A comparative study of error distributions in the GARCH model through a Monte Carlo simulation approach

Citation for published version:

Digital Object Identifier (DOI):
10.1016/j.sciaf.2023.e01988

Link:
Link to publication record in Heriot-Watt Research Portal

Document Version:
Publisher's PDF, also known as Version of record

Published In:
Scientific African

Publisher Rights Statement:
© 2023 The Author(s).

General rights
Copyright for the publications made accessible via Heriot-Watt Research Portal is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

Take down policy
Heriot-Watt University has made every reasonable effort to ensure that the content in Heriot-Watt Research Portal complies with UK legislation. If you believe that the public display of this file breaches copyright please contact open.access@hw.ac.uk providing details, and we will remove access to the work immediately and investigate your claim.
A comparative study of error distributions in the GARCH model through a Monte Carlo simulation approach

Samuel Ampadu, Eric T. Mensah, Eric N. Aidoo, Alexander Boateng, Daniel Maposa

A R T I C L E   I N F O

Editor: DR B Gyampoh

JEL classification: C14 C15

Keywords:
Error distributions
GARCH model
Financial time series
Skewed generalized error distribution

A B S T R A C T

Financial time series data are known to exhibit volatility clustering, which implies that the volatility of financial returns tends to persist over time. This phenomenon has significant implications for risk management and financial decision-making. To capture this volatility clustering effect, the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model is widely used. Through a Monte Carlo simulation experiment, the performance of different GARCH-type models with five different error distribution specifications were compared. The study found that the Skewed Generalized Error Distribution (SGED) was the most efficient and consistent distribution for modelling financial time series. The SGED outperformed the Normal, Student’s t, GED, and skewed Student’s t distributions in terms of goodness-of-fit, MSE, MAE and different sample sizes. The study suggests that the SGED may be more suitable for modelling financial time series with a GARCH-type specification in general contexts.

I n t r o d u c t i o n

Financial time series data provide useful information for financial analysts and decision-makers regarding the behaviour of the financial market over a given period. Modelling and forecasting this type of data in relation to its future behaviour is important for financial investors and traders for planning and decision-making [16,22]. However, modelling and forecasting this type of time series data remains a challenging task as such data is known to have noisy dynamics, high volatility, excess kurtosis, volatility clustering, heavy tail and non-stationary properties [2,7,15]. These features of financial data are influenced by several forces including economic situations, public health emergencies, speculative information, investors’ expectations, financial crises, and political events, [19,28].

In an attempt to model and predict the volatility of financial time series, different models with different statistical properties have been proposed [3,10,30]. From the existing literature, the generalized autoregressive conditional heteroskedastic (GARCH) model proposed by Bollerslev [3] has been frequently used to characterise the volatility of most financial time series. The GARCH model extends the autoregressive conditional heteroskedastic (ARCH) model introduced by [10] to solve the problem of higher order specifications to ensure more parsimonious model. Volatility modelling in financial time series using the GARCH model is somewhat motivating as some studies have discovered good in-sample performance compared to other models.
The standard GARCH model assumed that the error is normally distributed with zero mean and unit variance. However, a GARCH model with a such single assumed distribution may not be suitable for modelling the various characteristics of financial time series as described earlier. The GARCH model occasionally fails to capture the fat-tail characteristic of financial time series data [11,29]. In most scenarios, extra kurtosis remains in the standardized residuals. According to Cerqueti et al. [6] the assumption of normally distributed error for GARCH model in volatility modelling is not a reliable choice and more sophisticated probabilistic assumption may be required. Although, the GARCH model in some cases produce consistent estimates even when the true distribution is heavier-tailed [20], Bollerslev [4] suggested that the GARCH model with a corresponding true distribution can produce accurate results. This is especially true for predicting financial volatility, which is essential for portfolio risk management tasks such as risk assessment using Value-at-Risk and Conditional-Tail-Expectation models. In addition, inappropriate error distribution can lead to unreliable volatility forecasts, as the model’s assumptions regarding the conditional variance may not align with the data. A common solution to this problem is to utilise some fat-tailed distributions such as general error and the Student’s t distributions [8,31].

