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Towards Diagnostics of Random Polarization: How Stokes Correlations Provide Information Content of Stochastic Optical Fields

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ABSTRACT

In this paper, we reviewed our recent work on the correlations of the Stokes parameters and their applications to diagnose random polarization in stochastic optical fields. With the aid of the ensemble-average van Cittert-Zernike theorem for the propagation of polar-coherence, we investigated the autocorrelation functions and power spectra of the Stokes parameters to expose the dependence of the polarization-related scale-size distributions. A generalized concept of the Stokes ensemble-average coherence areas has been introduced to deal with the polarization-related average areas associated with polarization speckle. Noting the fact that the Stokes parameters cannot be measured at an ideal point, we also made investigation of the means and variances of the integrated Stokes parameters in polarization speckle after introduction of four parameters, i.e., the numbers of degrees of freedom for the Stokes detection. Furthermore, a new scheme of interferometry referred to as the Stokes Vector Interferometry has been proposed as an extension of intensity interferometry by taking the vector nature of electromagnetic field into account. Through the cross-covariance computations for the fluctuating Stokes parameters, the squared moduli of the generalized Stokes parameters are reconstructed for the first time.

Keywords: Polarization speckle, random polarization, Correlation of Stokes parameters, statistical optics

1. INTRODUCTION

Since continuous-wave lasers became commercially available in the early 1960s, extensive studies have been made on their basic properties and applications of laser speckle [1-3]. The term *speckle patterns* is usually associated with the fine-scale granular distribution of a light intensity pattern that arises from the interference of coherently superposed multiple random optical fields. In the majority of studies on speckle phenomena, these random optical fields have been treated as scalar optical fields, and the main interest has been in the statistical properties and applications of the intensity distribution of the speckle patterns. Recently, statistical properties of random electric vector fields have come to attract new interest of researchers because of their importance in wide areas of practical applications such as biology and metrology. Statistical phenomena of random electric vector fields referred to as *Polarization Speckle* have relevance to the theories of speckles, polarization and coherence [4]. Much effort is now being made by researchers to establish a new realm of statistical optics based on a unified theory on speckles, coherence and polarization [5]. Clearly it is far beyond the ability of the authors to cover all the subjects related to the phenomena of stochastic electric fields. We will therefore restrict ourselves to the narrow-scope review on some of our recent works on the correlations of the Stokes parameters and their applications to study the statistical properties of polarization-related speckle phenomena, along with the autocorrelation functions and power spectra of the Stokes parameters [6], and the integrated Stokes parameters for detection of polarization speckle [7]. Furthermore, we will also introduce a new scheme of interferometry referred to as the Stokes Vector Interferometry to explore the higher-order polar-coherence properties of polarization speckle [8].

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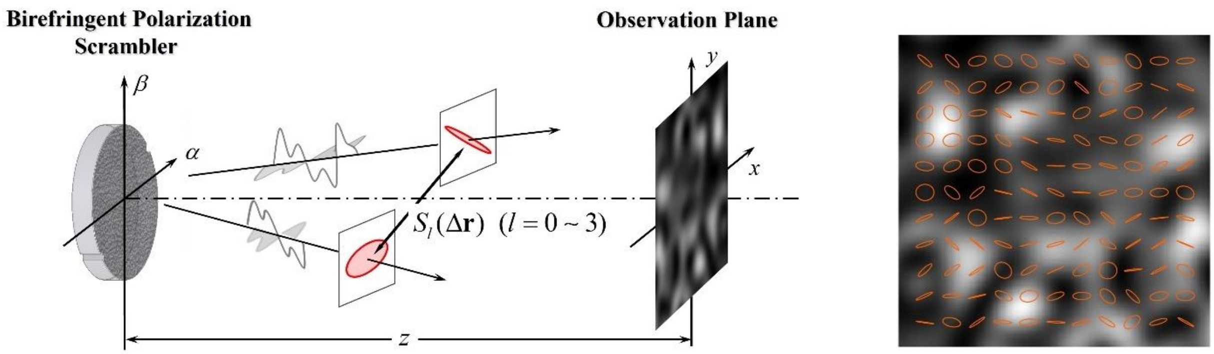
2. AUTOCORRELATION FUNCTIONS AND POWER SPECTRA OF THE STOKES PARAMETERS OF PLARIZATION SPECKLE

Let the light from an ideally stabilized monochromatic continuous-wave (CW) laser illuminate a stationary birefringent polarization scrambler [9,10], and consider the polarization and coherence properties of light some distance beyond the depolarizing diffuser, as shown in Fig. 1(a). Due to the complicated and unknown birefringent microstructure of the diffuser itself, the optical field behind the diffuser is intricate and its intensity and state-of-polarization give an unpredictable pattern, as shown in Fig.1(b). Note the fact that the spatial variations of both intensity and state-of-polarization in a polarization speckle pattern are random, our interest here is in the coarseness of polarization-related spatial structures and the distributions of scale sizes in their random spatial fluctuations of the Stokes parameters. In this section, we consider some important aspects of the spatial structure of polarization speckle, namely, the autocorrelation functions and the power spectral densities of the Stokes parameters of polarization speckle in a free-space scattering geometry.



(a) (b)

Figure 1 (a) Generation of polarization speckle by an ideally stabilized CW laser and a stationary birefringent polarization scrambler. (b) Example of polarization speckle with the spatial variations of polarization ellipses and the fluctuations of intensity.

We consider the case when a coherent source illuminates a birefringent polarization scrambler, and the scattered light is observed some distance z from that surface. The birefringent polarization scrambler is assumed to be stationary in time without specifying its rough surface or birefringent microstructure. Over an ensemble of ideally rough surfaces with a very short lateral correlation width of surface height fluctuations, there is little relationship between the phase differences of two polarization components of the light scattered from two closely spaced surface elements, at least until the spacing becomes close to exceeding the wavelength of the illuminating light. From an ensemble-averaging point of view, the ensemble-average generalized Stokes vector of the light scattered by a depolarizing diffuser, and observed very close to that surface, is Dirac delta-correlated with essentially the same correlation extent as the generalized Stokes vector of an incoherent source. Mathematically, we can represent the ensemble-average generalized Stokes vector of light just leaving the surface by $\mathbf{S}^{\mathrm{Sca}}(\alpha_1,\beta_1;\alpha_2,\beta_2)=\kappa\mathbf{S}^{\mathrm{Sca}}(\alpha_1,\beta_1)\delta(\alpha_1-\alpha_2,\beta_1-\beta_2)$, where κ is a constant with dimensions meters squared, $\mathbf{S}^{\mathrm{Sca}}$ with its superscript Sca indicates the ensemble-average Stokes vector of the scattered light just leaving the rough surface of the depolarizing diffuser and δ is the 2D delta function. With the aid of the van Cittert-Zernike theorem, we can express the propagation of the ensemble-average polar-coherence in terms of the generalized Stokes parameters. That is [5]

$$\mathbf{S}^{\mathrm{Obs}}(x_1,y_1;x_2,y_2)=\frac{\kappa e^{-j\psi}}{(\lambda z)^2}\iint_{-\infty}^{\infty}\mathbf{S}^{\mathrm{Sca}}(\alpha,\beta)\exp\left[\frac{j2\pi}{\lambda z}(\Delta x\alpha+\Delta y\beta)\right]d\alpha d\beta, \tag{1}$$

where the superscript Obs indicates the observation plane, $\psi=\pi[(x_2^2+y_2^2)-(x_1^2+y_1^2)]/(\lambda z)$, and $\Delta x=x_2-x_1,\Delta y=y_2-y_1$, and λ is the wavelength of the incident radiation. Up to a scaling constant, the ensemble-average generalized Stokes vector is given by Fourier transforms of the distributions of the ensemble-average Stokes vector leaving the surface of the birefringent polarization scrambler.

