),\end{aligned} \tag{19}$$

further showing the unravelings connection.

### D. Pseudomode as effective memory

The interpretation of Eq. (11) is particularly interesting in the light of the NMQJ method, where negative decay rates lead to reversed quantum jumps. Consider, for example, an atom initially prepared in a generic superposition of ground and excited state performing a quantum jump to its ground state at a certain time $t'$. If the decay rate $\gamma(t)$ becomes negative at $t>t'$, the superpositions destroyed by the earlier normal jump can be restored by a reversed jump. In fact a reversed jump takes the atom to the state into which it would have evolved if the previous quantum jump had not occurred. Thus in the NMQJ framework one can characterize the period of negativity of the decay rate as the period of time in which memory effects and restoration of quantum superpositions occur through reversed quantum jumps. Stated another way, the reverse jumps describe the process through which the system recovers part of the information that leaked into the environment. This is confirmed by the dynamics of the von Neumann entropy of the atom which

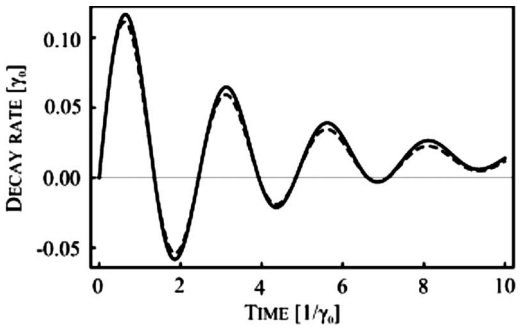


FIG. 2. The solid line is the decay rate for a two-level atom in a Lorentzian structured reservoir. The dashed line is the compensated rate of change of the pseudomode population $d|b_1(t)|^2/dt + \Gamma|b_1(t)|^2$. The units for the rates on the vertical and horizontal axis are $\gamma_0$ and $1/\gamma_0$ with $\gamma_0 = 4\Omega_0^2/\Gamma$ Markovian decay rate of the atom. We have taken the values $\Gamma = 0.6\gamma_0$, $\Omega_0 = \sqrt{0.15}\gamma_0$, and $\omega_c - \omega_0 = 4\Gamma$.

shows an oscillatory behavior following the oscillations of the atomic decay rate $\gamma(t)$. In particular the atomic von Neumann entropy decreases when the decay rate is negative indicating a temporary reduction in the mixedness of the atomic state and a recovery of coherence.

Equation (11) states that if the decay rate is negative then the compensated rate of change of the pseudomode population is negative as well, as clearly shown by Fig. 2. So whenever the atom increases its excited-state population the pseudomode must deplete. This equation, therefore, establishes a link between the restoration of coherence, typical of a reversed jump, and the pseudomode depletion.

This observation suggests an interpretation of the pseudomode as that part of the reservoir from which the atom receives back information and probability due to memory effects. In other words the pseudomode can be seen as an effective description of the reservoir memory. This is further shown by the dynamics of the mutual information [2,15] between atom and pseudomode which perfectly follows the oscillation of the pseudomode population. However the pseudomode is not a perfect storage place since the efficiency of the information restoration from the pseudomode to the atom depends on the pseudomode loss rate $\Gamma$.

## III. MEMORY STRUCTURE AND PSEUDOMODE ARCHITECTURE

We now generalize our study to a more complicated spectral distribution, namely, the inverted Lorentzian model describing in a simplified way photonic band gaps [14]

$$D(\omega) = \frac{W_1\Gamma_1}{(\omega-\omega_c)^2 + (\Gamma_1/2)^2} - \frac{W_2\Gamma_2}{(\omega-\omega_c)^2 + (\Gamma_2/2)^2}. \quad (20)$$

The negative Lorentzian introduces a dip into the density of states leading to the inhibition of atomic spontaneous emission in the region of the dip. In particular, for $W_1/\Gamma_1 = W_2/\Gamma_2$, the spectral distribution in Eq. (20) presents a perfect gap, $D(\omega_c) = 0$. The exact pseudomode master equation is given by [8]

$$\frac{d\rho}{dt} = -i[H_0,\rho] - \frac{\Gamma_1'}{2}[a_1^\dagger a_1\rho - 2a_1\rho a_1^\dagger + \rho a_1^\dagger a_1] - \frac{\Gamma_2'}{2}[a_2^\dagger a_2\rho - 2a_2\rho a_2^\dagger + \rho a_2^\dagger a_2], \quad (21)$$