Several empirical studies have been conducted to explore suitable error distributions for GARCH model in volatility modelling and forecasting. For instance, Cerqueti et al. [6] explored the use of skewed non-Gaussian GARCH models for modelling volatility of cryptocurrencies. The results of the study revealed that GARCH-type model with skewed and skewed generalized error distributions improves cryptocurrencies volatility specification and forecast accuracy. Similarly, Abdullah et al. [1] compared GARCH-type models with normal and student’s t error distributions for modelling and forecasting exchange rate volatility in Bangladesh. The results of the study revealed that GARCH-type models with student’s t error distribution provide better fit and forecast for the data. Liu and Morley [21] explore the ability of GARCH-type models to forecast the volatility in Hang Seng index. From the study, it was found that the GARCH model with non-normal conditional volatility yielded more efficient out-of-sample estimates. GARCH model with non-normal error distribution may provide a better basis for volatility modelling and forecasting. However, the choice of a particular non-normal error distribution may be data dependent.

A suitable GARCH model with non-normal error distribution that is more robust to different conditions can be determined through Monte Carlo simulation comparison. However, such model comparison via Monte Carlo simulation is limited in the existing literature. This leaves the gap on how to choose the right error distribution for a given data. This study seeks to compare the forecasting accuracy of five different error distributions commonly used in GARCH modelling: Normal, Student’s t, Generalized Error Distribution (GED), skewed Student’s t, and skewed GED via Monte Carlo Simulation. Monte Carlo simulation is a technique used to generate a probability space by taking independent random samples from the space according to the probability distribution [24]. This is essential in comparing error distributions because it gives a robust and adjustable framework for evaluating different error distributions in modelling uncertainty [26]. The results of the study can inform practitioners and researchers in finance in selecting the appropriate error distribution for GARCH modelling in their applications. Generally, the study contributes to the existing literature on GARCH modelling by providing a comprehensive analysis of the impact of different error distributions on the performance of the GARCH model.

Materials and methods

The methodology employed in this study involves the use of GARCH-type models, which are not only popular for their simplicity and efficiency but also for their ability to generalize various volatility measures. In this section, we provide a concise overview of the conditional volatility models used. When discussing volatility in financial time series, several noteworthy characteristics emerge, such as fat-tail (leptokurtic) distributions of risky asset returns, asymmetry, time-varying volatility, volatility clustering, pronounced persistence, mean reversion, and the interrelated movements of volatilities across different assets and financial markets. The GARCH class of conditional volatility models, perhaps the most widely used family of volatility models, is specifically designed to address these issues.

Autoregressive conditional heteroskedasticity (ARCH) model

The family of ARCH models was established by Engle [10] to accommodate the changing aspects of conditional heteroskedasticity in time series [13]. Many researchers have employed ARCH models extensively in various aspects of financial time series research. Let $\varepsilon_t (t = 1, \ldots, T)$ represent the random shocks or error term of a return relative to the mean. This $\varepsilon_t$ can be divided into two components: a stochastic component denoted as $\eta_t$ and a time-varying standard deviation denoted as $\sigma_t$, which have the following relation:

$$\varepsilon_t = \sigma_t \eta_t$$

(1)

The random $\eta_t$ is a strong white noise process. The series $\varepsilon_t$ is expressed using the ARCH model of order quasusly written as ARCH($q$) and defined by:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \ldots + \alpha_q \varepsilon_{t-q}^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2$$

(2)

where $\sigma_t^2$ represents the conditional variance (volatility), $\alpha_0$ represents a constant term in the model and $\alpha_i (i = 1, \ldots, q)$ denote the ARCH coefficients, which represent the effect of the squared residuals from previous periods ($\varepsilon_{t-1}^2, \ldots, \varepsilon_{t-q}^2$) on the recent conditional variance $\sigma_t^2$. The parameters must satisfy the conditions $\alpha > 0$ and $\alpha_i \geq 0$ for $i \geq 1$ when the conditional variance is positive. Estimating the ARCH coefficients, $\alpha_i$, is mostly done using the maximum likelihood estimation (MLE).
Generalized autoregressive conditional heteroskedasticity (GARCH) model

