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Spatial Resolution of Polarization Imaging System for illumination of Arbitrary Coherence

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ABSTRACT

The two-point resolution under the illumination of partially coherent light has been discussed previously and concluded that the measurement of two-point separation depends strongly upon the coherence of the illumination. However, its polarization properties that arise from the vector nature of the electromagnetic field have been ignored in previous studies. Considering multidimensional physical optical field information in the polarized imaging system, we study the spatial resolution namely separations of two points in normalized Stokes parameters based on the unified theory of polarization and coherence. Research results show that the ratio of the measured to the true separation is not equal to unity except for particular values of the actual separation. Some interesting phenomena are found, namely that two resolvable points with certain Stokes parameters will not always be resolved for other Stokes parameters.

Keywords: Partially coherent light, polarization imaging system, spatial resolution

1. INTRODUCTION

As a quality factor of the optical imaging system, spatial resolution describes the ability to distinguish two close points, which plays a crucial role in the fields of astronomy, biomedical imaging, and lithography for integrated circuit engineering [1-2]. Based on the Rayleigh criterion and the Sparrow criterion, numerous studies of two-point resolution for imaging systems are investigated under partially coherent illumination. However, most of them have been done on the basis of scalar theory, except for [3-5], a few studies that have been done on the resolving power for polarization imaging under the illumination of partially coherent light [6-7]. We study the unique measurable quantity namely spatial resolution in a polarization imaging system with a set of normalized factors referred to as the degree of the generalized Stokes parameters.

2. THEORETICAL INTENSITY DISTRIBUTION

The object under consideration consists of two equally bright points separated by a distance 2ε in the ξ direction under simultaneous illumination of partially coherent and partially polarized light. The generalized Stokes parameters in the object plane are given by:

$$\begin{cases} S_0(\xi_1,\eta_1;\xi_2,\eta_2) = <E_x^*(\xi_1,\eta_1)E_x(\xi_2,\eta_2)> + <E_y^*(\xi_1,\eta_1)E_y(\xi_2,\eta_2)> \\ S_1(\xi_1,\eta_1;\xi_2,\eta_2) = <E_x^*(\xi_1,\eta_1)E_x(\xi_2,\eta_2)> - <E_y^*(\xi_1,\eta_1)E_y(\xi_2,\eta_2)> \\ S_2(\xi_1,\eta_1;\xi_2,\eta_2) = <E_x^*(\xi_1,\eta_1)E_y(\xi_2,\eta_2)> + <E_y^*(\xi_1,\eta_1)E_x(\xi_2,\eta_2)> \\ S_3(\xi_1,\eta_1;\xi_2,\eta_2) = i[<E_y^*(\xi_1,\eta_1)E_x(\xi_2,\eta_2)> - <E_x^*(\xi_1,\eta_1)E_y(\xi_2,\eta_2)>] \end{cases} \tag{1}$$

where, (ξ,η) is the coordinate in the object space, <…> indicates ensemble average, E_x and E_y are the components of the complex electric field vector represented by analytic signals. The mutual intensity function in the object plane is given by:

$$S_l^{\,o}(\xi_1,\eta_1;\xi_2,\eta_2) = S_l^{\,s}(\xi_1,\eta_1;\xi_2,\eta_2)[\delta(\xi_1-\varepsilon,\eta_1)+\delta(\xi_1+\varepsilon,\eta_1)][\delta(\xi_2-\varepsilon,\eta_2)+\delta(\xi_2+\varepsilon,\eta_2)],\quad l=0\sim 3 \tag{2}$$

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where, 2ε is the separation of two points. $S_l(\xi_1,\eta_1;\xi_2,\eta_2)$, $(l=0,1,2,3)$ with their superscripts "*o*" and "*s*" indicating object and source illumination, respectively. The intensity $S_l(u_1,v_1;u_2,v_2)$ of the polarization image for a spatially shift-invariant imaging system is given by

$$S_l^{\,i}(u_1,v_1;u_2,v_2)=\iiint\!\!\int_{-\infty}^{\infty} S_l^{\,o}(\xi_1,\eta_1;\xi_2,\eta_2)h(u_1-\xi_1,v_1-\eta_1)h^{*}(u_2-\xi_2,v_2-\eta_2)\mathrm{d}\xi_1\mathrm{d}\eta_1\mathrm{d}\xi_2\mathrm{d}\eta_2 \tag{3}$$

where the superscript "*i*" indicates the image plane and (*u*, *v*) is the coordinate in the image space, $h(u_1,v_1;\xi_1,\eta_1)$ is the complex-amplitude impulse response of the imaging system. After substitution of Eq. (2) into Eq. (3) we obtain the distribution of the Stokes parameters for polarization imaging that can be estimated for various separations of the two emitters and various degrees of polarization coherence. Those are:

