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Entanglement trapping in structured environments

Bruno Bellomo,[1] Rosario Lo Franco,[1,*] Sabrina Maniscalco,[2] and Giuseppe Compagno[1]
[1]*CNISM and Dipartimento di Scienze Fisiche ed Astronomiche, Università di Palermo, via Archirafi 36, 90123 Palermo, Italy*
[2]*Department of Physics, University of Turku, Turun yliopisto, FIN-20014 Turku, Finland*
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We highlight the direct link between the time-dependent entanglement and single-qubit excited-state population for two independent qubits, each coupled to a zero-temperature bosonic environment. We show that, in environments structured so as to inhibit spontaneous emission, entanglement trapping and thus prevention of entanglement sudden death occur. We explicitly show that, for photonic band-gap materials as environment, high values of entanglement trapping can be achieved. We finally discuss how, under these conditions, coherent quantum operations can be implemented.

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Entanglement preservation is an important challenge in quantum information and computation technologies [1]. Realistic quantum systems are affected by decoherence and entanglement losses because of the unavoidable interaction with their environments [2]. For example, in Markovian (memoryless) environments, in spite of an exponential decay of the single-qubit coherence, the entanglement between two qubits may completely disappear at a finite time [3,4]. This phenomenon, known as "entanglement sudden death" and experimentally proven to occur [5,6], limits the time when entanglement can be exploited for practical purposes. It is therefore of interest to examine the possible conditions that lead to preservation of entanglement. In the case of environments with memory (non-Markovian), such as imperfect cavities supporting a mode resonant with the atomic transition frequency, revivals of two-qubit entanglement have been found [7–9]. These revivals, although effectively extending the possible usage time of entanglement, decrease with time and eventually disappear after a certain critical time. Moreover, when the qubits interact with a common environment, it has been shown that entanglement can be preserved by means of the quantum Zeno effect [9].

In this paper we continue the investigation of physical systems and physical effects that may lead to effective long-time entanglement protection. The main aim of this work is thus to analyze if and to what extent entanglement initially present may be preserved for a general class of states of noninteracting qubit pairs. In particular, we highlight the direct connection between the time-dependent entanglement and single-qubit excited-state population. One is then led to investigate situations where population trapping occurs. This can happen in structured environments where the density of states presents a dip which can inhibit spontaneous emission in the region of the dip [10]. Among realistic physical situations, this effect is known to occur for atoms embedded in photonic band-gap (PBG) materials [11,12]. We finally analyze how such a system may be realistically manipulated to perform quantum computation and quantum-information processing.

*lofranco@fisica.unipa.it; http://www.fisica.unipa.it/~lofranco

We start by considering a system composed by two independent parts each made of a two-level atom (qubit) placed inside a zero-temperature bosonic environment. The two qubits A and B, with ground and excited states $|0\rangle,|1\rangle$ and transition frequency ω_0, are identical and initially entangled. For this system we may exploit a procedure which allows us to obtain the two-qubit reduced density matrix at the time t from knowledge of that of a single qubit [7]. The assumption of independent parts implies that the total time evolution operator can be factorized into two terms relative to the two parts; this leads to the fact that, once the dynamics of each qubit is expressed in the form $\rho^A_{ii'}(t)=\Sigma_{ll'}A^{ll'}_{ii'}(t)\rho^A_{ll'}(0)$, $\rho^B_{jj'}(t)=\Sigma_{mm'}B^{mm'}_{jj'}(t)\rho^B_{mm'}(0)$, the dynamics of the two-qubit system is simply given by [7]

$$\rho_{ii',jj'}(t)=\sum_{ll',mm'}A^{ll'}_{ii'}(t)B^{mm'}_{jj'}(t)\rho_{ll',mm'}(0), \tag{1}$$

where the indices $i,j,l,m=0,1$. This procedure can also be applied to the case of a common environment provided that the atoms are sufficiently separated in order to neglect the dipole-dipole interaction and at a distance larger than the spatial correlation length of the reservoir.