The GARCH model proposed by Bollerslev [3] is an extension of the ARCH model to solve the problem of higher order ARCH model specifications to ensure more parsimonious model. The

Volatility \( \sigma_t^2 \) is expressed using the GARCH model of order \( p \) and \( q \) usually written as GARCH\((p, q)\) and defined by:

\[
\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2
\]

where \( \beta_j (j = 1, \ldots, p) \) represent the GARCH coefficients, which denote the effect of the past conditional variances \( \sigma_{t-j}^2 \) on the recent conditional variance \( \sigma_t^2 \). The parameters of the GARCH model satisfy the following constraints: \( \alpha > 0, \alpha_i \geq 0, \beta_j \geq 0 \) for \( j = 1, \ldots, p \) and \( \sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1 \). In the conventional GARCH model, the random shocks \( \varepsilon_t^2 \) are assumed to be normally distributed with zero mean and variance \( \sigma_t^2 \) \( (\varepsilon_t \sim N(0, \sigma_t^2)) \). To account for excess high kurtosis, fat-tails and other properties associated with financial time series volatility, different distributions other than the normal distribution can be assumed for \( \varepsilon_t^2 \). In this study, the five different distributions were assumed for the random shocks. These are the generalized error distribution (GED), skewed generalized error distribution (SGED), Student’s t distribution (SD), skewed Student’s t distribution (SSD) and the normal distribution. These distributions are appropriate to capture the excess kurtosis and the skewness in the volatility series. The description of each of the distributions is presented in the next subsection.

Generalized error distribution (GED)

The is a family of distributions distribution which was introduced by Subbotin [27] and has been used by different authors under different name and parametrization. Nelson [25] advised considering the family of GEDs since it accounts for skewness and kurtosis and these features are significant in financial applications. The probability density function for the error term, \( \varepsilon_t \) in the GARCH process under GED is given by:

\[
f(\varepsilon_t) = \frac{\eta \exp \left( -\frac{|\varepsilon_t|^\eta}{\lambda} \right)}{2^{(\eta-1)/2} \Gamma(1/\eta) \lambda^{1/2}}
\]

where

\[
\lambda = \left[ \frac{2^{-2/\eta} \Gamma(1/\eta)}{\Gamma(3/\eta)} \right]^{1/2}
\]

where \( \eta > 0 \) is a parameter that controls the shape of the distribution, \( \sigma_t \) represents the conditional standard deviation of the residuals, and \( \Gamma(\cdot) \) represents the gamma function [11]. The GED distribution converges to the normal distribution when \( \eta = 2 \) and a fat-tail distribution when \( \eta < 2 \).

Skewed generalized error distribution (SGED)

The skewed generalized error distribution (SGED) is a complex distribution that combines skewness with the flexibility of the generalized error distribution (GED). The probability density function of \( \varepsilon_t \) under the SGED is defined as:

\[
f(\varepsilon_t) = \phi \exp \left( -\frac{|\varepsilon_t + \delta|^{\theta}}{1 + \text{sign}(\varepsilon_t + \delta) \varphi |\varepsilon_t|^{\theta}} \right)
\]

where

\[
\phi = \frac{k}{2\theta} \Gamma \left( \frac{1}{k} \right)^{-1}, \quad \theta = \Gamma \left( \frac{1}{k} \right)^{0.5} \Gamma \left( \frac{3}{k} \right)^{-0.5} S(\varphi)^{-1},
\]

\[
S(\varphi) = \sqrt{1 + 3\varphi^2 - 4\delta^2}, \quad \delta = \frac{2\varphi A}{S(\varphi)} A = \Gamma \left( \frac{2}{k} \right) \Gamma \left( \frac{1}{k} \right)^{-0.5} \Gamma \left( \frac{2}{k} \right)^{-0.5}
\]

where \( k > 0 \) controls the tails and height of the distribution, and \( -1 < \varphi < 1 \) controls the skewness of the density, sign represents the sign function. For \( k = 2 \) and \( \varphi = 0 \), the SGED converges to the standard normal distribution [18].