Just as the autocorrelation function of intensity has been widely adopted to characterize the average scale-size of a scalar speckle, a suitable description for the fluctuating state-of-polarization in polarization speckle will be the autocorrelation functions of the Stokes parameters at two points, ensemble-average quantities, which are defined by

$$\Gamma_{S_l}(\Delta x, \Delta y) = \overline{S_l(x_1, y_1) S_l(x_2, y_2)}, \tag{2}$$

for $l = 0 \sim 3$ and the overbar represents the ensemble average. When Eq. (2) is written, we have made use of an assumption that the autocorrelation functions of the Stokes parameters depend only on the difference of observation coordinates. Note that two polarization components of a polarization speckle field are complex Gaussian random processes. From the complex Gaussian moment theorem [11], we have

$$\begin{aligned}
\Gamma_{S_0}(\Delta x, \Delta y) &= \overline{S_0}^2 + \tfrac{1}{2}[|S_0(\Delta x, \Delta y)|^2 + |S_1(\Delta x, \Delta y)|^2 + |S_2(\Delta x, \Delta y)|^2 + |S_3(\Delta x, \Delta y)|^2] \\
\Gamma_{S_1}(\Delta x, \Delta y) &= \overline{S_1}^2 + \tfrac{1}{2}[|S_0(\Delta x, \Delta y)|^2 + |S_1(\Delta x, \Delta y)|^2 - |S_2(\Delta x, \Delta y)|^2 - |S_3(\Delta x, \Delta y)|^2] \\
\Gamma_{S_2}(\Delta x, \Delta y) &= \overline{S_2}^2 + \tfrac{1}{2}[|S_0(\Delta x, \Delta y)|^2 - |S_1(\Delta x, \Delta y)|^2 + |S_2(\Delta x, \Delta y)|^2 - |S_3(\Delta x, \Delta y)|^2] \\
\Gamma_{S_3}(\Delta x, \Delta y) &= \overline{S_3}^2 + \tfrac{1}{2}[|S_0(\Delta x, \Delta y)|^2 - |S_1(\Delta x, \Delta y)|^2 - |S_2(\Delta x, \Delta y)|^2 + |S_3(\Delta x, \Delta y)|^2],
\end{aligned} \tag{3}$$

where $\overline{S_l}$ and $S_l(\Delta x, \Delta y)$ for $l = 0 \sim 3$ are the ensemble-average Stokes parameters and the ensemble-average generalized Stokes parameters of light in the observation plane, respectively. Here, the above Stokes autocorrelations expressed in terms of the (generalized) Stokes parameters have revealed the complicated relationship between the local polarization (provided by the Stokes parameters at a certain point) and all the possible polar-coherence (provided by the generalized Stokes parameters at two points). The first term in each expression stems from the contribution of the corresponding ensemble-averaged Stokes parameter. While the second term, consisting of a combination of the generalized Stokes parameters, indicates that each Stokes autocorrelation depends not only on its corresponding generalized Stokes parameter but also on other generalized Stokes parameters. Therefore, the second term(s) give the autocoavariances, where the DC-vaues are subtracted. Note from Eq. (3) that there is a high redundency among four terms with similar expressions. To simplify the expressions and highlight the underlying physics of the Stokes autocorrelations, we define four linear transforms as follows:

$$\begin{aligned}
&\mathcal{L}_0\{g_0, g_1, g_2, g_3\} = g_0 + g_1 + g_2 + g_3, \quad \mathcal{L}_1\{g_0, g_1, g_2, g_3\} = g_0 + g_1 - g_2 - g_3, \\
&\mathcal{L}_2\{g_0, g_1, g_2, g_3\} = g_0 - g_1 + g_2 - g_3, \quad \mathcal{L}_3\{g_0, g_1, g_2, g_3\} = g_0 - g_1 - g_2 + g_3.
\end{aligned} \tag{4}$$

with g_l (for $l = 0 \sim 3$) being an expression. Let us also introduce formally the normalized ensemble-average, generalized Stokes vector by the Frobenius norm of the ensemble-average Stokes vector

$$\boldsymbol{\gamma}_S(\Delta x, \Delta y) = \mathbf{S}(\Delta x, \Delta y) \big/ \|\mathbf{S}(0,0)\| = \mathbf{S}(\Delta x, \Delta y) \big/ \left(\overline{S_0}\sqrt{1+\mathcal{P}^2}\right), \tag{5}$$

where the denominator $\|\mathbf{S}(0,0)\| = \|\overline{\mathbf{S}}\|$ is the Frobenius norm of the ensemble-average Stokes vector. When Eq. (5) is derived, we have made use of the definition for the degree of ensemble-average polarization: $\mathcal{P} = \sqrt{\overline{S_1}^2 + \overline{S_2}^2 + \overline{S_3}^2} \big/ \overline{S_0}$. With the aid of the normalized ensemble-average Stokes vector, $\hat{\overline{\mathbf{S}}} = \overline{\mathbf{S}} / \|\overline{\mathbf{S}}\|$, we are now ready to present the autocorrelation functions of the Stokes parameters of the polarization speckle in the observation plane:

$$\Gamma_{S_l}(\Delta x, \Delta y) = (1+\mathcal{P}^2)\overline{S_0}^2 \left[\hat{\overline{S_l}}^2 + 2^{-1}\mathcal{L}_l\left\{|\gamma_{S_0}(\Delta x, \Delta y)|^2, |\gamma_{S_1}(\Delta x, \Delta y)|^2, |\gamma_{S_2}(\Delta x, \Delta y)|^2, |\gamma_{S_3}(\Delta x, \Delta y)|^2\right\} \right], \tag{6}$$

for $l = 0 \sim 3$.

The power spectral densities of the Stokes parameters $\mathcal{G}_{S_l}(f_x, f_y)$ for $l = 0 \sim 3$ in a polarization speckle pattern are also ensemble-average quantities, representing the spatial power distributions of fluctuating Stokes parameters over the 2D frequency plane, and are given by the Fourier transforms of the corresponding autocorrelation functions:

$$\mathcal{G}_{S_l}(f_x, f_y) = \iint_{-\infty}^{\infty} \Gamma_{S_l}(\Delta x, \Delta y) \exp[j2\pi(\Delta x f_x + \Delta y f_y)] d\Delta x d\Delta y. \quad (7)$$