$$\left\{\begin{array}{l}
S_0=I\left\{\left|\frac{2\sigma J_1\left[k\sigma(\rho-\varepsilon')/f_q\right]}{\left[k\sigma(\rho-\varepsilon')/f_q\right]}\right|^2+\left|\frac{2\sigma J_1\left[k\sigma(\rho+\varepsilon')/f_q\right]}{\left[k\sigma(\rho+\varepsilon')/f_q\right]}\right|^2+2\,\mathrm{Re}\left\{\mu(2\varepsilon')\frac{2\sigma J_1\left[k\sigma(\rho-\varepsilon')/f_q\right]}{\left[k\sigma(\rho-\varepsilon')/f_q\right]}\frac{2\sigma J_1\left[k\sigma(\rho+\varepsilon')/f_q\right]}{\left[k\sigma(\rho+\varepsilon')/f_q\right]}\right\}\right\}\\
S_1=I\left\{s_1\left[\left|\frac{2\sigma J_1\left[k\sigma(\rho-\varepsilon')/f_q\right]}{\left[k\sigma(\rho-\varepsilon')/f_q\right]}\right|^2+\left|\frac{2\sigma J_1\left[k\sigma(\rho+\varepsilon')/f_q\right]}{\left[k\sigma(\rho+\varepsilon')/f_q\right]}\right|^2\right]+2\,\mathrm{Re}\left\{\mu(2\varepsilon')\frac{2\sigma J_1\left[k\sigma(\rho-\varepsilon')/f_q\right]}{\left[k\sigma(\rho-\varepsilon')/f_q\right]}\frac{2\sigma J_1\left[k\sigma(\rho+\varepsilon')/f_q\right]}{\left[k\sigma(\rho+\varepsilon')/f_q\right]}\right\}\right\}\\
S_2=I\left\{s_2\left[\left|\frac{2\sigma J_1\left[k\sigma(\rho-\varepsilon')/f_q\right]}{\left[k\sigma(\rho-\varepsilon')/f_q\right]}\right|^2+\left|\frac{2\sigma J_1\left[k\sigma(\rho+\varepsilon')/f_q\right]}{\left[k\sigma(\rho+\varepsilon')/f_q\right]}\right|^2\right]+2\,\mathrm{Re}\left\{\mu(2\varepsilon')\frac{2\sigma J_1\left[k\sigma(\rho-\varepsilon')/f_q\right]}{\left[k\sigma(\rho-\varepsilon')/f_q\right]}\frac{2\sigma J_1\left[k\sigma(\rho+\varepsilon')/f_q\right]}{\left[k\sigma(\rho+\varepsilon')/f_q\right]}\right\}\right\}\\
S_3=I\left\{s_3\left[\left|\frac{2\sigma J_1\left[k\sigma(\rho-\varepsilon')/f_q\right]}{\left[k\sigma(\rho-\varepsilon')/f_q\right]}\right|^2+\left|\frac{2\sigma J_1\left[k\sigma(\rho+\varepsilon')/f_q\right]}{\left[k\sigma(\rho+\varepsilon')/f_q\right]}\right|^2\right]+2\,\mathrm{Im}\left\{\mu(2\varepsilon')\frac{2\sigma J_1\left[k\sigma(\rho-\varepsilon')/f_q\right]}{\left[k\sigma(\rho-\varepsilon')/f_q\right]}\frac{2\sigma J_1\left[k\sigma(\rho+\varepsilon')/f_q\right]}{\left[k\sigma(\rho+\varepsilon')/f_q\right]}\right\}\right\}
\end{array}\right. \tag{4}$$

Where, s_l is the degree of polarization (l=1,2,3), σ is the radius of the exit pupil, $J_1(\ldots)$ is first order the Bessel Function of the first kind, f_q is the distance from the exit pupil to the image plane, $k=2\pi/\lambda$ is the wavevector, λ is the wavelength of the illuminating quasi-monochromatic light, ρ is the radial coordinate distance, ε' is the distance from the image point to the optical axis, μ is the degree of coherence, and $\mathrm{Re}\{\ldots\}$ and $\mathrm{Im}\{\ldots\}$ denote the real and imaginary parts, respectively.