Student’s t distribution

The probability density function of the random variable \( \varepsilon_t \) under the Student’s t distribution is given by [9,14]:
Skewed Student’s t distribution

The skewed Student’s t distribution proposed by Hansen [14] extends the standardized Student’s t distribution to account for skewness. The probability density function of the random variable \( \varepsilon_t \) under the skewed Student’s t distribution is given by [14]:

\[
f(\varepsilon_t) = \begin{cases} 
bc \left(1 + \frac{1}{\eta - 2} \frac{a + b \varepsilon_t}{1 - m}\right)^{-\eta/2} & \varepsilon_t < -a/b \\
bc \left(1 + \frac{1}{\eta - 2} \frac{a + b \varepsilon_t}{1 + m}\right)^{-\eta/2} & \varepsilon_t \geq -a/b 
\end{cases}
\]

where the constants \( a, b, \) and \( c \) are given by:

\[
a = 4mc \left(\frac{\eta - 2}{\eta - 1}\right), \quad b^2 = 1 + 3m^2 - a^2, \quad c = \frac{\Gamma\left(\frac{\eta}{2}\right)}{\sqrt{\pi (\eta - 2)} \Gamma(\eta/2)}
\]

where \( 2 < \eta < \infty \) represents the degree of freedom, and \( -1 < \eta < 1 \) represent the skewness parameter.

Normal distribution

The probability density function of the random variable \( \varepsilon_t \) under the normal distribution is given by:

\[
f(\varepsilon_t) = \frac{1}{\sqrt{2\pi}\sigma_t^2} \exp\left(-\frac{\varepsilon_t^2}{2\sigma_t^2}\right)
\]

where \( \sigma_t^2 \) is the conditional variance and the mean is set to zero [11].

Model comparison

The choice of a model is an important aspect of statistical analysis. By considering information criteria, the suitable distribution selection leads to the correct error distribution. The use of information criteria to select the best-fitting distribution from a group of competing distributions is known as model selection [12]. The lower the value of the information criteria, the more suited the distribution.

The Akaike information criterion (AIC) and the Bayesian information criterion (BIC) were used in this study. The AIC and BIC are defined as:

\[
AIC = -2\tilde{L} + 2K
\]

\[
BIC = -2\tilde{L} + K\log(T)
\]

where \( \tilde{L} \) denote the estimates of the loglikelihood function of the fitted model and \( K \) denote the number of estimated parameters in the model and \( T \) represents the number of observations. The AIC contains a penalty that accumulates in comparison to the number of estimated parameters, as well as encourages goodness-of-fit (as measured by the loglikelihood function). Overfitting is discouraged by the penalty, which is desirable because increasing the number of parameters in a model almost always enhances the goodness of fit. On the other hand, the BIC accounts for the effect of sample size in the penalization. According to Markon and Krueger [23], AIC’s model selection ability performs well for small sample sizes whilst the BIC appears to perform better with large sample sizes.

Another metrics which was used to measure the performance of the GARCH model amongst the competing error distributions were the mean squared error (MSE) and mean absolute error (MAE). The MSE measures the average squared difference between the true values and the estimated values while the MAE quantifies the average absolute difference between estimated values and the true values. The latter is useful when you want to examine the difference without considering the direction of the deviations. The MSE and MAE are defined as:

\[
MSE = \frac{1}{N} \sum_{t=1}^{N} (A_t - \hat{E}_t)^2
\]
$MAE = \frac{1}{N} \sum_{i=1}^{N} |A_{r} - \hat{E}_{r}|$ (12)

where $A_{r}$ and $\hat{E}_{r}$ represent the true and estimated values of the parameter space, respectively, and $N$ represents the number of samples.