With the help of the autocorrelation theorem [12], the power spectral densities of the Stokes parameters can be reduced to

$$\begin{aligned} \mathcal{G}_{S_l}(f_x, f_y) &= {}_{\mathrm{I}}\mathcal{G}_{S_l}(f_x, f_y) + {}_{\mathrm{II}}\mathcal{G}_{S_l}(f_x, f_y) \\ &= (1+\mathcal{P}^2)\overline{S_0}^2 \left[\overline{S_l}^2 \delta(f_x, f_y) + (\lambda z)^2 / (2\mathcal{Q}) \mathcal{L}_l \left\{ \mathcal{R}_{S_0^{\mathrm{Sca}}}(f_x, f_y), \mathcal{R}_{S_1^{\mathrm{Sca}}}(f_x, f_y), \mathcal{R}_{S_2^{\mathrm{Sca}}}(f_x, f_y), \mathcal{R}_{S_3^{\mathrm{Sca}}}(f_x, f_y) \right\} \right], \end{aligned} \quad (8)$$

where $\mathcal{R}_{S_l^{\mathrm{Sca}}}(f_x, f_y) = \iint_{-\infty}^{\infty} S_l^{\mathrm{Sca}}(\alpha, \beta) S_l^{\mathrm{Sca}}(\alpha - \lambda z f_x, \beta - \lambda z f_y) d\alpha d\beta$ and $\mathcal{Q} = \sum_{m=0}^{3} \left[\iint_{-\infty}^{\infty} S_m^{\mathrm{Sca}}(\alpha, \beta) d\alpha d\beta \right]^2$. The first terms in Eq. (8) (denoted by the symbols ${}_{\mathrm{I}}\mathcal{G}_{S_l}$) are the Dirac delta functions corresponding to the zero-frequency discrete powers contributed by the ensemble-averaged Stokes parameters S_l^{Sca} of the scattering spot. The second terms (denoted by the symbols ${}_{\mathrm{II}}\mathcal{G}_{S_l}$), consisting of different combinations of the normalized and scaled autocorrelation functions of the Stokes parameters $S_l^{\mathrm{Sca}}(\alpha, \beta)$ of the scattering spot, represent the distributions of varying powers over spatial frequency for the fluctuating parts of the Stokes parameters in the polarization speckle pattern.

To illustrate the results above, we use an example of circular uniform distributions of the Stokes parameters at the scattering spot with its diameter D . With a proper normalization, the autocorrelation functions of the Stokes parameters of the polarization speckle in the free-space scattering geometry become

$$\Gamma_{S_l}(\Delta x, \Delta y) = (1+\overline{\mathcal{P}}^2)\overline{S_0}^2 \left\{ \overline{S_l}^2 + \tfrac{1}{2}\mathcal{L}_l \left\{ \overline{S_0}^2, \overline{S_1}^2, \overline{S_2}^2, \overline{S_3}^2 \right\} \left| 2J_1\left(\pi D \sqrt{\Delta x^2 + \Delta y^2} / (\lambda z) \right) \Big/ \left[\pi D \sqrt{\Delta x^2 + \Delta y^2} / (\lambda z) \right] \right|^2 \right\}, \quad (9)$$

where $J_1(\cdots)$ is a Bessel function of the first kind, of order one. It is interesting to note that the polarization speckle maintains the degree of polarization during its free-space propagation. The corresponding power spectral densities of the Stokes parameters of polarization speckle can then be shown to be

$$\mathcal{G}_{S_l}(f_x, f_y) = (1+\mathcal{P}^2)\overline{S_0}^2 \left\{ \overline{S_l}^2 \delta(f_x, f_y) + \tfrac{4(\lambda z)^2}{\pi^2 D^2} \mathcal{L}_l \left\{ \overline{S_0}^2, \overline{S_1}^2, \overline{S_2}^2, \overline{S_3}^2 \right\} \left[\arccos(\lambda z |\vec{f}| / D) - (\lambda z |\vec{f}| / D) \sqrt{1 - (\lambda z |\vec{f}| / D)^2} \right] \right\} \quad (10)$$

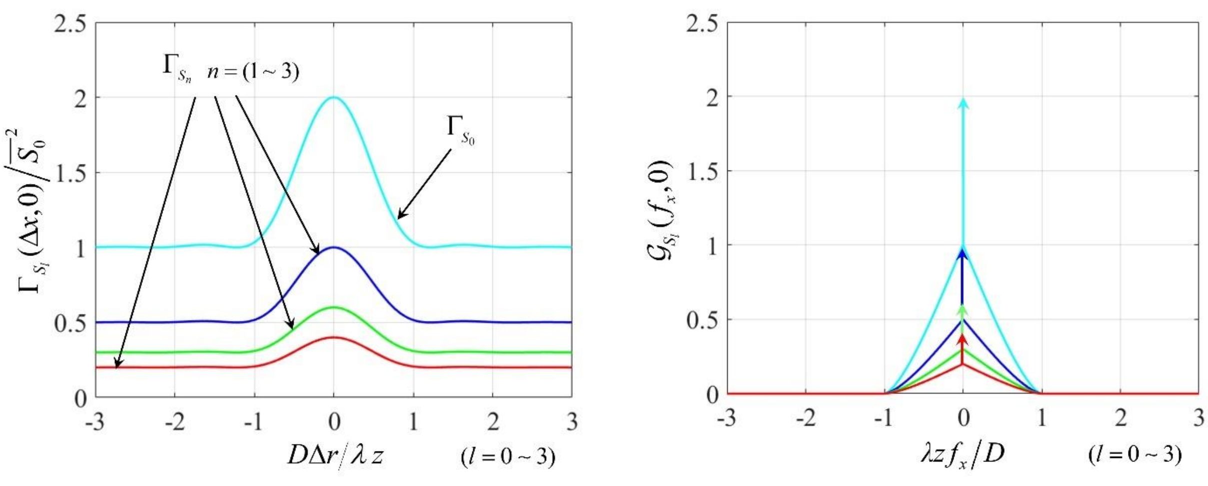


Figure 2 Cross sections of (a) autocorrelation functions and (b) power spectral densities of the Stokes parameters of polarization speckle.

Figure 2 shows cross-sections of the autocorrelations and power spectral densities of the Stokes parameters for polarization speckle with $\mathcal{P}=1$. In this example, a set of $\overline{S_l}$ (i.e., $\overline{S_0}=\sqrt{0.5},\overline{S_1}=-\sqrt{0.25},\overline{S_2}=\sqrt{0.15},\overline{S_3}=-\sqrt{0.1}$) has been chosen. We conclude that, in any polarization speckle, large-scale (low frequency) sizes are the most populous, and no scale sizes greater than a certain cutoff frequency are present. Due to the different means of the Stokes parameters in a polarization speckle, the areas under the Dirac delta functions at zero spatial frequency are different. Meanwhile, the exact power spectral densities of the Stokes parameters in a polarization speckle pattern also depend on the geometric shape of the scattering spot.

