Assessing The Load-Carrying Capacity Of RC Beams Under Impact Loading

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ABSTRACT: Data obtained from drop-weight tests reveals that the response exhibited by reinforced concrete (RC) beam specimens under impact loading differs significantly from that established during equivalent static testing. This shift in structural behaviour predominantly takes the form of an increase in the maximum sustained load as well as a reduction in the portion (span) of the RC beam reacting to the imposed action which tends to concentrate around the area of impact. However, measurements obtained from drop-weight tests concerning certain important aspects of RC structural response (e.g. maximum sustained load or deflection) often correspond to a specimen physical-state characterised by high concrete disintegration in combination with low residual load-bearing capacity and stiffness. This stage of structural response has little practical significance as it depends heavily on post-failure mechanisms for transferring the applied load to the specimen supports. In view of the above, the available test data cannot provide insight into the mechanisms underlying RC structural response nor can it identify the true ultimate limit state of the subject specimens when subjected to impact loading. To achieve insight into the mechanics underlying RC structural response under impact two well established structural analysis packages (ADINA and ABAQUS) are employed in the present study. Both packages are capable of carrying out three-dimensional dynamic nonlinear finite element analysis while realistically accounting for the nonlinear behaviour of concrete and the characteristics of the problem at hand i.e. a wave propagation problem within a highly nonlinear medium. The numerical predictions obtained concerning various aspects of RC structural response are initially validated against relevant data obtained from drop-weight tests. A parametric investigation is then carried out aiming to study the dynamic response exhibited by RC beams when subjected to specific rates and intensities of impact loading. The latter investigation reveals that the true load-bearing capacity is often significantly lower than the maximum sustained load recorded experimentally. In fact, the higher the loading rate and intensity of the impact load the larger the latter difference becomes.

Keywords: Reinforced concrete, beams, finite elements, nonlinear dynamic analysis, loading rate, impact, drop-weight tests.

Authors:

N. Madjlessi and**. Behinaein** are PhD students at Heriot Watt University investigating experimentally and numerically the mechanics underlying RC structural response under impact loading.

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INTRODUCTION

It has been established, experimentally [1-5] and numerically [6-9], that the dynamic response exhibited by reinforced concrete (RC) beams under impact loading exhibits significant departures from that recorded during equivalent static testing as certain thresholds of applied loading rate are surpassed. The analysis of the available published experimental and numerical data reveals that the observed shift in structural response is owed to the combined effect of the inertia forces developing along the element span and the exhibited localised response [6-8]. More specifically, it has been established that the length of the element span (effective length, L_{eff}) reacting to the applied load reduces with increasing loading rates. This can be explained when viewing the problem at hand as a wave propagation problem within a highly nonlinear medium. In such cases the deformation exhibited by RC beams when subjected to impact loads is dependent on: (i) the intensity and speed of the stress waves generated during impact, which travel away from the impact region towards the supports of the structural member considered as well as (ii) the level of damage (cracking) sustained which locally reduces the stiffness of the RC element [6-8]. Under high loading rates, structural failure can be exhibited prior to the stress waves reaching the specimen supports resulting in localised response. The higher the loading-rate the more localised the response becomes as the distance within which the stress-waves travel prior to failure gradually reduces, concentrating around the area of impact [6-8]. This reduction of the element span reacting to the imposed load can be used to explain the observed increase in stiffness and load-carrying capacity exhibited by RC beams when subjected to impact loads characterised by increasing loading rates and intensities [6-8].

Present work forms an extension to already published studies investigating numerically the effect of loading rate on the response of RC beams under concentrated loads applied 'monotonically' to failure [6-8]. The aim of this investigation is to determine the *'true'* load-bearing capacity of the RC beam specimens when subjected to specific rates and intensities of impact loading. In an attempt to assess individually the effect of the loading rate and the intensity characterising the imposed impact load on the behaviour of the beams, the form of the latter load is assumed to be described by the simplified force time history shown in Figure1. This function is considered to consist of an ascending and a descending branch in which the rate of loading (associated with the ascending branch) is assumed equal to the rate of unloading (associated with the descending branch). Two well established three-dimensional (3D) dynamic nonlinear finite element analysis (NLFEA) packages are employed (ADINA and ABAQUS) [10,11]which are both capable of realistically accounting for the brittle nature characterising concrete material behaviour as well as the characteristics of the problem at hand: a wave propagation problem with a highly nonlinear medium. Emphasis is presently focused on studying certain important aspects of RC structural response such as the mode of failure as well as the deformation and cracking profiles exhibited throughout the loading process. The predictions obtained from the latter study reveal that the actual load-bearing capacity exhibited under impact loading is often significantly lower than the value of the maximum sustained load recorded experimentally. More specifically, it is observed that the latter difference increases as the loading rate and intensity of the impact load become higher.

