Statistics of random polarization in sea clutter with Weibull-distribution-intensity

Citation for published version:
Luo, Z, Pan, Z, Wang, Y & Wang, W 2024, Statistics of random polarization in sea clutter with Weibull-distribution-intensity. in OV Angelsky & CY Zenkova (eds), *Sixteenth International Conference on Correlation Optics.*, 129380O, Proceedings of SPIE, vol. 12938, SPIE, 16th International Conference on Correlation Optics 2023, Chernivtsi, Ukraine, 18/09/23. https://doi.org/10.1117/12.3009732

Digital Object Identifier (DOI):
10.1117/12.3009732

Link:
Link to publication record in Heriot-Watt Research Portal

Document Version:
Publisher's PDF, also known as Version of record

Published In:
Sixteenth International Conference on Correlation Optics

Statistics of random polarization in sea clutter with Weibull-distribution-intensity

Zuishuang Luo, [a] Zhongbao Pan, [a] Ying Wang, [a] and Wei Wang [b,*]
[a] Department of Physics, Jiangsu University of Science and Technology, Zhenjiang, China 212003;
[b] School of Engineering and Physical Sciences, Heriot-Watt University, Edinburgh, UK EH14 4AS
Corresponding Author: w.wang@hw.ac.uk

ABSTRACT

In some applications, including but not limited to synthetic-aperture radar (SAR) imagery and wireless communication, the Weibull distribution is more flexible than the exponential distribution for these purposes, because it fits particularly well at the heavy tail of the sea clutter. In this paper, we conduct a statistical analysis of Stokes parameters based on random polarization phasor sums. We provide a comprehensive analysis for the statistical properties of the Stokes parameters in stochastic electric field with a Weibull-distribution intensity. The presented statistics of the Weibull-distribution optical field will be useful to optical scattering from random birefringent medium and polarization speckle imaging, which can also be extended to deal with the characterization of the scattering matrix in radar polarimetry.
Keywords: Weibull distribution, polarization phasor sum, statistics of the Stokes parameters

1. INTRODUCTION

The multivariate models are generally compatible with the compound Gaussian model, which has received wide attention in the recent past due to its theoretical and physical justification. Moreover, on-field measurements have also confirmed that clutter in HRRs (high-resolution radars, possibly at low grazing angles) can be modeled as a compound Gaussian process. This process consists of a zero-mean Gaussian signal, known as the speckle component [1]. Additionally, in K-distributed SAR images, the different polarimetric channels may not exhibit homogeneous texture [2]. Furthermore, the speckle may not follow a Gaussian distribution; instead, it can exhibit a Weibull distribution [3]. A requirement arises for statistical characteristics of polarization in general clutter, such as Weibull clutter [4]. The study's statistical examination of the Stokes parameters in random light fields, resulting from the cumulative phase of finite Weibull distribution amplitude steps as derived from random vector fields, carries significant importance in the context of modeling optical signal processing and addressing related issues.

2. STATISTICAL PROPERTIES OF STOKES PARAMETERS FOR RANDOM PHASE SUMS

The Weibull distribution can have various distribution forms, depending on the parameter values taken. In the context of speckle theory [5], the Weibull density function has non-zero values only in its definition domain, so the probability density function whose intensity follows the Weibull distribution is therefore expressed as follows:

Sixteenth International Conference on Correlation Optics, edited by
Oleg V. Angelsky, Claudia Yu. Zenkova, Proc. of SPIE Vol. 12938,
129380O · © 2024 SPIE · 0277-786X · doi: 10.1117/12.3009732

$$p_I(I)=\frac{\Gamma(1+\alpha^{-1})\alpha}{\bar{I}}\left(\frac{\Gamma(1+\alpha^{-1})I}{\bar{I}}\right)^{\alpha-1}\exp\left[-\left(\Gamma(1+\alpha^{-1})\frac{I}{\bar{I}}\right)^{\alpha}\right] \tag{1}$$

We let $\bar{I}=\beta\cdot\Gamma(1+\alpha^{-1})$ in this paper, and the typical parameter of $\alpha=1$.