**Simulation procedure**

To determine the performance of GARCH models under the five error distributions, a Monte Carlo simulation experiment was conducted. In the simulation process, 5000 replications and four different sample sizes ($T = 50, 1000, 5000,$ and $8000$) were generated. During the data generation process, five forms of error distributions in the GARCH model were specified. These data were simulated as follows:

i. Specify a univariate GARCH(1,1) model with the following specification: the conditional variance, $\sigma^2_t = \Omega + \alpha \varepsilon^2_{t-1} + \beta \sigma^2_{t-1}$ and the parameter values are $\alpha = 0.15$, $\beta = 0.65$, and $\Omega = 0.02$, where $\Omega$ represent the constant coefficient of the variance equation, $\alpha$ represent the autoregressive coefficients, and $\beta$ represent the variance coefficients.

ii. Use the information obtained in stage 1 (i) to simulate data from the GARCH process with a defined error distribution. The error distribution was alternated amongst the normal, GED, Skewed GED, Student’s $t$, or skewed Student’s $t$.

iii. Fit and estimate the parameters of the GARCH model to the simulated data obtained in (ii) using the following distributions: GED, Skewed GED, Student’s $t$, skewed Student’s $t$ and normal distribution.

iv. Repeat the simulation process from (i) to (iii) in 5000 times for different samples $T = 50, 1000, 5000,$ and $8000$ and estimate the parameters accordingly. Compute the appropriate accuracy statistics and determine the average over 5000 replications.

**Results and discussion**

The performance of GARCH models under five different error distributions were compared through a Monte Carlo simulations experiment. The model at each instance was varied for four different sample sizes $T = 50, 1000, 5000,$ and $8000$. The GARCH (1, 1) process were set within the assumption of stationary which indicates that $\alpha + \beta < 1$. The results from the GARCH model with the five different error distributions were compared using their standard errors, MSE, MAE, AIC, and BIC.

**Simulation results for GARCH model when true error distribution is normal**

The simulation results for the GARCH model when the true error distribution is normal are presented in Table 1. The AIC and BIC of
the fitted GARCH model with the normal errors were less compared to the models with other distributions suggesting a good performance. The results were consistent for the varying samples sizes. This was expected since the data was simulated from such distribution. However, in terms of standard error, the SGED provides a close to the normal distribution particularly in small samples. In addition, the SGED provides similar MSE and better MAE than the normal distribution. These results are consistent for an increasing
sample size. For a sample size of 1000 and more, the results for GED, SGED and the normal distribution are similar. In general, the Student's t and skewed Student's t distribution provides the worse results in terms of goodness-of-fit, MSE and MAE compared to the other distributions particularly for a sample size of 50.

Simulation results for GARCH model when true error distribution is GED

The simulation results for the GARCH model when the true error follows the generalized error distribution are presented in Table 2. The results show that the GARCH model with GED error specification has a better fit according to the AIC whilst the model with normal error specification does better according to the BIC. In addition, the standard error from the GARCH with GED specification were less for the parameters associated with the ARCH and GARCH terms but not for the constant term in the model. The MSE for GARCH specification with normal errors was lower than a model with GED errors even though the simulated data comes from GED specification.

According to the MAE the GED model specification is better only for sample size above 1000. For sample size 1000 and below the normal GARCH model specification was better. In general, the skewed Student's t distribution does worse in terms of model specification and parameter estimates when the true error distribution is GED. In general, the Student's t distribution provides the worse results in terms of goodness-of-fit, MSE, and MAE compared to the other distributions.

Simulation results for GARCH model when true error distribution is skewed GED

The simulation results for the GARCH model when the true error follows the generalized error distribution are presented in Table 3. The results show that the goodness-of-fit as measured by AIC and BIC for the GARCH model with SGED specification were better compared to the other four GARCH model specification. The fit is generally better for high for an increasing sample size. The standard error of the estimated parameters for the ARCH and GARCH terms were better for the SSD compared to the other distributions whilst the normal distribution also produces low standard errors for the constant term in the model for a sample size of 50 and similar results to that of SGED and GED. In addition, the MSE associated with the estimated parameters of the SGED were generally low. However, the MAE for the ARCH and GARCH parameters generally favours the GED model specification. The MAE further shows that the SGED was better only for the constant term in the model.