EXPERIMENTAL BACKGROUND

A large number of drop weight tests have been conducted to date on a wide range of RC beam specimens [1-5]. During such tests the load is applied though a steel striker (drop-weight) which is allowed to fall freely onto the mid-span region of the specimen from a predefined height (depending on the desired rate of loading). The layout of the drop-weight testing setup currently employed at Heriot Watt University (HWU) is shown in Figure 2. Different pads (e.g. steel, rubber, plywood) are usually employed to moderate the level of damage (cracking) sustained locally by the specimen at the impact area and control (to a certain extent) the loading rate and intensity of the contact force generated. During testing measurements concerning the variation of displacement and strain at certain points along the specimen span, the acceleration of the steel impactor, as well as the support reactions and the contact force generated are recorded. In addition, crack formation and propagation up to failure, is closely monitored as it provides an indication of the corresponding internal stress state of the beam throughout the loading process. Typical load-deflection curves obtained from drop-weight tests describing the behaviour of RC beams under impact and equivalent static loading applied at mid-span are shown in Figure3 [5]. These curves reveal that as the rate of loading increases, the beams sustain higher values of loading. In addition, the exhibited crack patterns (see Figure4) suggest that with increasing loading rates the span of the beam mostly affected by the impact load tends to shorten and concentrate around the region where the load is applied (impact region).

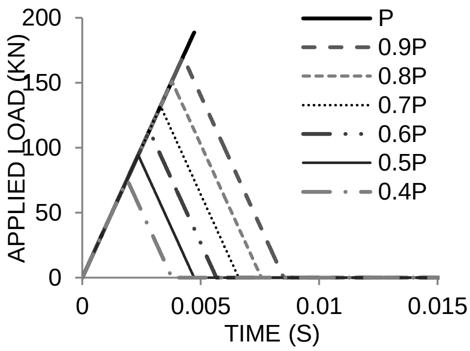


Figure 1 Force time history describing the contact force generated during impact.

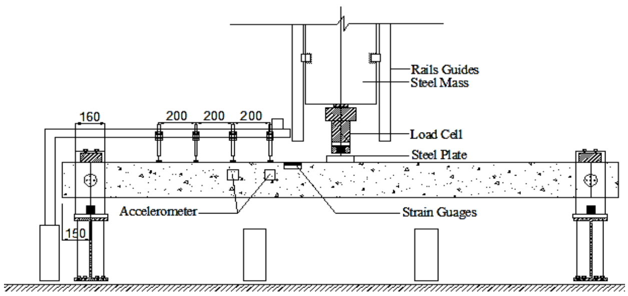


Figure 2 Drop-weight test setup employed at HWU

It should be noted, that drop-weight tests are difficult to conduct as the intensity of the loads generated during impact increase rapidly (in a few milliseconds) from zero to a maximum value (characterised by significantly higher values compared to those recorded during equivalent static testing) often leading to explosive (brittle) forms of failure which can in turn damage the instruments employed. Data obtained from such tests is characterised by a large scatter (see Figure 5) due to a wide range of parameters (associated with the different experimental techniques used, the variation of the size and shape of the impactor and the design details of the RC specimens) which differ from test to test [6-8]. Furthermore, it should be noted that the experimental information available does not usually provide a detailed description of the response exhibited by the RC specimens throughout the loading process. Instead, the available published data, concerning crack patterns and deformation profiles, is usually measured after (and not throughout) the application of the impact load. As a result it is difficult to correlate the measured responses obtained from drop weight tests to the actual physical state of the specimens as the measured maximum value of imposed load frequently corresponds to a specimen physical-state characterised by high concrete disintegration in combination with low residual load-bearing capacity and stiffness. This stage of structural response has little (if any) practical significance as it depends heavily on post-failure mechanisms (e.g. catenary and dowel action) for transferring the applied loads to the specimen supports. Based on the above it appears that the true load-carrying capacity is likely to be significantly lower than the maximum value of the contact force measured during testing. In addition it can be also concluded that the available test data cannot provide detailed insight into the mechanisms underlying RC structural response; it can, however, provide a qualitative description of the effect of loading-rate on certain important aspects of specimen behaviour.