2.1 Statistics of S_0 and S_1

The partially polarized speckle is composed of two normal speckles with orthogonal polarization directions, and the light intensity of the partially polarized speckle I can be expressed as the sum of the light intensity of the above two normal speckles,i.e [6]

$$I=I_x+I_y \tag{2}$$

The degree of polarization $\mathcal{P}$ is introduced and the relationship between the intensity statistics of light polarized in different directions, we can obtain the respective probability density functions of the two components I_x and I_y :

$$\begin{aligned}p_{I_x}(I_x)&=\frac{\alpha}{I_x}\left(\frac{2I_x\cdot\Gamma(1+\alpha^{-1})}{(1+P)\overline{S_0}}\right)^{\alpha}\exp\left[-\left(\Gamma(1+\alpha^{-1})\frac{2I_x}{(1+P)\overline{S_0}}\right)^{\alpha}\right]\\ p_{I_y}(I_y)&=\frac{\alpha}{I_y}\left(\frac{2I_y\cdot\Gamma(1+\alpha^{-1})}{(1-P)\overline{S_0}}\right)^{\alpha}\exp\left[-\left(\Gamma(1+\alpha^{-1})\frac{2I_y}{(1-P)\overline{S_0}}\right)^{\alpha}\right]\end{aligned} \tag{3}$$

The Stokes parameter S_0 is the total intensity in the speckle field. Its probability density function can be obtained by the convolution of two components of the probability density function p_{Ix} and p_{Iy}. That is:

$$p_{S_0}=\int_0^{S_0}p_{I_x}(u)p_{I_y}(S_0-u)du \tag{4}$$

Figure 1 shows the probability density function of S_0 after normalization at the different number of $\mathcal{P}$.

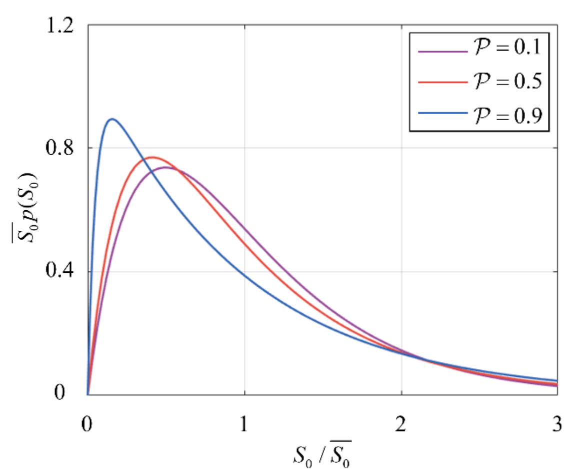

Figure1. The intensity probability density function of S_0 with the different number of $\mathcal{P}$.

The Stokes parameter S_1 characterizes whether the light is more closely polarized in the x-direction or the y-direction. For

S_1, we have the following equation:

$$p_{S_1}(S_1) = \begin{cases} \int_{S_1}^{\infty} p_{I_x}(u) p_{I_y}(u - S_1) du, & S_1 \geq 0 \\ \int_{0}^{\infty} p_{I_x}(u) p_{I_y}(u - S_1) du, & S_1 < 0 \end{cases} \tag{5}$$

Substitute Eq. (5) into Eq. (7), after the integration operation, we can obtain the probability density function of S_1 with the different number of $\mathcal{P}$:

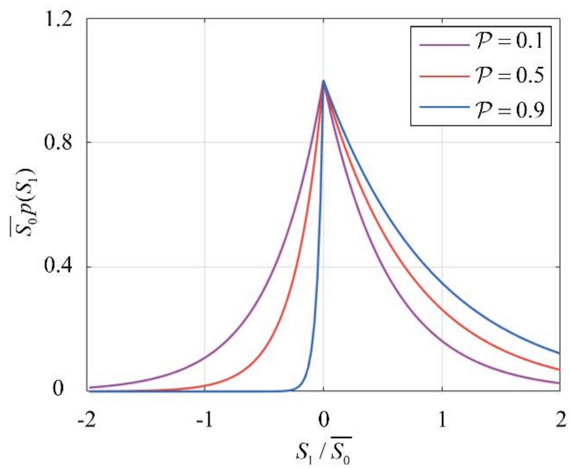

Figure2. The intensity probability density function of S_1 with the different number of $\mathcal{P}$.