where

$$H_0 = \omega_0\sigma_+\sigma_- + \omega_c a_1^\dagger a_1 + \omega_c a_2^\dagger a_2 + \Omega_0[a_2^\dagger\sigma_- + a_2\sigma_+] + V(a_1^\dagger a_2 + a_1 a_2^\dagger), \quad (22)$$

where $a_1$ and $a_2$ are the annihilation operators of the two pseudomodes decaying with decay rates $\Gamma_1' = W_1\Gamma_2 - W_2\Gamma_1$ and $\Gamma_2' = W_1\Gamma_1 - W_2\Gamma_2$, respectively. The two pseudomodes are coupled and $V = \sqrt{W_1 W_2}(\Gamma_1 - \Gamma_2)/2$ is the strength of the coupling. Figure 1(b) shows the atom-pseudomodes architecture in this case. The atom interacts coherently with the second pseudomode, which is in turn coupled to the first one. Both pseudomodes are leaking into independent Markovian reservoirs. In the case of a perfect gap only the second pseudomode leaks. The set of ordinary differential equations associated to the master equation (21) is

$$i\frac{d}{dt}c_1 = \omega_0 c_1 + \Omega_0 a_2,$$

$$i\frac{d}{dt}a_1 = z_1' a_1 + V a_2,$$

$$i\frac{d}{dt}a_2 = z_2' a_2 + V a_1 + \Omega_0 c_1, \quad (23)$$

where $c_1$, $a_1$, and $a_2$ are the complex amplitudes for the states with one excitation in the atom, one excitation in the first pseudomode, and one excitation in the second pseudomode, respectively. The position of the true poles is $z_1' = \omega_c - i\Gamma_1'/2$ and $z_2' = \omega_c - i\Gamma_2'/2$.

Similarly to the calculations for the Lorentzian case one can show that, after tracing out the two pseudomodes in Eq. (21), and with the help of Eqs. (23), one obtains the non-Markovian master equation (7) for the atom, where $A(t)$ and $B(t)$ are given by the same expressions in Eq. (8) and (9) provided that we replace $b_1(t)$ with $a_2(t)$. The non-Markovian dynamics of the atom is linked to the coherent variation of both pseudomodes by the following equation:

$$\frac{d|a_1(t)|^2}{dt} + \Gamma_1'|a_1(t)|^2 + \frac{d|a_2(t)|^2}{dt} + \Gamma_2'|a_2(t)|^2 = \gamma(t)|c_1(t)|^2. \quad (24)$$

This equation generalizes Eq. (11) to the more complex reservoir structure considered here. Moreover, using Eqs. (23) we obtain a relation connecting the dynamics of the first and second pseudomodes,

$$\frac{d|a_1(t)|^2}{dt} + \Gamma_1'|a_1(t)|^2 = 2V\frac{\mathrm{Im}\{a_2(t)a_1^*(t)\}}{|a_2(t)|^2}|a_2(t)|^2. \quad (25)$$

Having in mind that $\gamma(t) = 2\Omega_0\,\mathrm{Im}\{c_1(t)b_1^*(t)\}/|c_1(t)|^2$ one sees that Eq. (25) has the same structure as Eq. (11). In fact

the compensated rate of change of the first pseudomode population equals a time dependent coefficient times the second pseudomode population. Therefore the first pseudomode acts as a memory storage for the second one. Such a storage of memory of the second pseudomode into the first one is perfect in the case of perfect gap where $\Gamma_1'=0$.

## IV. CONCLUSION

Both the pseudomodes and the reversed quantum jumps describe the non-Markovian back action of a reservoir on a quantum system. Our results show that the picture of open quantum systems as comprised of a system and environment is a commonly held view that does not reflect the dynamics when the coupling is "strong;" the environment divides into memory and nonmemory parts. Establishing a connection in the case of a general structured reservoir, i.e., for a generic form of the spectrum is an extremely complicated issue because both the number of pseudomodes and their equations of motion depend on the details of the reservoir structure. In this sense our results constitutes a first step in the direction of an understanding of memory in non-Markovian systems.

It is likely that a measurement on the reservoir will in general affect any memory that it contains, and so far it has not been possible to attach any measurement scheme to non-Markovian reservoirs, as Ref. [12] demonstrates. However, since we have seen that it is possible to make a separation of the reservoir into a memory part and a Markovian reservoir part, there remains the possibility that we could make a measurement scheme that would address only the Markovian part of the reservoir. In the cavity-atom situation, this might correspond to a measurement on "outside" modes, which have a significant amplitude outside the cavity. In such a case it is clear that the reservoir memory for the atom actually resides in inaccessible "inside" mode, or pseudomode, belonging to the cavity.

## ACKNOWLEDGMENTS

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