Similar to the case of conventional scalar speckle, where the normalized correlation function of speckle intensity has been adopted as the averaged speckle size, here we would like to develop a similar concept for measures of the average "sizes" for the Stokes parameters in a polarization speckle from the above example of calculation of the autocorrelation functions and power spectral densities. Since the normalized covariance function of the Stokes parameter S_0 (intensity of polarization speckle) is related to the autocorrelation function of the Stokes parameter S_0 by $[\Gamma_{S_0}(\Delta x,\Delta y)-\overline{S_0}^2]\big/[\Gamma_{S_0}(0,0)-\overline{S_0}^2]$. The equivalent area of S_0 detection in a polarization speckle is given by

$$\begin{aligned}\mathcal{A}_{S_0}^{\mathrm{C}}&=\iint_{-\infty}^{\infty}[\Gamma_{S_0}(\Delta x,\Delta y)-\overline{S_0}^2]\big/[\Gamma_{S_0}(0,0)-\overline{S_0}^2]\,d\Delta x d\Delta y\\&=\iint_{-\infty}^{\infty}\Big[|\gamma_{S_0}(\Delta x,\Delta x)|^2+|\gamma_{S_1}(\Delta x,\Delta x)|^2+|\gamma_{S_2}(\Delta x,\Delta x)|^2+|\gamma_{S_3}(\Delta x,\Delta x)|^2\Big]d\Delta x d\Delta y.\end{aligned}\tag{11}$$

Here, we call $\mathcal{A}_{S_0}^{\mathrm{C}}$ the Stokes S_0 correlation area or the ensemble-average coherence area of the Stokes parameter S_0 for a polarization speckle. Similarly, we can also introduce ensemble-average coherence areas for other Stokes parameters in a polarization speckle. Therefore, all the Stokes correlation areas $\mathcal{A}_{S_l}^{\mathrm{C}}$ for $l=0\sim3$ can be defined in a unified manner. They are

$$\mathcal{A}_{S_l}^{\mathrm{C}}=\iint_{-\infty}^{\infty}\mathcal{L}_l\left\{|\gamma_{S_0}(\Delta x,\Delta x)|^2,|\gamma_{S_1}(\Delta x,\Delta x)|^2,|\gamma_{S_2}(\Delta x,\Delta x)|^2,|\gamma_{S_3}(\Delta x,\Delta x)|^2\right\}d\Delta x d\Delta y.\tag{12}$$

As expected, we may have different correlation areas for detection of fluctuating Stokes parameters in a polarization speckle.

3. INTEGRATED STOKES PARAMETERS FOR DETECTION OF POLARIZATION SPECKLE

The Stokes parameters, observables in physics, cannot be measured at an ideal point or at an instant in time. In a variety of problems, including the experimental measurement of the Stokes parameters in a polarization speckle pattern, the detector aperture must of necessity be of finite size rather than an ideal point. Hence the measured Stokes parameters are always a somewhat smoothed or integrated version of the ideal point-Stokes parameters, and the statistics of the measured Stokes parameters will be somewhat different from the ideal statistics developed for stochastic electric fields. In addition, an entirely analogous problem arises in the study of polarization-sensitive photon-counting statistics for partially polarized thermal light, where finite-time integrals of instantaneous Stokes parameters occur. A similar effect is observed when the polarization speckle is for some reason blurred, as could be the case, for example, if the birefringent polarization scrambler (depolarizing diffuser) were in motion during the measurement. In this section, we will review our recent work on the integrated Stokes parameters for polarization speckle [7]. After introducing four parameters, referred to as the numbers of degrees of freedom for the Stokes detection, we derive the means and variances of the integrated Stokes parameters.

As the observables for representation of light polarization, the Stokes parameters S_l (for $l=0\sim3$) can be obtained from the measured intensities when different polarizing filters are inserted in front of a detector [3,5], i.e., $S_0=I_x+I_y$, $S_1=I_x-I_y$, $S_2=I_{45}-I_{135}$, and $S_3=I_{lc}-I_{rc}$, with the subscript: $x,y,45$ or 135 indicating the inserted linear polarizer with its axis of transmission being horizontal, vertical, at 45^o or 135^o with the horizontal, respectively, and the subscript: lc or rc being left or right circular polarizer, respectively. In the case of stationary speckle measured by a uniform detector of finite size, the measured intensity has been defined by $I^{\mathrm{Int}} = \mathcal{A}_D^{-1}\iint_{-\infty}^{\infty} D(x,y)I(x,y)dxdy$, where I^{Int} with its superscript Int represents the integrated intensity, $D(x,y)$ is a real and positive weighting function representing the distribution of the detector's photosensitivity over space, $\mathcal{A}_D$ is the area of the detector, i.e., $\mathcal{A}_D = \iint_{-\infty}^{\infty} D(x,y)dxdy$, and $I(x,y)$ is the intensity distribution of the stationary speckle pattern that is being detected. Therefore, the integrated Stokes parameters S_l^{Int} (for $l = 0 \sim 3$) of our prime interest here can be defined by

$$\begin{aligned} S_l^{\mathrm{Int}} = I_p^{\mathrm{Int}} \pm I_q^{\mathrm{Int}} &= \frac{1}{\mathcal{A}_D}\iint_{-\infty}^{\infty} D(x,y)I_p(x,y)dxdy \pm \frac{1}{\mathcal{A}_D}\iint_{-\infty}^{\infty} D(x,y)I_q(x,y)dxdy \\ &= \frac{1}{\mathcal{A}_D}\iint_{-\infty}^{\infty} D(x,y)[I_p(x,y) \pm I_q(x,y)]dxdy = \frac{1}{\mathcal{A}_D}\iint_{-\infty}^{\infty} D(x,y)S_l(x,y)dxdy, \end{aligned} \tag{13}$$

where two subscripts: p and q, (for $p,q = x, y, 45, 135, lc, rc$) represent the corresponding polarizing filters to be inserted before a detector， and $S_l(x,y)$ (for $l = 0 \sim 3$) are the spatial distributions of the Stokes parameters to be detected. For a uniformly sensitive photodetector, the weighting function has the form

$$D(x,y) = \begin{cases} 1 & \text{in the sensitive area} \\ 0 & \text{outside the sensitive area.} \end{cases} \tag{14}$$

When Eq. (13) is written, we have made use of an assumption that identical portions of a stationary polarization speckle have been detected for the integrated Stokes measurement. After giving the definition of the spatially integrated Stokes parameters, we aim to find exact expressions for the means $\overline{S_l^{\mathrm{Int}}}$ and variances $\sigma^2_{S_l^{\mathrm{Int}}}$ with $l = 0 \sim 3$ of the integrated quantities for polarization speckle.

Calculation of each mean value of S_l^{Int} is quite straightforward. The expected values of Eq. (13) are obtained by interchanging the orders of integrations and expectations, yielding

$$\overline{S_l^{\mathrm{Int}}} = \frac{1}{\mathcal{A}_D}\iint_{-\infty}^{\infty} D(x,y)\overline{S_l}(x,y)dxdy = \overline{S_l}, \tag{15}$$

where $\overline{S_l}$ for $l = 0 \sim 3$ are the mean values of the Stokes parameters of the incident beam. Thus, each mean of the detected Stokes parameter is identical with the true mean of the corresponding Stokes parameter incident on a detector.