3. SPATIAL RESOLUTION FOR POLARIZATION IMAGE

To give a complete investigation of polarization imaging with all the Stokes parameters and to highlight the unique characteristics of the polarization imaging with S_l (l=0, 1,2,3), we studied a fixed-point separation imaging system where both the degree of coherence and the degree of polarization were varied. Figure 1 shows the image intensity distribution of the Stokes parameter S_0 S_1, S_2, and S_3 in polarization image of two points for various values, from μ=-1.0 to 1.0 in steps of 0.2 when s_0 is 1, s_1 is $1/\sqrt{2}$, s_2 is $-1/\sqrt{3}$, and s_3 is $1/\sqrt{6}$ respectively. In this example, the separation has been chosen as $1.3\lambda f_q/\sigma$ and the locations of the image for the two points are given by geometrical optics. Fig. 1(a) shows the polarization image of S_0, which exhibits similar characteristics of scalar optics case and is always positive. Note that when μ=-1.0, the two points are illuminated coherently but with a π phase difference. The resolution decreases with increasing μ until, μ=0.6, where the distribution curve for the resultant S_0 is essentially flat with no indication of a dip in illuminance. Unlike the Stokes parameter S_0, the rest of Stokes parameters S_l (l=1,2,3) are not necessarily positive. When the S_l is negative, we can resolve two points from the two negative-valued valleys, and find that when flipping the illumination condition from (μ, s_l) to ($-\mu$, $-s_l$), the two groups of curves are symmetric about zero. It is interesting to note that no matter what μ is, all distribution curves of Stokes parameters S_3 overlap each other and can be resolved.

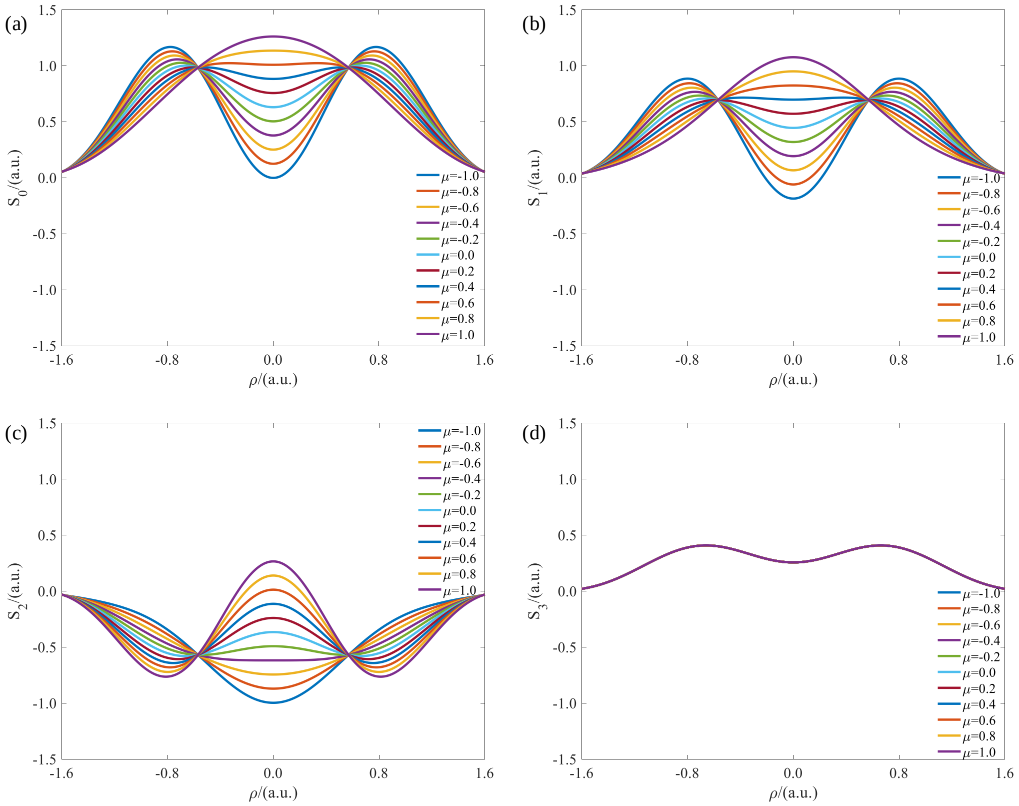

Fig. 1 Image illuminance distribution of S_0, S_1, S_2, and S_3 in polarization image of two points for various values from μ=-1.0 to 1.0 in steps of 0.2 when s_0 is 1(a), s_1 is $1/\sqrt{2}$ (b), s_2 is $-1/\sqrt{3}$ (c), and s_3 is $1/\sqrt{6}$ (d) respectively.