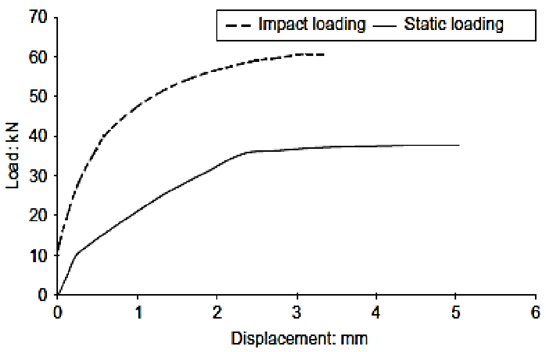


Figure 3 Typical load-deflection curves recorded during static and drop-weight testing of RC beams [5]

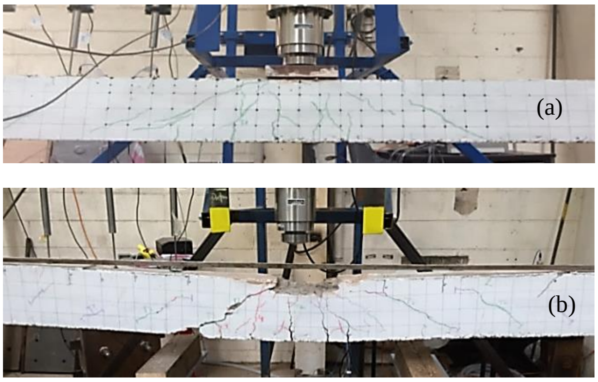


Figure 4 Crack patterns developing on RC beams under different rates of impact loading: (a) 25 kN/ms (b) 100 kN/sec;

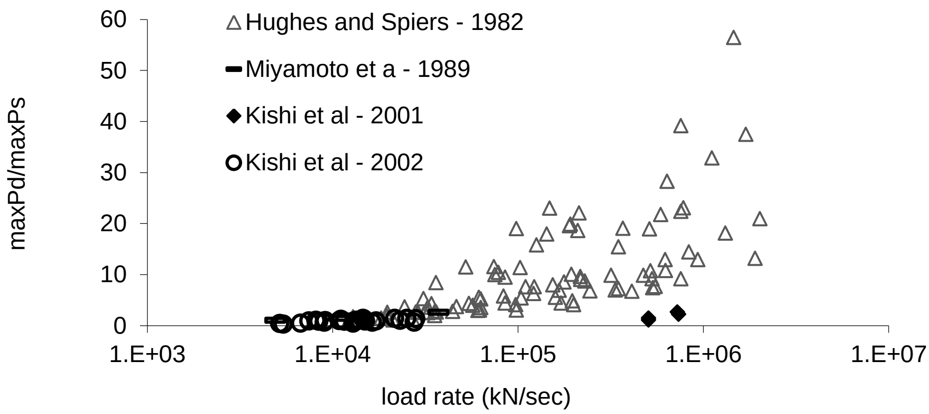


Figure 5 Variation of maximum sustained load during impact (*max*P_d) normalised by the static load-carrying capacity (*max*P_s) with increasing values of loading rate

LIMITATIONS OF EXISTING NLFEA PACKAGES

To date a range of NLFEA packages have been employed to study the RC structural response under impact loading. The use of such packages allows the study of more complex structural forms (compared to the simple structural configurations studied experimentally) while providing a more detailed description of the exhibited response (i.e. stress and strain distribution, deformation profiles, failure modes and crack patterns) throughout the loading process. However, the majority of the available NLFEA packages incorporate models of concrete behaviour, the derivation of which has been based on the regression analysis of test data obtained from static uniaxial compression and tension tests on plain concrete specimens [12-13]. Furthermore, they often assume that concrete material behaviour is strain-rate dependent (sensitive) and employ laws (usually in the form of dynamic increase factors) describing the variation (increase) of key material properties (e.g. modulus of elasticity, concrete compressive and tensile strength, yield and ultimate stress of steel) with strain-rate. The analytical formulation of these material models includes a number of parameters which are mainly linked to post-peak concrete characteristics such as strain softening, tension stiffening, and shear-retention ability. Such parameters are defined at the structural, rather than at the material level and attribute ductile characteristics to concrete behaviour not compatible with its brittle nature and not justified by the available test data [12-13]. As a result, the use of such parameters can affect the objectivity of the numerical predictions obtained since they require recalibration depending on the type of problem investigated.

GENERAL ASPECTS OF THE FE MODEL PRESENTLY ADOPTED

ADINA[11] shares a number of characteristics with RC-FINEL [14-15]which has been found capable of realistically predicting the response of a wide range of RC structural configurations under static and dynamic loading. In addition, ABAQUS [10] is also employed for purposes of comparison. Both packages are capable of carrying out three-dimensional (3-D) dynamic nonlinear finite element analysis (NLFEA) while realistically accounting for the brittle nature characterising concrete material behaviour and the characteristics of the problem at hand: a wave propagation problem within a highly nonlinear medium. In both cases the equation of motion – which governs structural response – is solved numerically through the use of an implicit Newmark integration scheme. The choice of 3D dynamic NLFEA is dictated by (a) the nonlinear behaviour of concrete under triaxial stress conditions, which invariably develop prior to local failure (i.e. cracking), (b) the introduction of non-homogeneity and stress redistribution after the occurrence of cracking and (c) the development of significant inertia forces.

