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Coherence-based investigation of stellar objects:

A speckle-based approach.

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ABSTRACT

Based on canonical transforms, we develop the space-time cross covariance from incoherent fields emitted from distant objects. We simplify the results to stress the aspects qualitatively, thereby losing exact quantities. Specifically, we argue that fourth order field correlations in the detector plane (the Earth) can reveal the difference in axial velocities (Stellar objects) for two – or several – unresolvable objects. Besides, we derive the angular velocity for a rotating cloud of objects with rotational axes, pointing towards the observer, as can the spatial cross covariance for a single object rotating about a transverse axis. We neglect relativistic effects and the influence of optical turbulence. The analysis resembles the ones performed for objects illuminated with a coherent field, although here the spatial coherence arises due to the extreme distances between object and observer, and the temporal coherence is facilitated by spectral filtering augmented optionally by adjusting the detector separation.

The results are applicable for radiation in a broader spectrum including microwaves and gamma radiation.

Key Words: Stellar observation, coherence theory, Doppler shift, angular momentum.

1. INTRODUCTION

Optical observations of stellar objects have a long history. Here, we will approach the issue by examining applications involving distant objects that cannot be resolved by telescopic means. Therefore, we will analyze the space-time cross covariance between the time lagged intensities for two laterally separated point detectors. We argue that by virtue of the Gaussian hypothesis for the emitted fields from the objects, the intrinsic fourth-order field correlation can be broken down to second-order field correlations. This will facilitate the possibility for deriving various information from the emitting structures. Specifically, we will as examples address the possibility of finding the difference in axial velocity for two closely spaced objects by their difference in Doppler shift. Next, we will indicate how the rotation of a cloud of particles can be found by probing the angular momentum of the emitted field.

Doppler shift from single stellar emitters have previously been deduced based on the vectorial components of the emitted radiation[1] or by observing the fine-structure of emission lines either in the Fourier domain[2] or in the wavelength regime[3].

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The case of detecting rotation of a cloud of particles, an abundance of high-level investigations has been presented. A modal expansion of the rotational Doppler shift has been treated theoretically and experimentally verified[45]. Both methods rely on determining the angular modes, indicating that the object can be resolved. A profound overlook of the measurements of orbital angular momentum of starlight by telescopes has been presented[6]. These references to previous treatment of this subject are far from thorough but can serve as a portal for further studies.

The present discussion commences with a simplification of the space-time lagged cross covariance (STCC) for the incoherently emitted field from the two distinct particles. This formalism will be used to get a quantitative description of the STCC first for a single detector, next for two spatially separated detectors. Next, the STCC for a rotating cloud is derived, and finally a discussion of further investigations is provided.

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2. SPACE TIME COVARIANCE

Figure 1 shows the parameters used for the derivation of the STCC in the case of axially moving objects.

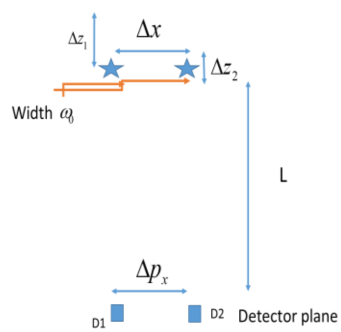

Figure 1. Two incoherent emitters transversely displaced a distance Δx each having the (Gaussian) width ω_0 are observed by two separated detectors at a large distance L.

The STCC for a single wavelength for the two emitters is given by:

$$\begin{aligned} C[\mathbf{p_1},\mathbf{p_2};\Delta\tau] &\equiv \langle I[\mathbf{p_1},0]\,I[\mathbf{p_2},\Delta\tau]\rangle - \langle I[\mathbf{p_1},0]\,I[\mathbf{p_2},0]\rangle = \\ &\left|\left\langle \left(U_1[\mathbf{p_1},0]+U_2[\mathbf{p_1},0]\right)\left(U_1[\mathbf{p_2},\Delta\tau]^* + U_2[\mathbf{p_2},\Delta\tau]^*\right)\right\rangle\right|^2 \end{aligned} \tag{1}$$