To find the variances of S_l^{Int} for $l = 0 \sim 3$, we first find their second moments,

$$\overline{(S_l^{\mathrm{Int}})^2} = \frac{1}{\mathcal{A}_D^2}\iint_{-\infty}^{\infty}\iint_{-\infty}^{\infty} D(x_1,y_1)D(x_2,y_2)\overline{S_l(x_1,y_1)S_l(x_2,y_2)}dx_1dy_1dx_2dy_2, \tag{16}$$

where, again, the orders of ensemble averaging and integration have been interchanged. For a wide-sense stationary polarization speckle pattern, the average of the product of the Stokes parameters depends only on the differences of coordinates $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$, and the equation for the second moment can be directly reduced to

$$\overline{(S_l^{\mathrm{Int}})^2} = \frac{1}{\mathcal{A}_D^2}\iint_{-\infty}^{\infty} \Gamma_D(\Delta x, \Delta y)\Gamma_{S_l}(\Delta x, \Delta y)d\Delta x d\Delta y, \tag{17}$$

where the ensemble-averaged quantity Γ_{S_l} is the autocorrelation function of the instantaneous Stokes parameter S_l, and Γ_D is the autocorrelation function of the deterministic function $D(x,y)$, which takes the form

$$\Gamma_D(\Delta x, \Delta y) = \iint_{-\infty}^{\infty} D(x_1,y_1)D(x_1+\Delta x, y_1+\Delta y)dx_1dy_1. \tag{18}$$

Substituting Eq. (9) into Eq. (18), and making use of the property $\mathcal{A}_D^2 = \iint_{-\infty}^{\infty} \Gamma_D(\Delta x, \Delta y) d\Delta x d\Delta y$, we obtain all the variances of S_l^{Int}. They are

$$\begin{aligned} \sigma_{S_l^{\text{Int}}}^2 &= \overline{(S_l^{\text{Int}})^2} - \overline{S_l^{\text{Int}}}^2 \\ &= (2\mathcal{A}_D^2)^{-1} \iint_{-\infty}^{\infty} \Gamma_D(\Delta x, \Delta y) \mathcal{L}_l \left\{ |S_0(\Delta x, \Delta y)|^2, |S_1(\Delta x, \Delta y)|^2, |S_2(\Delta x, \Delta y)|^2, |S_3(\Delta x, \Delta y)|^2 \right\} d\Delta x d\Delta y. \end{aligned} \tag{19}$$

With the aid of the definition for the degree of polarization and the normalized version of the generalized Stokes vector introduced before, we can rewrite the variances of the integrated Stokes parameters $\sigma_{S_l^{\text{Int}}}^2$ in Eq. (19) as

$$\sigma_{S_l^{\text{Int}}}^2 = (1+\mathcal{P}^2)\overline{S_0}^2 \big/ (2\mathcal{M}_{S_l}), \tag{20}$$

where four parameters $\mathcal{M}_{S_l}$ for $l = 0 \sim 3$ can be introduced as follows:

$$\mathcal{M}_{S_l} = \left[\mathcal{A}_D^{-2} \iint_{-\infty}^{\infty} \Gamma_D(\Delta x, \Delta y) \mathcal{L}_l \left\{ |\gamma_{S_0}(\Delta x, \Delta y)|^2, |\gamma_{S_1}(\Delta x, \Delta y)|^2, |\gamma_{S_2}(\Delta x, \Delta y)|^2, |\gamma_{S_3}(\Delta x, \Delta y)|^2 \right\} d\Delta x d\Delta y \right]^{-1}. \tag{21}$$

The parameters $\mathcal{M}_{S_l}$ referred to as the *numbers of degrees of freedom for detection of the Stokes parameters* are of fundamental importance in determining the statistics of the detected Stokes parameters. To gain some insight into their physical meanings, we first consider two limiting cases: one where the detection area is very large compared with the average size of a polarization speckle, and the other the opposite - a detection area that is small compared with the average size of a polarization speckle. In the first case, the function $\Gamma_D(\Delta x, \Delta y)$ is much wider than the functions $|\overline{\gamma_{S_l}}|^2$, and therefore, we can factor $\Gamma_D(0,0)$ outside the integrals, with the results

$$\mathcal{M}_{S_l} \approx \left[\mathcal{A}_D^{-2} \Gamma_D(0,0) \iint_{-\infty}^{\infty} \mathcal{L}_l \left\{ |\gamma_{S_0}(\Delta x, \Delta y)|^2, |\gamma_{S_1}(\Delta x, \Delta y)|^2, |\gamma_{S_2}(\Delta x, \Delta y)|^2, |\gamma_{S_3}(\Delta x, \Delta y)|^2 \right\} d\Delta x d\Delta y \right]^{-1}. \tag{22}$$

where $\Gamma_D(0,0) = \iint_{-\infty}^{\infty} D^2(x_1, y_1) dx_1 dy_1$ has the dimensions of area. It follows that $\mathcal{A}_D^2 \big/ \Gamma_D(0,0)$ has been named the effective measurement area $\mathcal{A}_M$ since it has the dimensions of area [3]. Note that, in the usual case of a uniformly sensitive detector, D is unity inside the detector aperture and zero outside that aperture, in which case $\mathcal{A}_M = \mathcal{A}_D$. With the definitions of the Stokes correlation areas for the polarization speckle in Eq. (12), we see that the parameters $\mathcal{M}_{S_l}$ for $l = 0 \sim 3$ are given by

$$\mathcal{M}_{S_l} \approx \mathcal{A}_M \big/ \mathcal{A}_{S_l}^{\text{C}} \qquad (\mathcal{A}_M \gg \mathcal{A}_{S_l}^{\text{C}}). \tag{23}$$

Thus, in the limit of a large measurement area compared with the speckle size, each value of $\mathcal{M}_{S_l}$ (for $l = 0 \sim 3$) increases in direct proportion to the corresponding ratio $\mathcal{A}_M$ to $\mathcal{A}_{S_l}^{\text{C}}$. Therefore, $\mathcal{M}_{S_l}$ can be interpreted as the average number of effective polarization speckles influencing the measurement of the Stokes parameter S_l.

For the opposite case of the average sizes of the Stokes correlation areas $\mathcal{A}_{S_l}^{\text{C}}$ much wider than the measurement area $\mathcal{A}_M$, we can place $\mathcal{L}_l \left\{ |\overline{\gamma_{S_0}}|^2, |\overline{\gamma_{S_1}}|^2, |\overline{\gamma_{S_2}}|^2, |\overline{\gamma_{S_3}}|^2 \right\}$ outside these integrals. Note that $\boldsymbol{\gamma}_S(0,0) = \overline{\mathbf{S}}(0,0) \big/ \| \overline{\mathbf{S}} \| = \hat{\overline{\mathbf{S}}}$ with the property: $\sum_{n=0}^{3} \overline{S_n}^2 = 1$. Therefore, we can rewrite Eq. (12) for the case $(\mathcal{A}_M \ll \mathcal{A}_{S_l}^{\text{C}})$,

$$\mathcal{M}_{S_l} \approx [\mathcal{L}_l \{ \overline{S_0}^2, \overline{S_1}^2, \overline{S_2}^2, \overline{S_3}^2 \}]^{-1}, \tag{24}$$

with $\mathcal{M}_{S_0} \approx 1$. When the expressions above are derived, we have made use of the fact that $\mathcal{A}_D^2 = \iint_{-\infty}^{\infty} \Gamma_D(\Delta x, \Delta y) d\Delta x d\Delta y$. In these cases, each $\mathcal{M}_{S_l}$ can never fall below unity, which is consistent with the interpretation of $\mathcal{M}_{S_l}$ as being the average numbers of effective polarization speckles influencing the measurement of the Stokes parameter S_l. No matter how small the measurement aperture becomes, a minimum of one polarization speckle will influence the results. Due to the unique property of fluctuating state of polarization, even the same polarization speckle may have different Stokes correlation areas and therefore give different influences to the measurement of the Stokes parameters.