Simulation results for GARCH model when true error distribution is student's t

The AIC and BIC of the simulation from Table 4 shows that the GARCH model specification with normal errors has good fit compared to the other models even though the data were simulated from the Student's t distribution. The performance of the model consistently increases for an increasing sample size. This may be influenced by the relationship between the normal and the Student's t.

<p>| Table. 4 | The average of the estimated standard error, MSE, MAE, AIC, and BIC for GARCH model with different error distributions and sample sizes. The true error distribution was assumed to be Student’s t. |</p>
<table>
<thead>
<tr>
<th>Sample size</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\Omega$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\Omega$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\Omega$</th>
<th>AIC</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.199</td>
<td>0.462</td>
<td>0.032</td>
<td>0.029</td>
<td>0.772</td>
<td>0.007</td>
<td>0.077</td>
<td>0.442</td>
<td>0.025</td>
<td>0.561</td>
<td>0.676</td>
</tr>
<tr>
<td>1000</td>
<td>0.383</td>
<td>0.090</td>
<td>0.007</td>
<td>0.026</td>
<td>0.253</td>
<td>0.000</td>
<td>0.062</td>
<td>0.079</td>
<td>0.024</td>
<td>0.488</td>
<td>0.503</td>
</tr>
<tr>
<td>5000</td>
<td>0.169</td>
<td>0.041</td>
<td>0.003</td>
<td>0.023</td>
<td>0.217</td>
<td>0.000</td>
<td>0.053</td>
<td>0.021</td>
<td>0.017</td>
<td>0.486</td>
<td>0.490</td>
</tr>
<tr>
<td>8000</td>
<td>0.134</td>
<td>0.034</td>
<td>0.002</td>
<td>0.015</td>
<td>0.122</td>
<td>0.000</td>
<td>0.047</td>
<td>0.012</td>
<td>0.015</td>
<td>0.485</td>
<td>0.485</td>
</tr>
<tr>
<td>GED</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.188</td>
<td>0.447</td>
<td>0.031</td>
<td>0.043</td>
<td>0.835</td>
<td>0.005</td>
<td>0.085</td>
<td>0.427</td>
<td>0.021</td>
<td>0.574</td>
<td>0.727</td>
</tr>
<tr>
<td>1000</td>
<td>0.038</td>
<td>0.098</td>
<td>0.007</td>
<td>0.026</td>
<td>0.253</td>
<td>0.000</td>
<td>0.062</td>
<td>0.078</td>
<td>0.012</td>
<td>0.489</td>
<td>0.509</td>
</tr>
<tr>
<td>5000</td>
<td>0.016</td>
<td>0.040</td>
<td>0.003</td>
<td>0.023</td>
<td>0.213</td>
<td>0.000</td>
<td>0.053</td>
<td>0.021</td>
<td>0.010</td>
<td>0.486</td>
<td>0.491</td>
</tr>
<tr>
<td>8000</td>
<td>0.013</td>
<td>0.320</td>
<td>0.002</td>
<td>0.014</td>
<td>0.202</td>
<td>0.000</td>
<td>0.016</td>
<td>0.012</td>
<td>0.009</td>
<td>0.485</td>
<td>0.489</td>
</tr>
<tr>
<td>SGED</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.156</td>
<td>0.385</td>
<td>0.030</td>
<td>0.027</td>
<td>0.611</td>
<td>0.004</td>
<td>0.107</td>
<td>0.376</td>
<td>0.022</td>
<td>0.570</td>
<td>0.761</td>
</tr>
<tr>
<td>1000</td>
<td>0.038</td>
<td>0.095</td>
<td>0.007</td>
<td>0.026</td>
<td>0.252</td>
<td>0.000</td>
<td>0.105</td>
<td>0.078</td>
<td>0.012</td>