The dynamics of each qubit coupled to the environment modes can be described by the Hamiltonian [2,13]

$$\hat{H}=\hbar\sum_k[(\omega_k-\omega_0)\hat{b}^\dagger_k\hat{b}_k+ig_k(\hat{\sigma}_-\hat{b}^\dagger_k-\hat{\sigma}_+\hat{b}_k)], \tag{2}$$

where ω_0 is the transition frequency, $\sigma_\pm$ are the raising and lowering operators of the qubit, $b^\dagger_k$ and b_k are the creation and annihilation operators, and g_k is the atom-field coupling constant of the mode k with frequency ω_k. The Hamiltonian of Eq. (2) is physically equivalent to the standard Hamiltonian of a two-level atom interacting with the radiation field and it is obtainable from the last one by unitary transformations [13]. When the environment is at zero temperature and the qubit is initially in a general superposition state of its two levels, the single-qubit reduced density matrix $\hat{\rho}^S(t)$ takes the form [2]

$$\hat{\rho}^S(t) = \begin{pmatrix} \rho_{11}^S(0)|q(t)|^2 & \rho_{10}^S(0)q(t) \\ \rho_{01}^S(0)q^*(t) & \rho_{00}^S(0) + \rho_{11}^S(0)(1-|q(t)|^2) \end{pmatrix}. \quad (3)$$

The single-qubit dynamics thus depends only on the function $q(t)$, whose explicit time dependence contains the information on the environment spectral density and the coupling constants. The results we shall present in the following hold in all cases where the single-qubit dynamics has the form of Eq. (3). Using Eq. (1), we can then determine the two-qubit dynamics. In particular, in the standard two-qubit basis $\mathcal{B}=\{|1\rangle\equiv|11\rangle,|2\rangle\equiv|10\rangle,|3\rangle\equiv|01\rangle,|4\rangle\equiv|00\rangle\}$, the diagonal elements of the reduced density matrix $\hat{\rho}(t)$ at time t result as

$$\rho_{11}(t) = \rho_{11}(0)|q(t)|^4,$$

$$\rho_{22}(t) = \rho_{11}(0)|q(t)|^2[1-|q(t)|^2] + \rho_{22}(0)|q(t)|^2,$$

$$\rho_{33}(t) = \rho_{11}(0)|q(t)|^2[1-|q(t)|^2] + \rho_{33}(0)|q(t)|^2,$$

$$\rho_{44}(t) = 1 - [\rho_{11}(t) + \rho_{22}(t) + \rho_{33}(t)], \quad (4)$$

and the nondiagonal elements

$$\rho_{12}(t) = \rho_{12}(0)q(t)|q(t)|^2, \quad \rho_{13}(t) = \rho_{13}(0)q(t)|q(t)|^2,$$

$$\rho_{14}(t) = \rho_{14}(0)q(t)^2, \quad \rho_{23}(t) = \rho_{23}(0)|q(t)|^2,$$

$$\rho_{24}(t) = \rho_{13}(0)q(t)[1-|q(t)|^2] + \rho_{24}(0)q(t),$$

$$\rho_{34}(t) = \rho_{12}(0)q(t)[1-|q(t)|^2] + \rho_{34}(0)q(t), \quad (5)$$

with $\rho_{ij}(t)=\rho_{ji}^*(t)$, $\hat{\rho}(t)$ being a Hermitian matrix. Equations (4) and (5) give the two-qubit density matrix evolution for any initial state dependent only on $q(t)$.

We use the concurrence C [14], which attains its maximum value 1 for maximally entangled states and vanishes for separable states, to analyze the two-qubit entanglement dynamics. We consider as initial states the Bell-like states

$$|\Phi\rangle = \alpha|01\rangle + e^{i\gamma}\sqrt{1-\alpha^2}|10\rangle,$$

$$|\Psi\rangle = \alpha|00\rangle + e^{i\gamma}\sqrt{1-\alpha^2}|11\rangle, \quad (6)$$

where α is real. For these initial conditions the concurrence at time t is given, respectively, by

$$C_\Phi(t) = 2\max[0, \alpha\sqrt{1-\alpha^2}|q(t)|^2],$$

$$C_\Psi(t) = 2\max(0, \sqrt{1-\alpha^2}|q(t)|^2\{\alpha - \sqrt{1-\alpha^2}[1-|q(t)|^2]\}). \quad (7)$$