Quantities of considerable interest are the signal-to-noise ratios associated with the integrated Stokes parameters indicating the level of the detected Stokes parameter with respect to the level of the fluctuations of that particular Stokes parameter. Note that the Stokes parameters S_n (for $n = 1 \sim 3$) may take zero or negative means. With the aid of Eqs. (16) and (20), the signal-to-noise ratios $(S/N)_{S_l^{\text{Int}}}$ in the power sense for $l = 0 \sim 3$ take the form

$$\left(\frac{S}{N}\right)_{S_l^{\text{Int}}}^{\text{Pow}} = \frac{\overline{(S_l^{\text{Int}})^2}}{\sigma_{S_l^{\text{Int}}}^2} = 1 + \frac{2\mathcal{M}_{S_l} \overline{S_l}^2}{(1+\mathcal{P}^2)\overline{S_0}^2}. \tag{25}$$

To specify the values of $\mathcal{M}_{S_l}$ for $l = 0 \sim 3$ in any particular case, it is necessary to know the detector aperture function $D(x, y)$ and the generalized Stokes parameters. In the case of a Gaussian-shaped intensity distribution across the scattering spot where radiation is passed through a depolarizing diffuser, we can apply the ensemble-average van Cittert-Zernike theorem to write

$$\gamma_{S_l}(\Delta x, \Delta y) = \overline{S_l} \exp\left[-\frac{\pi}{2\mathcal{A}}(\Delta x^2 + \Delta y^2)\right], \tag{26}$$

where the identical correlation areas $\mathcal{A}$ have been used for mathematical simplicity. Performing the required integrations in Eq. (21) yields

$$\mathcal{M}_{S_l} = [\mathcal{L}_l\{\overline{S_0}^2, \overline{S_1}^2, \overline{S_2}^2, \overline{S_3}^2\}]^{-1} \left[\sqrt{\frac{\mathcal{A}}{\mathcal{A}_M}} \text{erf}\left(\sqrt{\frac{\pi \mathcal{A}_M}{\mathcal{A}}}\right) - \left(\frac{\mathcal{A}}{\pi \mathcal{A}_M}\right)\left[1 - \exp\left(-\frac{\pi \mathcal{A}_M}{\mathcal{A}}\right)\right]\right]^{-2}. \tag{27}$$

where $\text{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-z^2} dz$ is the error function. When the expression in Eq. (27) is derived, we have assumed a square, uniform detector with its effective measurement area $\mathcal{A}_M$.

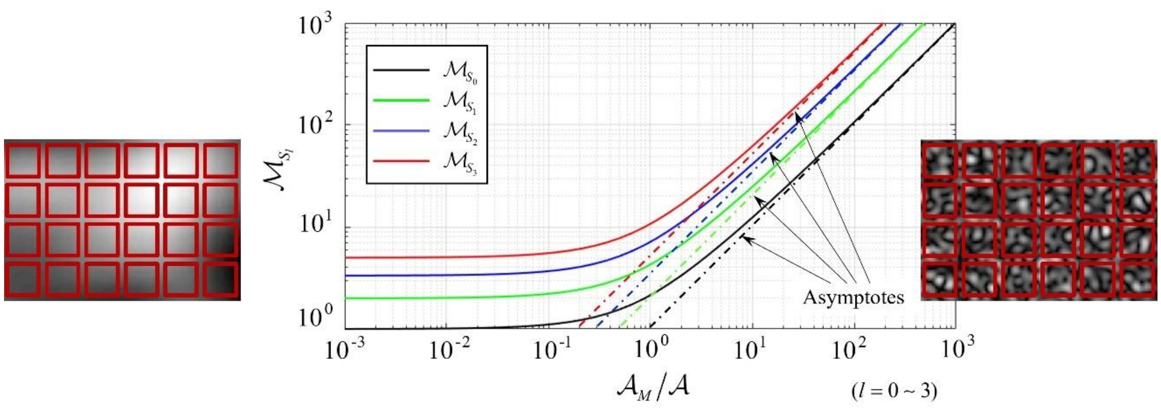


Figure 3 Plots of $\mathcal{M}_{S_l}$ versus $\mathcal{A}_M / \mathcal{A}$ with two limiting cases for detection of polarization speckle.

Figure 3 shows plots of $\mathcal{M}_{S_l}$ versus the ratio $\mathcal{A}_M/\mathcal{A}$ for an example of a set of $\overline{S_l}$, i.e., $\overline{S_0} = \sqrt{0.5}, \overline{S_1} = -\sqrt{0.25}, \overline{S_2} = \sqrt{0.15}$ and $\overline{S_3} = -\sqrt{0.1}$. It can be seen that each curve for the parameter $\mathcal{M}_{S_l}$ approaches its corresponding constant given by Eq. (24) when $\mathcal{A}_M/\mathcal{A}$ becomes small as illustrated in the left subplot. While all these $\mathcal{M}_{S_l}$ for $l = 0 \sim 3$ increase in proportion to $\mathcal{A}_M/\mathcal{A}$, when these ratios are large as indicated in the right subplot. As a special case of isotropic polarization speckle with $\mathcal{P} = 0$, the curves for the parameters $\mathcal{M}_{S_1}, \mathcal{M}_{S_2}$ and $\mathcal{M}_{S_3}$ merge with that for $\mathcal{M}_{S_0}$, since $\overline{S_0} = 1$ and $\overline{S_1} = \overline{S_2} = \overline{S_3} = 0$. On the other hand, we can only observe two coinciding curves for the parameters $\mathcal{M}_{S_0}$ and $\mathcal{M}_{S_1}$ for uniform polarization speckle linearly polarized along the $\hat{x}$ direction, since $\mathcal{M}_{S_2}$ and $\mathcal{M}_{S_3}$ become infinity due to the fact $\overline{S_0} = \overline{S_1} = \sqrt{2}/2$ and $\overline{S_2} = \overline{S_3} = 0$. Similarly, only two coinciding curves for $\mathcal{M}_{S_0}$ and $\mathcal{M}_{S_3}$ can be observed for a circularly polarized speckle.

In the case of partially polarized thermal light or a temporally changing polarization speckle (whose radiation consists of a large number of random and independent contributions and exhibits random fluctuations in time), the integrated Stokes parameters over the finite observation interval can be written in a similar way. That is

$$\begin{aligned} S_l^{\mathrm{Int}} &= I_p^{\mathrm{Int}} \pm I_q^{\mathrm{Int}} = \frac{1}{T}\int_{t_p}^{t_p+T} I_p(\xi)d\xi \pm \frac{1}{T}\int_{t_q}^{t_q+T} I_q(\eta)d\eta = \frac{1}{T}\int_0^T [I_p(t) \pm I_q(t)]dt \\ &= \frac{1}{T}\int_{-T/2}^{T/2} S_l(t)dt = \frac{1}{T}\int_{-\infty}^{\infty} \mathrm{rect}\left(\frac{t-T/2}{T}\right) S_l(t)dt, \end{aligned} \tag{28}$$

where T is an integration time, $\mathrm{rect}(t)$ represents the rectangular function, and $S_l(t)$ (for $l = 0 \sim 3$) is the temporally changing Stokes parameters. When Eq. (28) is written, we have made use of an assumption that the light in question is an ergodic (and, therefore, stationary) random process so that the statistics of polarization-sensitive integrated intensities do not depend on the particular observation time t_p or t_q.