Here angular brackets indicate *ensemble average,* and due to the field statistics being Gaussian we have reduced the inherent fourth order field correlations into products of second order field correlations[7]. Expansion of Eq.1 gives:

$$\begin{aligned} C[\mathbf{p_1},\mathbf{p_2};\Delta\tau] &\equiv \langle I[\mathbf{p_1},0]\,I'[\mathbf{p_2},\Delta\tau]\rangle - \langle I[\mathbf{p_1},0]\,I'[\mathbf{p_2},0]\rangle = \\ &\left| \begin{matrix} \langle U_1[\mathbf{p_1},0]U_1{}^*[\mathbf{p_1},\Delta\tau]\rangle + \langle U_2[\mathbf{p_2},0]U_2{}^*[\mathbf{p_2},\Delta\tau]\rangle \\ +\langle U_2[\mathbf{p_2},0]U_1{}^*[\mathbf{p_1},\Delta\tau]\rangle + \langle U_1[\mathbf{p_1},0]U_2{}^*[\mathbf{p_2},\Delta\tau]\rangle \end{matrix} \right|^2 \end{aligned} \tag{2}$$

We essentially have two different terms, namely:

$$\Psi_{1,2} \equiv \left\langle U_{1,2}\left[\mathbf{p_{1,2}},0\right]U_{1,2}{}^*\left[\mathbf{p_{1,2}},\Delta\tau\right]\right\rangle \text{ and } \Gamma_{1,2} \equiv \left\langle U_{2,1}\left[\mathbf{p_{2,1}},0\right]U_{1,2}{}^*\left[\mathbf{p_{1,2}},\Delta\tau\right]\right\rangle \tag{3}$$

The first term $\Psi_{1,2}$ is the "self-term" stemming from mixing of fields from the same object at the same detector, whereas the "distinct term" $\Gamma_{1,2}$ gives the interference between fields scattered from two separated objects. The mixed term is given by:

$$\iint \int \int d\mathbf{r}_1\, d\mathbf{r}_2\, dk_1\, dk_2\, G[\mathbf{r}_1,\mathbf{p}_1;k_1]G^*[\mathbf{r}_2,\mathbf{p}_2;k_2]^*\langle U_{2,1}[\mathbf{r}_1,0;k_1]U_{1,2}[\mathbf{r}_2,\tau;k_2]^*\rangle S[k_1]S[k_2]. \quad (4)$$

Here $G[\mathbf{r},\mathbf{p}] = \frac{-ik}{2\pi L} exp\left[-\frac{ik}{2L}(\mathbf{r}-\mathbf{p})^2\right]$ is the Greens function for free space connecting the fields in the input- and output planes, and $S[k]$ is the spectral wavenumber width determined by filtering, here assumed Gaussian with width Δk centered at k_0. The spectra are assumed delta-correlated, i.e. $\langle U_{2,1}[\mathbf{r}_1,0;k_1]U_{1,2}[\mathbf{r}_2,\tau;k_2]^*\rangle \propto \langle U_{2,1}[\mathbf{r}_1,0;k_1]U_{1,2}[\mathbf{r}_2,\tau;k_2]^*\rangle \delta(k_1-k_2)k_1$. Likewise, are the field emission assumed delta-correlated $\langle U_{2,1}[\mathbf{r}_1,0;k]U_{1,2}[\mathbf{r}_2,0;k]^*\rangle = 4/(\pi k^2)\, I[\mathbf{r}_1]\delta[\mathbf{r}_2-\mathbf{r}_1]$. The mixed term $\Gamma_{1,2}$ will vanish due to the lack of intensity overlap between the irradiance from the two sources. The only terms remaining are the self-terms $\Psi_{1,2}$ in Eq. 3. The ensemble average of the self- terms in Eq.3 becomes:

$$\Psi_{1,2} \equiv \langle U_{1,2}[\mathbf{p}_{1,2},0]U_{1,2}^*[\mathbf{p}_{1,2},\Delta\tau]\rangle =$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dk_1\; dk_2\; d\mathbf{r}_1\; d\mathbf{r}_2 S[k_1]S[k_2]\langle U_{1,2}[\mathbf{r}_1,0;k_1]U_{1,2}[\mathbf{r}_2,\tau;k_2]^*\rangle G[\mathbf{r}_1,\mathbf{p}_{1,2};k_1]G^*[\mathbf{r}_2,\mathbf{p}_{1,2};k_2]. \quad (5)$$