2.2 Statistics of S_2 and S_3

Next we perform the statistical calculation for S_2 and S_3. Here we use the monotone transformation $A = \sqrt{I}$ and bring it into Eq. (5) to obtain the probability density function of the amplitude component:

$$\begin{aligned} p_{A_x}(A_x) &= \frac{2\alpha}{A_x}\left(\Gamma(1+\alpha^{-1})\frac{2A_x^{\ 2}}{(1+P)\overline{S_0}}\right)^{\alpha} \exp\left[-\left(\Gamma(1+\alpha^{-1})\frac{2A_x^{\ 2}}{(1+P)\overline{S_0}}\right)^{\alpha}\right] \\ p_{A_y}(A_y) &= \frac{2\alpha}{A_y}\left(\Gamma(1+\alpha^{-1})\frac{2A_y^{\ 2}}{(1-P)\overline{S_0}}\right)^{\alpha} \exp\left[-\left(\Gamma(1+\alpha^{-1})\frac{2A_y^{\ 2}}{(1-P)\overline{S_0}}\right)^{\alpha}\right] \end{aligned} \tag{6}$$

By an integral transformation $\gamma = A_x A_y$, The probability density function of $p_\gamma(\gamma)$ can be expressed as:

$$p_\gamma(\gamma) = \int_0^\infty p_{A_x}(A_x) p_{A_y}(\gamma / A_x) A_x^{-1} dA_x \tag{7}$$

Then we can get the probability density function of S_2 by the following formula:

$$p_{S_2}(S_2) = \int_{-1}^{1} p_\gamma(S_2 / 2\xi)(2\pi|\xi|\sqrt{1-\xi^2})^{-1} d\xi \tag{8}$$

S_2 characterizes whether the light is closer to the positive 45 degree polarization or the negative 45 degree polarization; S_3 characterizes whether the light is closer to the right-handed circularly polarized light or the left-handed circularly polarized light, so they have the same probability density function distribution. As shown in the following figure:

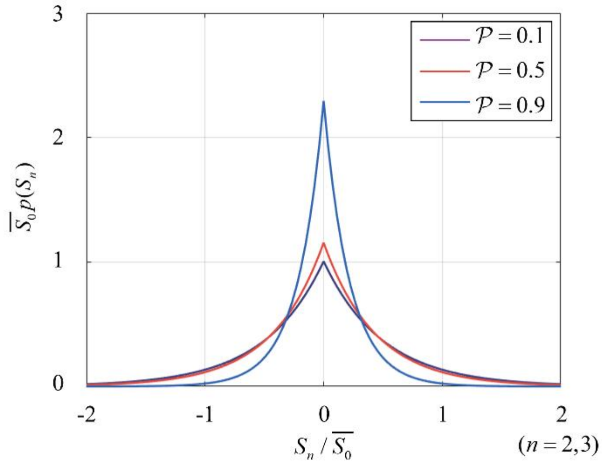

Figure3. The intensity probability density function of S_2 or S_3 with the different number of $\mathcal{P}$.

3. CONCLUSION

This study delves into the statistical characteristics of random light fields following a Weibull distribution and provides an accurate expression of the Stokes parameters for the random polarization phasor sum that characterizes the fluctuation of polarization. Through comparative analysis, the statistical theory of the random light fields is applied to the field of optics, which can provide a more reasonable level of accuracy for appropriate estimation. The new statistical theory can be seen as a development and extension of the traditional theory, offering greater insight into the evolution of random electromagnetic fields. The simple expression of the first normalized Stokes parameter makes it applicable to target detection and recognition. Applying the Weibull distribution model to radar detection can offer a more reasonable level of accuracy for appropriate estimation and contribute to the development of high-resolution radar.

REFERENCES

[1] E. Conte, A. De Maio, and C. Galdi, "Statistical analysis of real clutter at different range resolutions," IEEE Transactions on Aerospace and Electronic Systems 40(3), 903-918 (2004).

[2] S. H. Yueh, J. A. Kong, J. K. Jao, R. T. Shin, and L. M. Novak, "K-distribution and polarimetric terrain radar clutter," J. Electromagn. Waves Appl 3(8), 747-768(1989).

[3] V. Anastassopoulos, G. A. Lampropoulos, A. Drosopoulos, and M.Ray, "High resolution radar clutter statistics," IEEE Trans. Aerosp.Electron. Syst 35(1), 43–60 (1999).

[4] T. Liu, G. Huang, X. Wang, and S. Xiao, "Statistics of the Polarimetric Weibull-Distributed Electromagnetic Wave," IEEE Transactions on Antennas and Propagation 57(10), 3232-3248(2009).

[5] Goodman, J. W, "Speckle with a finite number of steps," Applied optics 47(4), 111-118 (2008),

[6] K. D. Ward, R. J. A. Tough, and S. Watts, "Sea clutter: Scattering, the K distribution and radar performance," Waves in Random and Complex Media 17(2), 233-234 (2007).