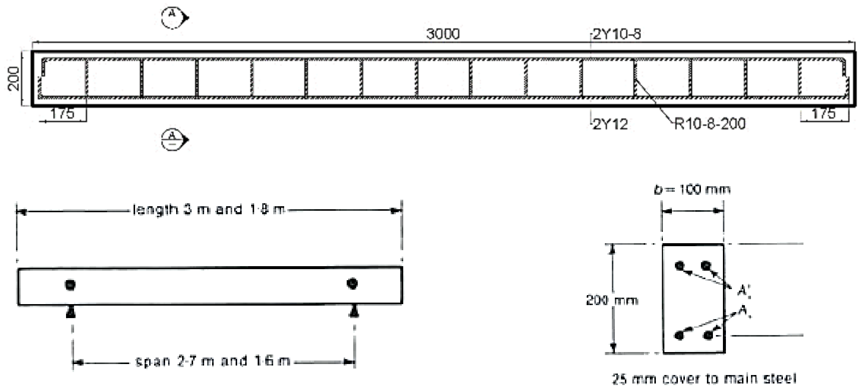
Material Modelling: The concrete material model employed by ADINA [11] stems from experimental data obtained from tests conducted on concrete cylinders under triaxial loading conditions [14,15]. It realistically accounts for the brittle nature and the triaxiality which characterises concrete material behaviour. Its formulation is characterised by both simplicity (fully brittle, with neither strain-rate nor load-path dependency, fully defined by a single material parameter - the uniaxial cylinder compressive strength f_c) and attention to the actual physical behaviour of concrete in a structure. The subject model has been successfully used to predict the behaviour of plain concrete prisms under increasing rates of uniaxial compressive and tensile loading [12,13].The predictions obtained from the latter studies suggest that the observed shift in plain concrete specimen behaviour under high rates of compressive and tensile loading is mainly attributed to parameters associated with structural response (i.e. inertia, the boundary conditions imposed and the geometry of the specimens) as well as the characteristics of the problem at hand (a wave propagation problem within a highly nonlinear medium) rather than to strain-rate sensitivity of the material properties of concrete. In the case of ABAQUS [10] a simple brittle model (termed “brittle cracking model”) is employed for describing concrete material behaviour which is purpose-built for materials the behaviour of which is dominated by tensile cracking. The latter assumption is largely true in the case of RC flexural structural elements where cracks form due to the development of tensile strains within the concrete medium. The predictions of the subject model have been validated for a wide range of RC structural configurations (beams, columns, joints) under static (monotonic and cyclic) loading conditions [16-19]. A simple bilinear elasto-plastic hardening model is employed for describing the behaviour of steel. Finally, concrete and steel material behaviour is assumed to be independent of the loading-rate and full bond is assumed in order to describe the interaction between the two materials.

Nonlinear Solution Strategy: During each time step the equation of motion governing the nonlinear dynamic problem considered is solved as a sequence of equivalent static problems through the use of the Newmark family of approximation methods. At the beginning of each iteration and based on the values of displacement, velocity and acceleration obtained from the previous iteration, the effective stiffness and load matrix are calculated and an equivalent static problem is formulated [15]. The equivalent static problem is solved through an iterative procedure based on the Newton-Raphson method [10, 11, 15]. During the solution process of the equivalent static problem every Gauss point is checked to determine whether loading or unloading takes place and to establish whether any cracks close or form. Depending on the results of the previous checks, changes are introduced to the stress-strain matrices of the individual FE's and to the global stiffness matrix representing the structure investigated. Convergence is checked locally at each Gauss point and once the values of the strain and the corresponding stress increments become less than a small predefined value (i.e. convergence criterion) then convergence is accomplished and the solution can move on to the next time step. When the convergence criterion is not achieved, the residual forces are calculated and are then re-imposed onto the FE model of the RC form investigated until convergence is finally achieved.

Modelling of Cracking: The smeared-crack approach is adopted for modelling cracking. A crack forms when the stress developing in a given part of the structure corresponds to a point in the principal stress space that lies outside the predefined failure surface of concrete material. This is then followed by an immediate loss of load-carrying capacity in the direction normal to the plane of the crack. At the same time, the shear stiffness is also reduced drastically to a small percentage (about 5 to 10%) of its previous value (before the occurrence of the crack). However, it is not set to zero in order to minimize the risk of numerical instability during the execution of the solution procedure, as explained elsewhere [15]. It should be noted that each integration point can develop up to three cracks.