For initial Bell (maximally entangled) states, $|\Phi\rangle=(|01\rangle\pm|10\rangle)/\sqrt{2}$ and $|\Psi\rangle=(|00\rangle\pm|11\rangle)/\sqrt{2}$, the concurrences at the time t take the simple form

$$C_\Phi(t) = |q(t)|^2, \quad C_\Psi(t) = |q(t)|^4, \quad (8)$$

independently of the relative phase γ. The previous equation shows a direct link between the time behavior of the single-qubit excited-state population $P(t)=\rho_{11}^S(0)|q(t)|^2$ and that the concurrence. The relevance of Eqs. (7) and (8) consists in their highlighting that, whenever single-qubit population trapping occurs, entanglement trapping follows. These results are quite general since, depending on $|q(t)|^2$, their form does not explicitly depend on the particular choice of the environment, but only on the Hamiltonian model of Eq. (2) and on the chosen initial state. Certainly, the explicit time dependence of concurrence will require the explicit form of the function $q(t)$ and thus will contain the information about the environment structure.

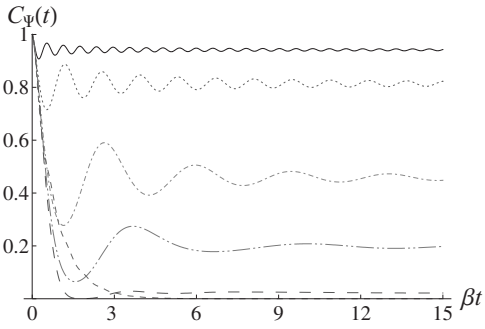

FIG. 1. Concurrence $C_\Psi(t)$ as a function of βt starting from the initial Bell state $|\Psi\rangle=(|00\rangle\pm|11\rangle)/\sqrt{2}$ for various values of detuning from the photonic band edge: $\delta=-10\beta$ (solid curve), -4β (dotted curve), $-\beta$ (long-short-dashed curve), 0 (long-short-short-dashed curve), β (long-dashed curve), and 10β (short-dashed curve).

As an application, we now consider as environment a zero-temperature three-dimensional periodic dielectric with isotropic photon dispersion relation ω_k [15]. By symmetrizing ω_k, one produces PBGs at the spheres $|\mathbf{k}|=m\pi/L$ ($m=1,2,\ldots$), where L is the lattice constant. In such ideal photonic crystals, a PBG is the frequency range over which the local density of electromagnetic states and the decay rate of the atomic population of the excited state vanish. Near the band-gap edges the density of states becomes singular [16], the atom-field interaction becomes strong, and one can expect modifications to the spontaneous emission decay. For $k\cong\bar{k}\equiv\pi/L$, the dispersion relation near the band edge ω_c can be approximated by $\omega_k=\omega_c+D(k-\bar{k})^2$, where $D\cong\omega_c/\bar{k}^2$ [16]. This dispersion relation is isotropic since it depends only on the magnitude k of the wave vector $\mathbf{k}$. In the case of a two-level atom embedded in such a material, the explicit form of $q(t)=f(\delta,\beta,t)$ has been obtained as a function of the detuning $\delta=\omega_0-\omega_c$ between the atomic transition frequency ω_0 and the band edge frequency ω_c and of a parameter β defined as $\beta^{3/2}=\omega_0^{7/2}d^2/6\pi\epsilon_0\hbar c^3$ with ϵ_0 being the Coulomb constant and d the atomic dipole moment [15]. The parameter $\beta^{3/2}$, being proportional to $\omega_0^2d^2$, represents the strength of the atom-reservoir coupling. The parameter β also scales the time evolution and for large times ($\beta t\gg 1$) one has $|q(t)|^2\to$const., that is, an asymptotic population trapping [15].