When the instantaneous Stokes parameters of partially polarized thermal light or time-varying polarization speckle are integrated over time, results are very similar to those derived above. In particular, we can apply temporal integrations this time with respect to the temporal fluctuations of the Stokes parameters. Again we can introduce $\mathcal{M}_{S_l}$ for $l = 0 \sim 3$ representing the numbers of polar-coherence cells of the light wave that influence the experimental outcome for detection of the Stokes parameters. By analogy with results obtained for spatial integrations, the numbers of degrees of freedom within the finite-time segment T relative to the coherence time τ for the temporally integrated Stokes parameters are

$$\mathcal{M}_{S_l} = \left[T^{-1}\int_{-\infty}^{\infty} \Lambda(\tau/T)\mathcal{L}_l\left\{|\gamma_{S_0}(\tau)|^2, |\gamma_{S_1}(\tau)|^2, |\gamma_{S_2}(\tau)|^2, |\gamma_{S_3}(\tau)|^2\right\}d\tau\right]^{-1}, \tag{29}$$

where $\Lambda(t) = 1 - |t|$ for $|t| \le 1$ and zero otherwise is a triangular function, and $\gamma_{S_l}(\tau)$ for $l = 0 \sim 3$ are the normalized generalized Stokes parameters providing temporal polar-coherence of the light. Analytical solutions are possible when the light has Gaussian spectral profiles. Since the normalized version of the generalized Stokes parameters γ_{S_l} provide a complete description of the polar-coherence effects of the stochastic electric field, therefore, these four parameters $\mathcal{M}_{S_l}$ (for $l = 0 \sim 3$) introduced in Eqs. (21) and (29) may be considered as a natural extension of two parameters $\mathcal{M}_1$ and $\mathcal{M}_2$ proposed by Goodman for the two independent polarization components [1,3].

4. THE STOKES VECTOR INTERFEROMETRY

Since the seminal work by Hanbury Brown and Twiss in the 1950s [13], the technique of intensity interferometry has attracted increasing interests with broad applications to different branches of physics, and provides a unique class of instruments to study the correlation and/or anticorrelation in intensities of a beam received by two independent detectors.

Noting the fact that polarization is an intrinsic property of light, we will propose the principle of a new kind of interferometry, referred to as the Stokes vector interferometry, to study the polarization-resolved Hanbury Brown-Twiss effect. Just as the covariance of the intensities at two points positions serves as the underlying principle for intensity interferometry, the proposed Stokes vector interferometry is named after the cross-covariances of the components of the Stokes vectors recorded at two observation points.

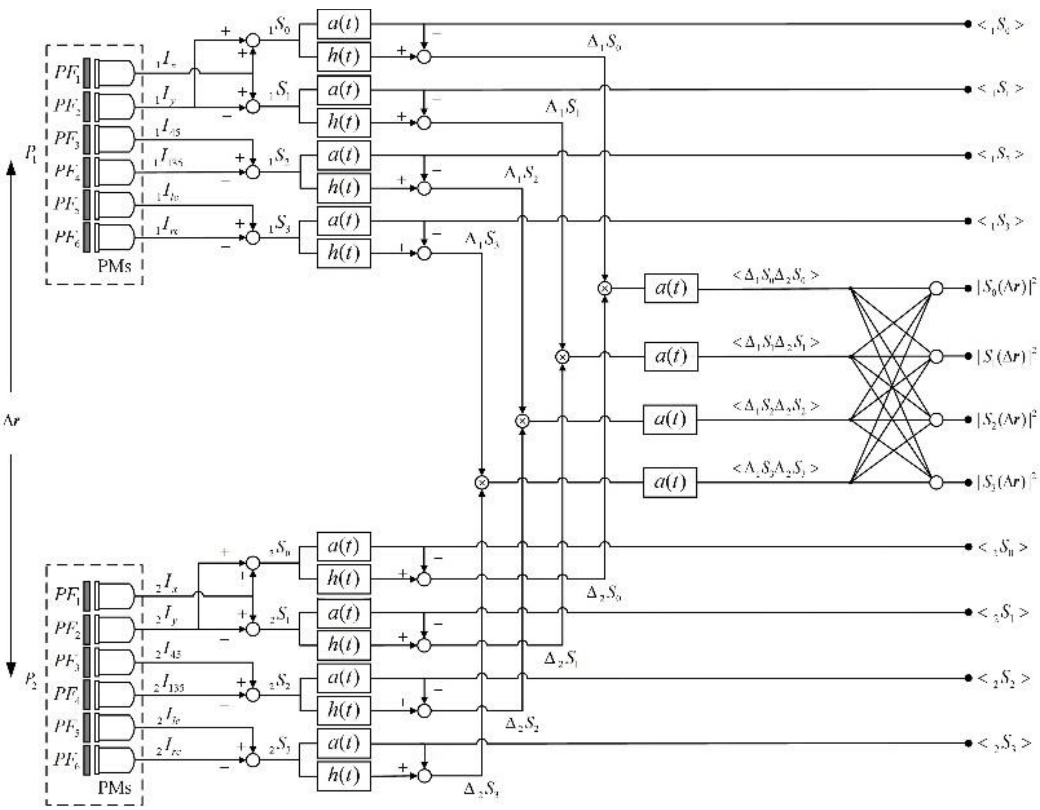


Figure 4. Block diagram of the Stokes vector interferometer (PDs: photoelectric detectors)

The general structure of the proposed scheme is illustrated in Fig. 4 in a simplified form. Two sets of highly sensitive and wideband photodetectors directly detect the intensity incident on two points P_1 and P_2 with a spatial separation of $\Delta\mathbf{r}$. For each set of photodetectors, six different polarization filters are inserted in front of the photoelectric detectors (PD) to reconstruct the Stokes parameters. Here, some unavoidable smoothing operations are represented by linear filters with their impulse responses $h(t)$, assumed for simplicity to be identical in the eight arms of the proposed interferometer.

Since only the fluctuations of the eight Stokes parameters about their corresponding direct currents (DCs) or average values carry polarization-related information about the polar-coherence of the light beams, the DC currents are removed before bringing these corresponding Stokes parameters together. Meanwhile, these DC components of the Stokes parameters are needed later for normalization purposes, so they are presented at the output of the electrical circuitry. The remaining fluctuating components, i.e., $\Delta S_l(\mathbf{r}_1,t)$ and $\Delta S_l(\mathbf{r}_2,t)$ for $l=0\sim 3$, are then subjected to averages. Next, these eight products of the electrical currents, i.e., $\Delta S_l(\mathbf{r}_1,t)\Delta S_l(\mathbf{r}_2,t)$ will be brought together again through linear combinations to retrieve information about the polar-coherence properties of light in terms of the generalized Stokes parameters.