STRUCTURAL FORM INVESTIGATED

The behaviour of the RC beam specimens considered herein (C2, D1, E1) has been experimentally investigated in the past [1] under static and impact loading. The design details of these specimens are presented in Figure6.The elasticity modulus (E_S), the yield stress (f_y), and the ultimate strength (f_u) of both the longitudinal and transverse reinforcement bars are 206 GPa, 460 MPa and 560 MPa, respectively. The uniaxial compressive strength (f_c) of concrete is 45 MPa. The subject beams were subjected to drop weight testing at their mid-span. Mild steel, rubber or ply pads were placed on the top face of the specimen in order to prevent or moderate local damage (cracking) in the impact area and to some extent control the rate of loading.



Beam type	Length	Tensile Steel A_s	Compression Steel $A_{s'}$	Stirrups
C2	3m	2x12Φ	2x6Φ	14x6
D1	3m	2x16	2x6	14x6
E1	1.8	2x12	2x6	8x6

Figure 6 RC beam investigated [1]

FE MODELLING OF THE PROBLEM AT HAND

In ADINA [11] concrete is modelled using a mesh of 27-noded brick elements which adopt a 3x3x3 integration rule. In ABAQUS [10] concrete is modelled through the use of a dense mesh of 8-node brick elements the formulation of which adopts a reduced integration scheme to avoid numerical instabilities due to locking. The steel reinforcement bars are modelled as 2-node truss elements of appropriate cross-sectional area which are embedded in the FE mesh representing the concrete medium. Due to the double symmetry of the problem at hand, only a quarter of each beam specimen is modelled with suitable boundary conditions, see Figure 7. The load is assumed to be applied onto the mid-span of the beam through a steel plate. For the case of static loading the load is applied monotonically until failure in the form of displacement increments (displacement control). In the dynamic case studies the load is imposed in the form of load increments applied either monotonically to failure (at a constant rate) or in the form of the pulse described by the force-time histories presented Figure 1.

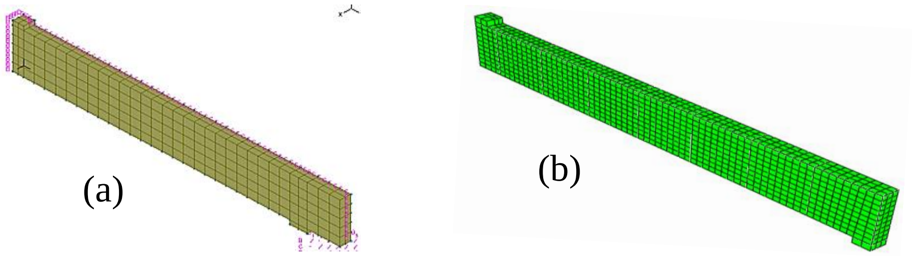


Figure 7 FE mesh adopted (a) ADINA and (b) ABAQUS

NUMERICAL PREDICTIONS

Static Case Study: For the case of static loading the predictions obtained describing the behaviour of the beam specimens are presented in Figure 8a in the form of a load-displacement curves which represent the relation between the applied load and the deflection at the load point (i.e. mid-span). The latter curves are in good agreement with their experimentally established counterparts [1]. Figure8b shows the deformation and cracking profiles predicted for the case of specimen C2 at different stages of the loading process. On the basis of these predictions flexural cracks begin to appear in the mid-span region of the specimen and, as the imposed load increases, they gradually spread towards the supports. Overall, the numerical predictions concerning the response of the RC beam specimens is in generally good agreement with that established experimentally. Both the experimental measurements and their numerically established counterparts show that all beams exhibited ductile behaviour, with failure occurring after yielding of the longitudinal reinforcement bars in the mid-span region of the specimen, resulting in the formation of extensive cracking that, ultimately, leading to loss of load-carrying capacity of the compressive zone at this location.

