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# Statistics modeling of random polarization in K-distributed sea clutter

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### ABSTRACT

K−distribution plays a significant role in describing the statistics of the electromagnetic scattering. The influence of shape factor on the K−distribution is very significant because different shape factors in K-distributed sea clutter represent different actual scenarios. In this paper, we investigate the impact of different shape factors in the K−distributed sea clutter and provide the statistics of the Stokes parameters in electromagnetic scattering of the K−distribution under different shape factors.

**Keywords:** K-distribution, polarization phasor sum, shape factor

## 1. INTRODUCTION

The K-distribution occupies an important position in the sea clutter model, which can describe the high-resolution low incidence residual angle well [1]. Up to now, speckle fields involving the use of the K-distribution have been treated as scalar fields ignoring the polarization properties arising from the vector nature of the electromagnetic fields [2]. We investigate the statistics of the random polarization of sea clutter waves with K-distribution and provide the probability density function of the Stokes parameter characterizing the fluctuating polarization [3]. The shape parameter is an important variable in the statistical model of sea clutter that indirectly reflects radar and environmental parameters. We investigate the impact of different shape factors in the K−distributed sea clutter and provide the statistics of the Stokes parameters in electric scattering of the K−distribution under different shape factors [4].

## 2. STATISTICAL PROPERTIES OF STOKES PARAMETERS FOR RANDOM PHASE SUMS

The Usually, the polarization state of the light can be described by the Stokes parameters, which can be given as [5]:

$$\begin{aligned} S_0 &= A_x^2 + A_y^2 = I_x + I_y \\ S_1 &= A_x^2 - A_y^2 = I_x - I_y \\ S_2 &= 2A_x A_y \cos(\theta_y - \theta_x) = 2A_x A_y \cos\delta \\ S_3 &= 2A_x A_y \sin(\theta_y - \theta_x) = 2A_x A_y \sin\delta \end{aligned} \tag{1}$$

where $I_x$ and $I_y$ ,are the intensities of two polarization components and $\delta$ is the random phase difference between two

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polarization components.

On the other hand, the K-distribution is a family of continuous probability distributions with parameters, so the probability density function whose amplitude follows the K-distribution is therefore expressed as follows [6]:

$$p_A(A) = \frac{2\beta}{\Gamma(1+\nu)}\left(\frac{\beta A}{2}\right)^{\nu+1} K_\nu(\beta A) \tag{2}$$

valid for $\nu > -1$ and $A \geq 0$. Here, $\nu$ and $\beta$ are two parameters used to fit the density function to experimental data, where $\nu$ is the shape factor and $K_\nu(\cdot\cdot)$ is the modified Bessel function of the second kind.

**2.1 Statistics of $S_0$ and $S_1$**

Finding the statistics of $S_0$ and $S_1$ are complicated that the probability density function of the intensity can be found with the transformation rule:

$$p_I(I) = \frac{2\beta^{(\nu+1)/2} I^{(\nu-1)/2}}{\Gamma(\nu)} K_{\nu-1}(2\sqrt{\beta I}) \tag{3}$$

The degree of polarization $\mathcal{P}$ is introduced and the relationship between the intensity statistics of light polarized in different directions, we can obtain the respective probability density functions of the two components $I_x$ and $I_y$:

$$p_{I_m}(I_m) = \frac{2}{I_m \Gamma(\nu)} (2\nu I_m / [(1 \pm \mathcal{P})\overline{S_0}])^{(\nu+1)/2} K_{\nu-1}(2\sqrt{2\nu I_m / [(1\pm\mathcal{P})\overline{S_0}}) \text{ , } m = x, y \tag{4}$$

The Stokes parameter $S_0$ is the total intensity in the speckle field. Its probability density function can be obtained by the convolution of two components of the probability density function $p_{I_x}$ and $p_{I_y}$. That is:

$$p_{S_0}(S_0) = \int_0^{S_0} p_{I_x}(u) p_{I_y}(S_0 - u) du \tag{5}$$

In order to study the influence of form factor $\nu$ on the K-distribution, substitute Eq. (4) into Eq. (5). Figure 1 shows the probability density function of $S_0$ after normalization at the different number of $\mathcal{P}$ for $\nu = 1$.

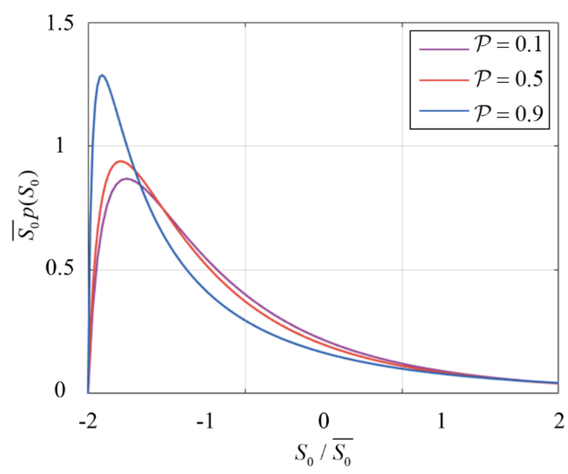


Figure1. For $\nu = 1$,the intensity probability density function of $S_0$ with the different number of $\mathcal{P}$.