Note the fact that the only measurable quantity spatial resolution in polarization imaging is the separation of two points in normalized Stokes parameters S_l (l=1,2,3). We use a dimensionless parameter R_{S_l} to present the two-point resolution for polarization imaging, R_{S_l} is defined as the ratio of the measured separation of the twin peaks of the resultant Stokes parameter to the true separation of the two image points formed by geometrical optics free from the effect of diffraction. Fig. 2 shows the ratio R_{S_l} as a function of the true separation of two image points for various values of μ where all these curves oscillate around the value $R_{S_l}=1$. When μ=-1.0, the ratio R never becomes zero regardless of their separation, indicating that these two points are always resolvable in polarization imaging. Compared with Fig. 2(b) and Fig.2(c), we found that when flipping the illumination condition from (μ, s_l) to (-μ, -s_l), the two curves for R_{S_l} are identical indicating the same spatial resolution for the Stokes parameters S_l (l=1,2). It is interesting to note that no matter what μ is, all ratio curves of Stokes parameters S_3 overlap each other.

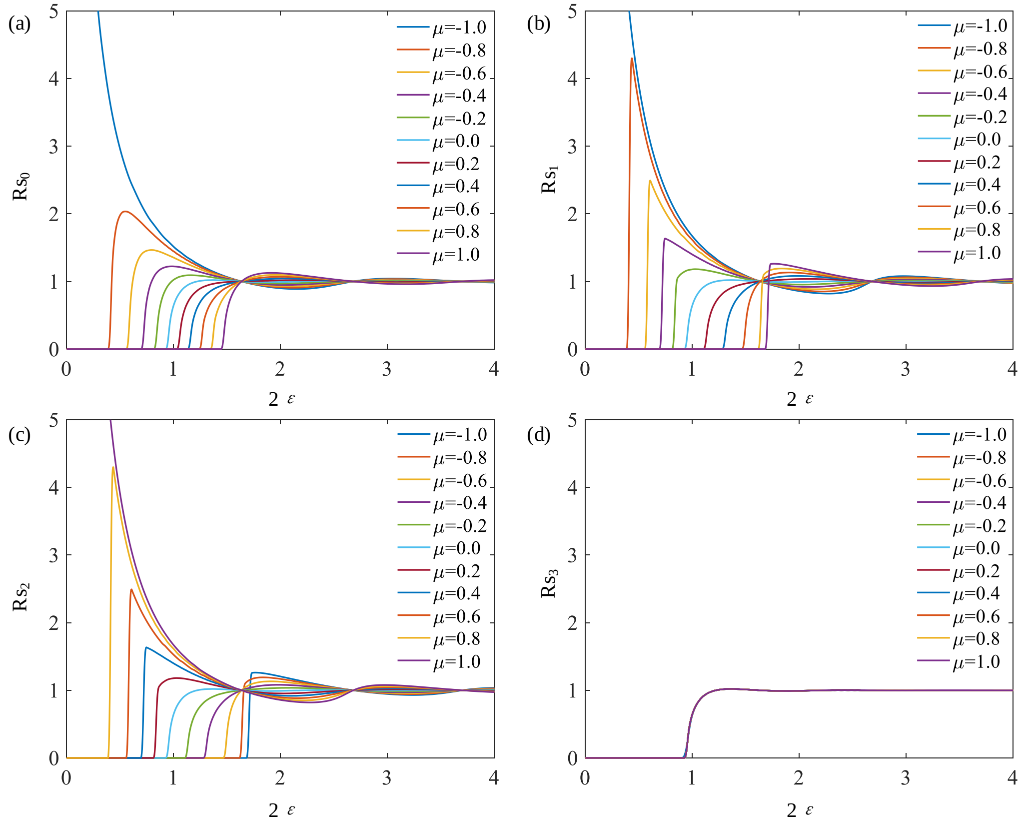

Fig. 2 The ratio Rs_0, Rs_1, Rs_2, and Rs_3, the ratio of the measured to the actual separation of two images as a function of the real separation 2ε for a diffraction-limited circularly symmetric system. The polarization image illuminance distribution varies with values from μ=-1.0 to 1.0 in steps of 0.2 when s_0 is 1(a), s_1 is $1/\sqrt{2}$ (b), s_2 is $1/\sqrt{3}$ (c), and s_3 is $1/\sqrt{6}$ (d) respectively.

4. CONCLUSIONS

The only measurable quantity of two-point resolution depends strongly upon the coherence and polarization of the illumination. Based on the unified theory of polarization and coherence, we provided a novel optical system for the full-field visualization of partially coherent and partial polarization tensors. Two points resolvable with certain Stokes parameters cannot always be resolved for other Stokes parameters. The ratio of the actual to the measured separation is not equal to unity except for special values of the actual separation. The criteria have been proposed for the accuracy for two-point resolution in partially coherent and partially polarized imaging system.

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