Monotonic high rate loading: The values of the applied loading rates considered in the numerical study range from 1 to 103 kN/ms which is in good agreement with the loading rates achieved experimentally [1]. The predicted load-displacement curves presented in Figure 9a reveal that an increase in the loading rate leads to an increase in stiffness and load-carrying capacity and a reduction of the maximum deflection exhibited at mid-span. As regards the cracking and deformation profiles, Figure 9b indicates that, under relatively low loading rates, beam behaviour is qualitatively similar to that exhibited under static loading. However, as the rate of loading increases, the portion (L_{eff}) of the beam essentially affected by the applied load reduces. More specifically, for high rates of loading, L_{eff} extends on either side of the mid-span cross section (area at which the impact load is applied) to a distance marked by the formation of vertical (flexural) cracking initiating at the upper face of the beam and extending downwards (see Figure9b), whereas the remainder of the beam (extending between the supports and the aforementioned cracking) practically remain unaffected by the applied load. Therefore, under high rates of loading, beam behaviour is essentially controlled by L_{eff}. The reduction of L_{eff} under increasing loading rates can explain the experimentally observed increase in stiffness and maximum sustained load. The variation of the dynamic increase factor (*DIF*), i.e. the ratio between the maximum load sustained ($maxP_d$) by the RC beams under high rate loading and the load-carrying capacity determined under static loading ($maxP_s$) ($DIF = maxP_d/maxP_s$), with increasing loading rates is presented in Figure10. The experimentally established values of the *DIF* are in good agreement with their counterparts predicted numerically by ADINA and ABAQUS for all three types of RC beam specimens considered herein.

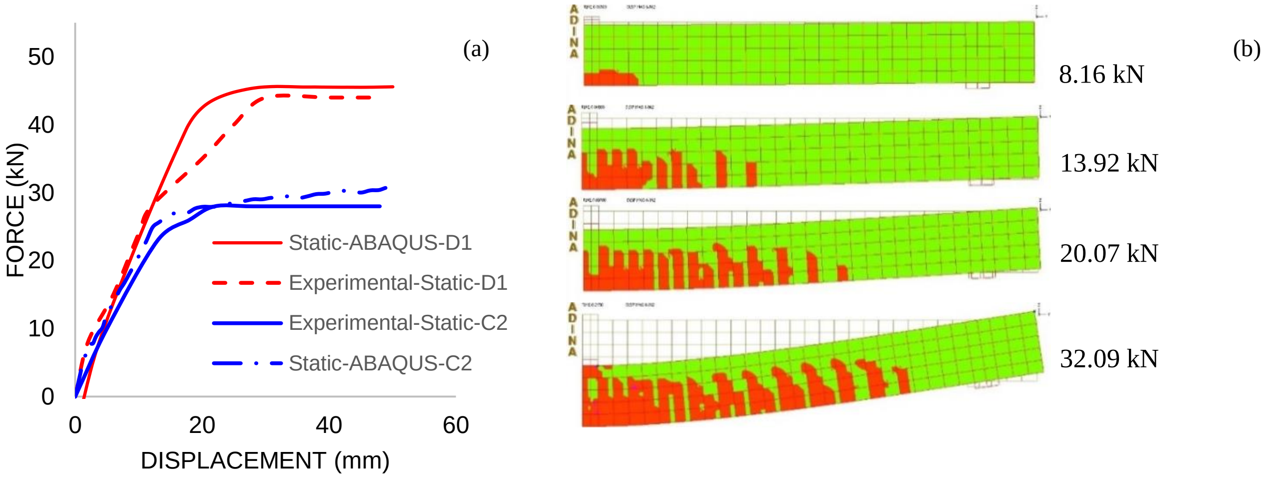


Figure 8 (a) Static load-deflection curves established experimentally and numerically by ABAQUS; (b) deformation and cracking profiles exhibited by beam C2 under static loading predicted by ADINA

High rate pulse loading: In an attempt to assess the effect of loading rate and intensity separately, high rate loading is imposed in the form of a concentrated pulse load (see Figure 1) at the mid-span region of the specimens presently considered (see Figure 6). In order to determine the actual load-bearing capacity of the RC beams for a specific value of loading rate ($\dot{P}$) a parametric study is carried out in which the intensity (*peak value*) of the imposed dynamic pulse load is varied as shown in Figure 1. Typical predictions obtained from these parametric studies are presented in Figure 11 for the case of beam C2 [1]. These predictions describe the variation of the mid-span deflection with time when the specimen is subjected to pulse loads characterised by different intensities (peak values) but the same loading rate: $\dot{P}=$ 400kN/msec. The latter curves reveal that when the *peak value* of the pulse load is higher than 25% of $maxP_d$ (obtained for the case of monotonic high rate loading) the RC beam fails during unloading. However, when the *peak value* of the pulse load becomes equal or less than 25% $maxP_d$, the RC beam does not fail, but continues to oscillate after unloading takes place. The latter critical value (≈ 25% $maxP_d$) can be considered as the true load bearing capacity of the RC beam for $\dot{P}=$ 400kN/msec.