The concurrence $C_\Psi(t)$ of Eq. (8), corresponding to the initial Bell state $|\Psi\rangle$, is shown in Fig. 1 as a function of the dimensionless time βt and for various values of detuning δ, $\delta<0$ corresponding to the case when the two-level atom frequency ω_0 is inside the band gap. The figure displays

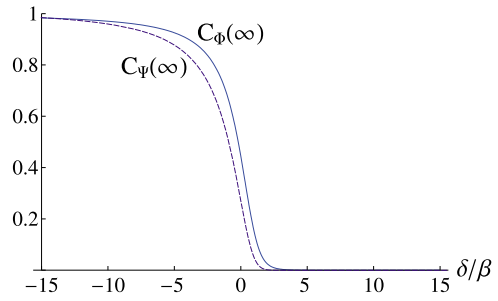

FIG. 2. (Color online) Asymptotic values of concurrences $C_\Phi(t=\infty)$ (solid line) and $C_\Psi(t=\infty)$ (dashed line) as a function of δ/β, starting, respectively, from the initial Bell state $|\Phi\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$ and $|\Psi\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$.

entanglement trapping. The concurrence is nearer to its maximum value 1 for atomic frequencies farther from the band edge and deeper inside the gap. The behavior of $C_\Phi(t)$ will be qualitatively similar to that of $C_\Psi(t)$. In Fig. 2 the asymptotic values of $C_\Phi(\infty)$ and $C_\Psi(\infty)$ are plotted as functions of the ratio δ/β, and it can be appreciated that $C_\Phi(\infty)$ is slightly larger than $C_\Psi(\infty)$. We see that, when $\delta/\beta<0$, these asymptotic values differ from zero and decrease very rapidly going near the edge. It is evident that the more the atomic transition frequency is far from the band edge and inside the gap ($\delta/\beta<0$), the higher is the preserved entanglement. An isotropic dispersion relation is known to lead also to the formation of atom-photon bound states [15,16], which in turn lead to entanglement trapping near the edge even for ω_0 outside the band gap ($\delta/\beta>0$), although with small asymptotic values (<0.4; see Fig. 2). By exploiting Eq. (7) it is possible to study the behavior of concurrence in terms of the initial degree of entanglement (represented by α). The result is that smaller initial entanglement leads to lower asymptotic values of concurrence and that the entanglement trapping phenomenon does not depend on the initial value of α. We stress that, in a zero-temperature Markovian environment, the initial Bell-like state $|\Psi\rangle$ of Eq. (6) would undergo entanglement sudden death for $\alpha<1/\sqrt{2}$ [4]; in contrast, on the basis of our results, in a PBG material entanglement sudden death and its consequent limit on the entanglement usage can be prevented.

The results obtained here are valid for ideal PBG materials; thus one may ask to what extent they hold for real crystals. In fact, in real crystals with finite dimensions a pseudogap is typically obtained where the density of states is much smaller than that of free space but is not exactly zero. In such gaps, however, the lowering of the local density of states can be so large that the spontaneous decay of an excited emitter is effectively inhibited by choosing suitably its position inside the PBG material [17–19]. Thus, memory and coherent control effects are admitted and similar to those occurring in an ideal PBG. For quantum dots inside a real PBG material a pronounced inhibition of spontaneous emission up to 30% has been observed [12]. This would correspond, in view of the relation between population and entanglement expressed by Eqs. (7) and (8), to a partial inhibition of entanglement decay.

One may ask how to entangle qubit pairs in structured environments. In PBG materials, entanglement can be generated when a pair of the atoms near-resonantly coupled to the edge of the PBG present direct dipole-dipole interaction [20]. In contrast, entangled independent atoms in PBG materials can be produced by considering a three-dimensional photonic crystal single-mode cavity with a high-quality Q factor where Rydberg atoms can freely travel through the connected void regions [21]. The atoms exchange photons with the cavity, represented by a single defect mode of the crystal resonant with the atomic transition frequency, which acts as an entanglement catalyst. In this situation, the atoms are independent because their transition frequency is inside the PBG and their relative distance permits neglect of the dipole-dipole interaction.