To understand the principle of operation and figure out what statistical information of stochastic light can be retrieved by the proposed scheme, we will simplify our theoretical analysis and develop a general model for the ideal output of the Stokes vector interferometer. When the means of the Stokes parameters are subtracted electronically, we obtain the fluctuating Stokes parameters: $\Delta S_l(\mathbf{r}_n,t)=S_l(\mathbf{r}_n,t)-<S_l(\mathbf{r}_n,t)>$ with $<\cdots>$ indicating temporal averaging, and then the auto-covariance functions of the Stokes parameters can be written in the form

$$C_{ll}(\mathbf{r}_1,\mathbf{r}_2) = <\Delta S_l(\mathbf{r}_1,t)\Delta S_l(\mathbf{r}_2,t)> \\ = <S_l(\mathbf{r}_1,t)S_l(\mathbf{r}_2,t)> - <S_l(\mathbf{r}_1,t)><S_l(\mathbf{r}_2,t)> . \tag{30}$$

After expressing the right-hand side of Eq. (30) in terms of the instantaneous electric field and making use of the complex Gaussian moment theorem, we have

$$C_{ll}(\mathbf{r}_1,\mathbf{r}_2) = 2^{-1}\mathcal{L}_l\left\{|S_0(\mathbf{r}_1,\mathbf{r}_2)|^2, |S_1(\mathbf{r}_1,\mathbf{r}_2)|^2, |S_2(\mathbf{r}_1,\mathbf{r}_2)|^2, |S_3(\mathbf{r}_1,\mathbf{r}_2)|^2\right\}, \tag{31}$$

where $S_l(\mathbf{r}_1,\mathbf{r}_2)$ for $l = 0 \sim 3$ are the generalized Stokes parameters. With the aid of the four linear transformations in Eq. (4), we can obtain the squared modulus for each generalized Stokes parameter. That is

$$|S_l(\mathbf{r}_1,\mathbf{r}_2)|^2 = 2^{-1}\mathcal{L}_l\left\{C_{00}(\mathbf{r}_1,\mathbf{r}_2), C_{11}(\mathbf{r}_1,\mathbf{r}_2), C_{22}(\mathbf{r}_1,\mathbf{r}_2), C_{33}(\mathbf{r}_1,\mathbf{r}_2)\right\}. \tag{32}$$

As shown in Fig. 4, we also have separate measurements of $<S_l(\mathbf{r}_1)>$ and $<S_l(\mathbf{r}_2)>$ at the output of the proposed interferometer. Normalization of the squared moduli of the generalized Stokes parameters by the product of the Frobenius norms of the ensemble-averaged Stokes vectors at these two points yields the result

$$|\gamma_{S_l}(\mathbf{r}_1,\mathbf{r}_2)|^2 = \frac{|S_l(\mathbf{r}_1,\mathbf{r}_2)|^2}{\left[\sum_{l=0}^{3} <S_l(\mathbf{r}_1)>^2\right]\left[\sum_{l=0}^{3} <S_l(\mathbf{r}_2)>^2\right]} \tag{33}$$

Under an assumption that the polarization speckle fields are ergodic, we experimentally demonstrate the validity of the proposed interferometry using space average. A birefringent polarization scrambler, which is a ground wave plate with a rough surface [9,10] is illuminated by He-Ne laser light through a quartz-wedge depolarizer (DP) to modify the spatial distribution of the state-of-polarization of the incident beam.

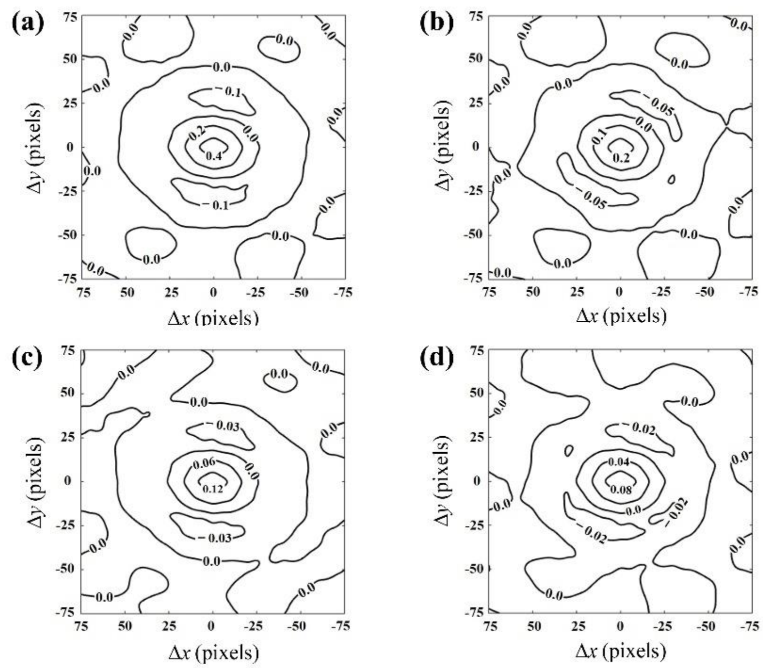


Figure 5 Contours of the spatial distributions for the squared moduli of the normalized generalized Stokes parameters: (a) $|\gamma_{S_0}|^2$, (b) $|\gamma_{S_1}|^2$, (c) $|\gamma_{S_2}|^2$, and (d) $|\gamma_{S_3}|^2$

The optical field behind this depolarizing diffuser is optically Fourier transformed by a lens to create a stationary random electric field on the detection plane [14]. To obtain the desired spatial distributions of the Stokes parameters from the recorded intensity distributions on the observation plane, each of the six polarization filters is inserted in turn in front of a CCD camera to record these polarization-related irradiance. From the central limit theorem, we may reasonably assume that the polarization speckle field obeys a complex

Gaussian random process so that we can apply the complex Gaussian moment theorem to calculate the cross-covariance functions of the reconstructed Stokes parameters in the observation plane.

From these polarization filtered irradiances $I_k(x,y)$ for $(k = x, y, 45, 135, lc, rc)$, we first reconstructed the distributions of the Stokes parameters $S_l(x,y)$ for $(l = 0 \sim 3)$ of the polarization speckle based on their definitions, and then conducted the calculations of cross-covariances of the Stokes parameters by spatial correlations. Based on the newly obtained autocovariance functions of the Stokes fluctuations $C_{ll}(\Delta x, \Delta y)$ and the average Stokes parameters, we reconstructed the squared moduli of the normalized generalized Stokes parameters with the aid of Eqs. (32) and (33). Figure 4(a)-(d) shows, respectively, the distributions of the squared moduli: $|\gamma_{S_0}|^2, |\gamma_{S_1}|^2, |\gamma_{S_2}|^2$ and $|\gamma_{S_3}|^2$ as the normalized version of the generalized Stokes parameters versus coordinate difference Δx and Δy in the observation plane.

5. CONCLUSIONS

In summary, we have reviewed our recent work on the correlations of the Stokes parameters and their applications to diagnose the random polarization in polarization speckle. With the aid of the complex Gaussian moment theorem, we investigated the autocorrelation functions and power spectra of the Stokes parameters to expose the dependence of the polarization-related scale-size distributions. A generalized concept of the Stokes ensemble-average coherence areas has been introduced to deal with the polarization-related average areas associated with polarization speckle. Since the Stokes parameters cannot be measured at an ideal point, we also made investigation of the means and variances of the integrated Stokes parameters in polarization speckle by introducing four parameters referred to as the numbers of degrees of freedom for the Stokes detection. As a development and extension of conventional intensity interferometry, we have proposed a new scheme of interferometry called the Stokes Vector Interferometry and reconstructed the squared moduli of the generalized Stokes parameters through the cross-covariance computations for the fluctuating Stokes parameters.

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