The variation of *DIF* associated with the *peak load* of the pulse load that will not result in failure of the RC beam during unloading (DIF = *peak value*$/maxP_s$) is presented in Figures10(a-c) for different loading rates for all specimens presently considered. These curves are compared to their counterparts established numerically for the case of monotonic high rate loading and experimentally [1]. It is interesting to note that the predictions concerning *DIF* for the case of pulse loading provided by ABAQUS are higher than their counterparts provided by ADINA. This is attributed to the differences in the formulation of the concrete material models adopted by the two packages, with the ADINA model considered to provide a more realistic description of concrete behaviour. Furthermore, the results obtained from drop-weight testing appear to be in good agreement with their counterparts predicted numerically for the case of the monotonically applied high rate load. This suggests that the relevant tests data are associated with post-failure behaviour and that that the true load-bearing capacity of the RC beams under impact loading is considerably lower than the latter values. The higher the loading rate and intensity of the impact load the larger the latter difference becomes. Figure 12 shows the predicted deformation profiles and the associated crack patterns developing on specimen C2 when subjected to pulse loads with $\dot{P}$= 400kN/msec but different intensities. From these figures it is observed that for a specific loading rate a reduction in the *peak value* of the pulse load applied will result in more global response as a larger portion of the span reacts to the applied load. This suggests that the L_{eff} and the mechanics underlying RC structural response will be affected by both the rate of applied loading and intensity of the imposed impact load.

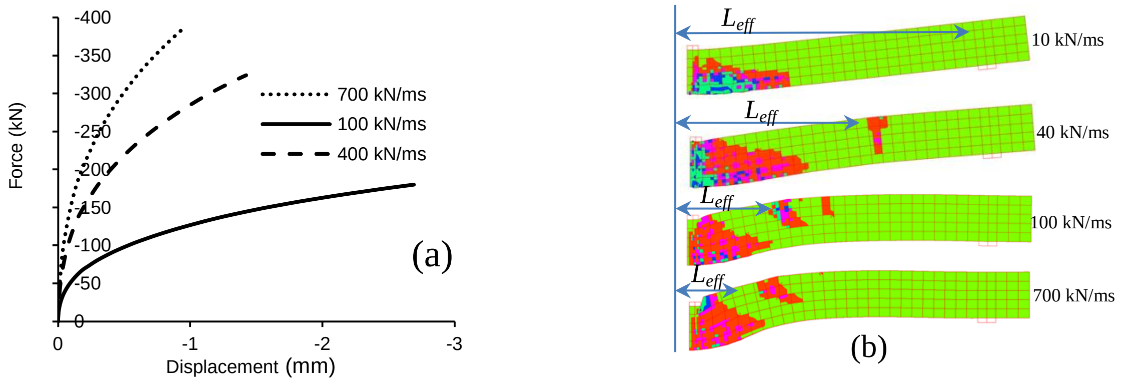
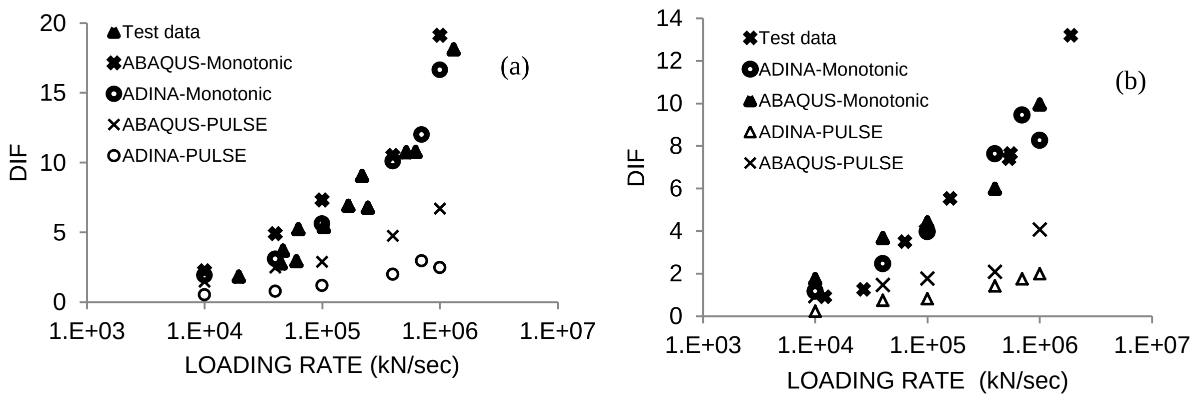


Figure 9 Beam C2 under loading applied at various rates: (a) Load-deflection curves; (b) Deformation and cracking profiles.