The Stokes parameter $S_1$ characterizes whether the light is more closely polarized in the x-direction or the y-direction. For $S_1$ , we have the following equation:

$$p_{S_1}(S_1) = \begin{cases} \int_{S_1}^{\infty} p_{I_x}(u) p_{I_y}(u - S_1) du, & S_1 \geq 0 \\ \int_{0}^{\infty} p_{I_x}(u) p_{I_y}(u - S_1) du, & S_1 < 0 \end{cases} \tag{6}$$

Substitute Eq. (4) into Eq. (6), after the integration operation, we can obtain the probability density function of $S_1$ with the different number of $\mathcal{P}$ for $\nu = 1$:

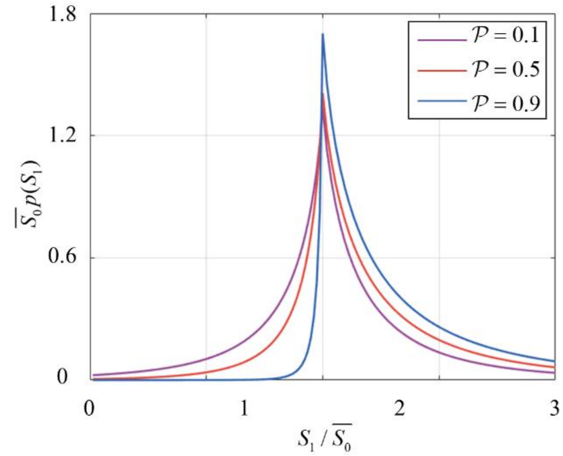


Figure2. For $\nu = 1$,the intensity probability density function of $S_1$ with the different number of $\mathcal{P}$.

**2.2 Statistics of $S_2$ and $S_3$**

Next we perform the statistical calculation for $S_2$ and $S_3$. Here we use the monotone transformation $A = \sqrt{I}$ and bring it into Eq. (5) to obtain the probability density function of the amplitude component:

$$p_{A_m}(A_m) = \frac{4}{A_m \Gamma(\nu)} (\sqrt{2\nu} A_m / \sqrt{(1-\mathcal{P})\overline{S_0}})^{(\nu+1)} K_{\nu-1}(2\sqrt{2\nu} A_x / \sqrt{(1-\mathcal{P})\overline{S_0}}), m = x, y \tag{7}$$

By an integral transformation $\gamma = A_x A_y$, The probability density function of $p_\gamma(\gamma)$ can be expressed as:

$$p_\gamma(\gamma) = \int_0^{\infty} p_{A_x}(A_x) p_{A_y}(\gamma / A_x) A_x^{-1} dA_x \tag{8}$$

Then we can get the probability density function of $S_2$ by the following formula:

$$p_{S_2}(S_2) = \int_{-1}^{1} p_\gamma(S_2 / 2\xi)(2\pi|\xi|\sqrt{1-\xi^2})^{-1} d\xi \tag{9}$$

$S_2$ characterizes whether the light is closer to the positive 45 degree polarization or the negative 45 degree polarization; $S_3$ characterizes whether the light is closer to the right-handed circularly polarized light or the left-handed circularly polarized light, so they have the same probability density function distribution. As shown in the following figure:

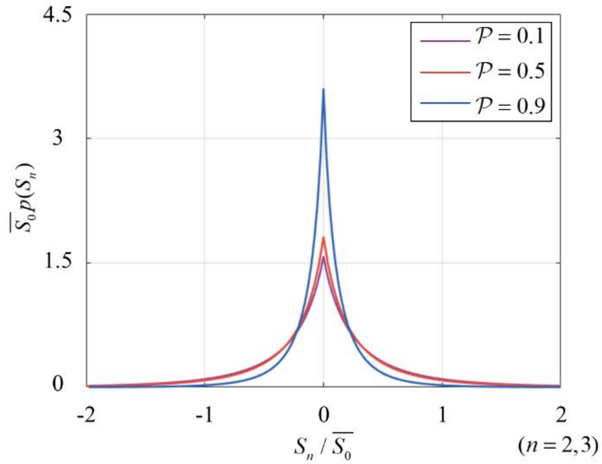


Figure3. For $\nu=1$,the intensity probability density function of $S_2$ or $S_3$ with the different number of $\mathcal{P}$.

# 3. CONCLUSION

The statistical properties of the random light field under the K-distribution are analyzed, and the accurate expression of the Stokes parameter of the random polarization phasor sum is obtained. When the degree of shape factor is 1, the probability distributions of intensity and amplitude of polarization in different directions are compared. In a certain class of applications, including synthetic-aperture radar imagery and wireless communications, the distribution plays a significant role in describing the statistical properties of the electromagnetic scattering. In this paper, we have investigated the development of the polarization speckle when the amplitudes of the contributing polarization phasors are distributed and have provided the probability density functions of the Stokes parameters charactering the fluctuating polarization. These results can be regarded as an extension of previous study on the statistical or probabilistic model with distribution for scalar fields and will provide a deeper insight into the evolution of random polarization for stochastic electromagnetic fields.

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