We have after discarding additional DC-terms:

$$C[\mathbf{p}_1,\mathbf{p}_2;\Delta\tau] \propto \mathrm{Re}\left[\langle U_1[\mathbf{p}_1,0]U_1[\mathbf{p}_1,\Delta\tau]^*\rangle\langle U_2[\mathbf{p}_2,0]U_2[\mathbf{p}_2,\Delta\tau]^*\rangle\right]. \quad (6)$$

This shows that mixing of the information from the two objects appears due to interference of the correlation functions for individual emitters, not due to interference between the scattered fields from the two sources, seemingly in contradiction with Young's interferometer. This discrepancy is due to the Gaussian assumption for the scattered field, used here.

3. DOPPLER SHIFT

The time-lagged cross covariance can be found from the STCC in Eq.5 by having only one detector by setting $\mathbf{p}_1 = \mathbf{p}_2 = \mathbf{p}$. We get for $\mathbf{p} = 0$, after having discarded further DC-values:

$$C[0,0;\Delta\tau] \propto \frac{\omega_0^4\,\Delta k_0^4}{\pi^2 L^4}\exp\left[-\frac{(\Delta z_1^2+\Delta z_2^2)\Delta k_0^2}{4}\right]\cos\left[(v_{z_1}-v_{z_2})\Delta\tau\, k_0\right]. \quad (7)$$

Here the axial velocities are v_{z_i}. The cosine term is responsible for the observed oscillations, whereas the exponential term controls the decorrelation, in effect the random phase shifting of the Doppler shift. The covariance gives a Doppler burst of frequency $f_{Doppler} = \frac{k_0|v_{z,1}-v_{z,2}|}{2\pi}$ and with a number of contiguous oscillations $N \propto \left(\frac{k_0}{\Delta k_0}\right)\left(\frac{|v_{z,1}-v_{z,2}|}{c}\right)$. Generally speaking, we need a finite number of contiguous oscillations in order to detect the difference in Doppler shift, which again calls for a narrow bandpass filter and/or large velocity difference. In optics, this might be difficult to fulfill, however in the case of microwaves, this requirement is alleviated. The full STCC for two detectors can be derived in like manner giving:

$$C\left[0,\Delta\mathbf{p};\Delta\tau\right]\propto\cos\left[k_0\left(\Delta z_1\left[\Delta\tau\right]-\Delta z_2\left[\Delta\tau\right]-\frac{\Delta p_x\,\Delta x}{L}\right)\right]. \tag{8}$$

The Doppler shift now includes a phase shift that depends on the angular ground separation between the two emitters and the detector separation. Two opportunities appear, namely a further artificial spectral narrowing can be obtained if the angular separation is known; or the angular separation of the emitters can be determined, even though the emitters cannot be resolved. Finally, the acquisition of the phase term will reduce the former demand for having a finite number of Doppler oscillations, as we can average several phase differences revealing the difference in Doppler shift.

An assembly of non-resolvable emitters (stars/planets) will likewise give rise to a compound space-time cross covariance:

$$C_\Sigma\left[\tau\right]\propto\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\omega_i^2\,\omega_j^2\Delta k_0^4}{\pi^2\,L^4}\exp\left[-\frac{\left(\Delta z_i^2\left[\tau\right]+\Delta z_j^2\left[\tau\right]\right)\Delta k_0^2}{4}\right]\cos\left[\left(\Delta z_i\left[\tau\right]-\Delta z_j\left[\tau\right]\right)k_0\right]\text{ for } i\neq j. \tag{9}$$

An alternative way of narrowing the governing spectral width Δk_0 could be to observe the shift in the spectral position of the narrow Fraunhofer lines. The theoretical evaluation of this would be slightly more delicate, but the theoretical considerations would be like the above.

4. IN-PLANE ROTATION

Next, we consider a rotating cloud of incoherent emitters that cannot be resolved in the detector plane, as shown in Fig.2.