Another aspect that must be considered is the possibility of performing, under the conditions here treated, two-qubit operations for quantum computing. When the qubit transition frequencies are inside the gap, the effect of any resonant external signal is impeded. Thus, a way to bring the qubit frequencies outside the gap is required for the time necessary to externally manipulate the qubits. This could be realized, e.g., by Stark-shifting the frequencies with a static electric field. The duration of this external control should be short enough to limit the decay of entanglement. For example, suppose that two Rb Rydberg atoms, having transition frequency $\omega_0 \sim 50$ GHz [22], are embedded in a photonic crystal built in such a way as to have PBGs in the range of gigahertz [23]. For such atoms the dipole moment is $d \sim 2 \times 10^{-26}$ C m [21], which in turn gives $\beta \sim 20$ kHz, while the typical Stark shifts are $\Delta \sim 200$ kHz [22]. This implies a dimensionless shift with respect to β of the order $\Delta/\beta \sim 10$. On the basis of the results displayed in Fig. 2, this shift is therefore large enough to move the atomic transition frequency, for example, from a ratio $\delta/\beta=-5$ inside the gap, corresponding to an entanglement trapping value ~ 0.9, to a ratio $\delta/\beta=5$ outside the gap, where the atoms can be manipulated by an external signal. The times required for turning the Stark shifts on and off to take the qubits out of the band are relevant for entanglement storage. They are typically $\tau \sim 1$ μs [22], corresponding to $\beta\tau \sim 10^{-2}$, and their influence on performing many gate operations remains small. In this Rydberg-atom context, by inserting a defect mode as a cavity inside the crystal, suitable atom-cavity interactions allow one to perform quantum logic gates, and other cavity QED-based procedures should be achievable [21].

In many experiments the qubits are mimicked by "artificial atoms" consisting in quantum dot structures embedded in the solid fraction of the PBG material [12,19]. Quantum dots permit the coherent manipulation of a single localized quantum system with the technological advantages of solid-state systems. In fact, the coherent control of an exciton wave function in a quantum dot, namely, the manipulation of the relative phases of the eigenstates in a quantum superposition, is experimentally achievable [24]. Typical values of dipole moment for InAs quantum dots are observed to be $d \sim 3.2 \times 10^{-28}$ C m [25] with transition frequencies ω_0 in the optical range ($\omega_0 \sim 100$ THz). This leads to a value of $\beta \sim 160$ MHz. Also, in the quantum dot scenario, one can tune the energy levels by the Stark effect with typical shifts

$\Delta \sim 1-10$ GHz [25,26], so that the dimensionless shift is $\Delta/\beta \sim 5-50$ and, once again, the required conditions for qubit coherent manipulations may be accomplished. These features make quantum dots incorporated in a PBG material good candidates for implementation of schemes of quantum computation and coherent information processing.

In conclusion, in this paper we have presented the phenomenon of entanglement trapping, and we have shown that it can be used to prevent entanglement sudden death. The physical interpretation of the entanglement trapping phenomenon rests on the direct link between the dynamics of entanglement and that of the single-qubit excited-state population which we have demonstrated here. Whenever population trapping occurs, entanglement trapping follows. As an application, we have shown that the entanglement initially present between two independent qubits embedded in a PBG material can be preserved when the qubit transition frequency is inside the band gap. Moreover, the qubits may be coherently manipulated by an external control field in order to perform some quantum computation or information protocol. A possible way to achieve this is by Stark-shifting the qubit transition frequencies outside the gap for a time necessary to the desired two-qubit operation. We have pointed out that, for this purpose, Rydberg atoms and quantum dots in photonic crystals may be suitable. This analysis can be simply extended to treat the cases both of N independent qubits [8], which is essential for quantum information, and of anisotropic PBG environments. In the latter case, although one expects a physics qualitatively similar to the isotropic case, quantitative differences may arise [16]. The results here obtained evidence that entanglement can be preserved by modifying the spectrum of the environment and highlight the potential of reservoir engineering for controlling and manipulating the dynamics of quantum systems.

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