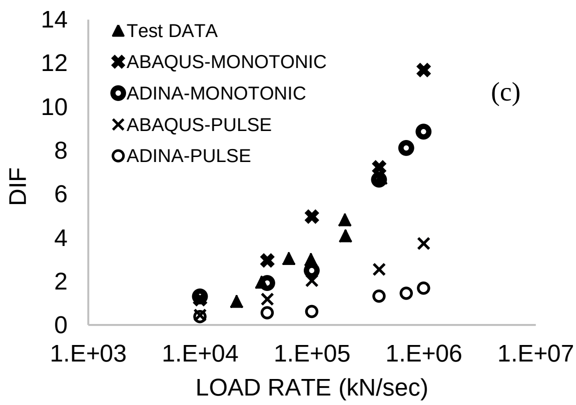


Figure 10 Comparison of experimental and numerical predictions expressing the variation of DIF with increasing values of loading rates observed for RC beam specimen (a) C2, (b) D1 and (c) E1

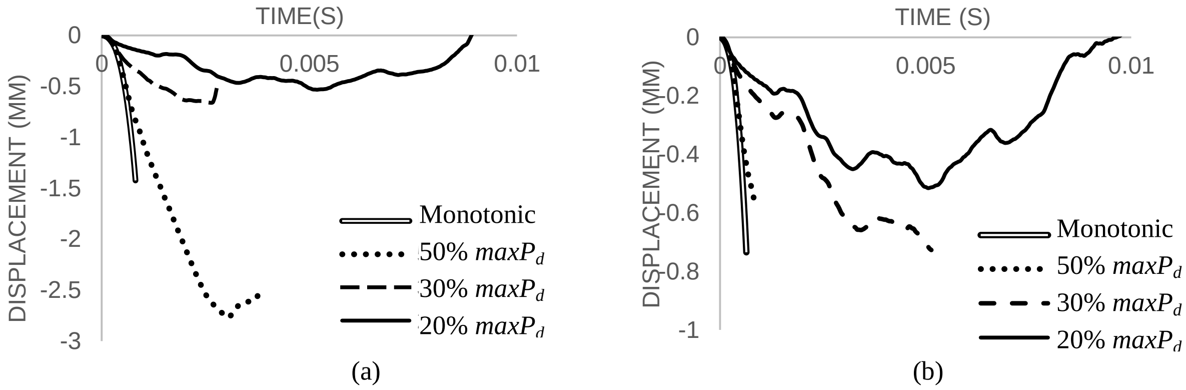


Figure 11 Predicted mid-span displacement time history obtained for pulse loads with $\dot{P}$=400kN/ms but different intensities for (a) Specimen C2 and (b) Specimen E1.

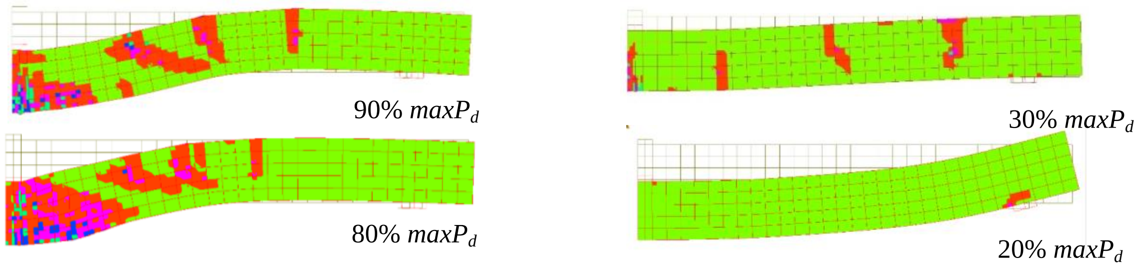


Figure 12 Predicted crack patterns and deformation profiles for specimen C2 for pulse loads with $\dot{P}$=400 kN/ms but different intensities

CONCLUDING REMARKS

The comparative study between the numerical predictions and their experimental counter parts reveals that the concrete material models presently employed by ADINA and ABAQUS are capable of providing realistic predictions concerning certain aspects of the response exhibited by RC beams under increasing rates and intensities of impact loading. The predictions obtained confirm the findings of previous published numerical studies [6-8] which suggest that effect of loading rate on RC structural response reflects the influence of inertia and the nature of the problem at hand: a wave propagation problem within a highly nonlinear medium. The numerical investigation also reveals that true load-bearing capacity is frequently significantly lower than the maximum sustained load recorded experimentally, the latter being associated with a specimen physical state characterised by considerable concrete disintegration and low residual stiffness and load-bearing capacity. The higher the loading rate and intensity of the impact load the larger the latter difference becomes. Further detailed experimental and numerical studies are currently being conducted in order to consider a wider range of RC structural elements (beams, columns, slabs, walls) and design parameters. The predictions obtained are presently forming the basis for the development of a new method for assessing the performance exhibited by RC structural elements under impact loading.

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