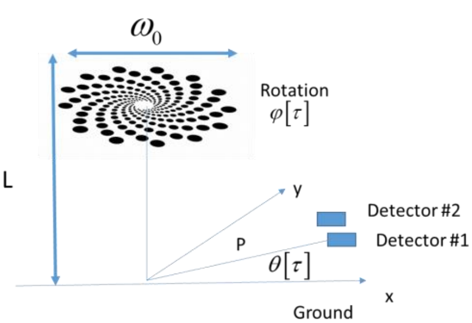

Figure 2. A rotating cloud of particles observed at a large distance L. The analysis in both planes are performed in cylindrical coordinates.

The Green's function connecting the field in the input plane $R,\varphi\left[t\right]$, with the field in the output plane $P,\theta\left[t\right]$, becomes:

$$G\left[R,\varphi\left[\tau\right],P,\theta\left[\tau\right]\right]=-\frac{i\,k_0\,R}{2\pi\,L}\exp\left[-\frac{i\,k\left(P^2+R^2-2\,R\,P\cos\left[\varphi\left[\tau\right]-\theta\left[\tau\right]\right]\right)}{2L}\right]. \tag{10}$$

The object is during the observation time assumed frozen, i.e., Taylor's hypothesis:

$$\left\langle U_{in}\left[R_1,\varphi_1\right]U_{in}{*}\left[R_2,\varphi_2\right]\right\rangle\propto 2\pi\,R_1\,I[R_1]\,\delta\left[R_1-R_2\right]\delta\left[\varphi_1-\varphi_2-\Delta\varphi\left[\tau\right]\right] \tag{11}$$

We perform the integrations and arrive at the result for the STCC for two detectors placed at an angular separation $\Delta\theta$ but with identical radial separation P:

$$C\left[P,0,\theta,\Delta\theta\right]\propto\left(\frac{\omega_0}{\rho}\right)^4\left(1-\frac{1}{2}\left(\frac{P}{\rho}\right)^2\left(1-\cos\left[\Delta\theta-\Delta\varphi\right]\right)\right)^2\exp\left[-\left(\frac{P}{\rho}\right)^2\left(1-\cos\left[\Delta\theta-\Delta\varphi\right]\right)\right] \tag{12}$$

where the speckle size is $\rho=\dfrac{L}{k\,\omega_0}$.

The angle of rotation of the object $\Delta\varphi$ is time dependent, whereas the azimuthal detector positions $\Delta\theta$ are the free parameter depending on the detector scheme. The cross covariance as a function of $\Delta\theta-\Delta\varphi$ and $\frac{P}{\rho}$ shows that a maximum exists for $\Delta\theta=\Delta\varphi$ with a decreasing width for increased detector separation P from the center of rotation in the detector plane, Fig. 3.

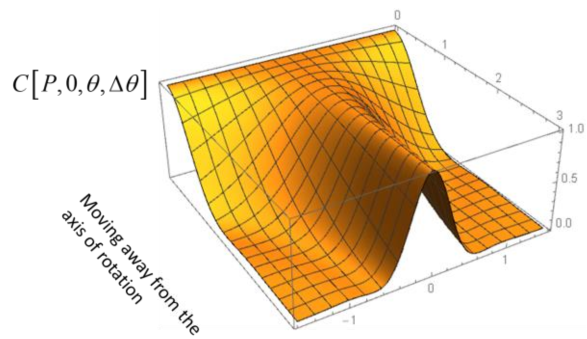

Fig.3. Cross covariance as a function of $\Delta\theta-\Delta\varphi$ and $\frac{P}{\rho}$.

An unambiguous determination of the object's rotation might call for having at least three detectors to distinguish between rotation and displacement.

5. CONCLUSIONS

It has been argued that information can be obtained for non-resolvable objects based on the space-time cross covariance between signals from two separated detectors. Expressions have been given in the case of the difference in Doppler shift for two axially moving objects, and for the result for an in-plane rotating cloud. We offer simplified results to highlight the idea using Canonical Transforms and neglecting the impact of optical turbulence and relativistic effects.

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