<td>0.490</td>
<td>0.514</td>
</tr>
<tr>
<td>5000</td>
<td>0.016</td>
<td>0.168</td>
<td>0.003</td>
<td>0.023</td>
<td>0.212</td>
<td>0.000</td>
<td>0.103</td>
<td>0.021</td>
<td>0.010</td>
<td>0.485</td>
<td>0.486</td>
</tr>
<tr>
<td>8000</td>
<td>0.013</td>
<td>0.013</td>
<td>0.002</td>
<td>0.014</td>
<td>0.202</td>
<td>0.000</td>
<td>0.096</td>
<td>0.012</td>
<td>0.009</td>
<td>0.484</td>
<td>0.485</td>
</tr>
<tr>
<td>SD</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.465</td>
<td>0.635</td>
<td>0.078</td>
<td>0.520</td>
<td>0.556</td>
<td>0.355</td>
<td>0.324</td>
<td>0.615</td>
<td>0.064</td>
<td>0.619</td>
<td>0.772</td>
</tr>
<tr>
<td>1000</td>
<td>0.045</td>
<td>0.110</td>
<td>0.009</td>
<td>0.023</td>
<td>0.241</td>
<td>0.000</td>
<td>0.154</td>
<td>0.091</td>
<td>0.011</td>
<td>0.502</td>
<td>0.526</td>
</tr>
<tr>
<td>5000</td>
<td>0.020</td>
<td>0.046</td>
<td>0.003</td>
<td>0.023</td>
<td>0.207</td>
<td>0.000</td>
<td>0.149</td>
<td>0.026</td>
<td>0.010</td>
<td>0.498</td>
<td>0.503</td>
</tr>
<tr>
<td>8000</td>
<td>0.015</td>
<td>0.036</td>
<td>0.002</td>
<td>0.013</td>
<td>0.117</td>
<td>0.000</td>
<td>0.134</td>
<td>0.064</td>
<td>0.009</td>
<td>0.496</td>
<td>0.500</td>
</tr>
<tr>
<td>SSD</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.677</td>
<td>0.558</td>
<td>0.092</td>
<td>0.660</td>
<td>0.965</td>
<td>0.481</td>
<td>0.549</td>
<td>0.538</td>
<td>0.549</td>
<td>0.626</td>
<td>0.817</td>
</tr>
<tr>
<td>1000</td>
<td>0.045</td>
<td>0.110</td>
<td>0.009</td>
<td>0.023</td>
<td>0.241</td>
<td>0.000</td>
<td>0.154</td>
<td>0.090</td>
<td>0.155</td>
<td>0.503</td>
<td>0.527</td>
</tr>
<tr>
<td>5000</td>
<td>0.020</td>
<td>0.046</td>
<td>0.003</td>
<td>0.022</td>
<td>0.206</td>
<td>0.000</td>
<td>0.149</td>
<td>0.260</td>
<td>0.139</td>
<td>0.497</td>
<td>0.504</td>
</tr>
<tr>
<td>8000</td>
<td>0.016</td>
<td>0.036</td>
<td>0.002</td>
<td>0.011</td>
<td>0.117</td>
<td>0.000</td>
<td>0.144</td>
<td>0.016</td>
<td>0.124</td>
<td>0.496</td>
<td>0.501</td>
</tr>
</tbody>
</table>
That is, as the sample size increases the Student’s $t$ converges to the normal distribution [17]. For sample size above 1000, the results of normal, GED and skewed GED were similar in terms of the AIC and BIC. The standard errors for the SGED were generally low compared to the other distributions. In addition, the SGED produced less MSE and MAE compared to the other distributions according to MSE but similar results with the normal distribution in terms of MAE.

Simulation results for GARCH model when true error distribution is skewed student’s $t$

The simulation results for the GARCH model when the true error follows the skewed Student’s $t$ distribution are presented in Table 5. The AIC and BIC suggest that the SGED fit the data better even though the data were simulated from the GARCH model with skewed Student’s $t$ distribution. The standard errors associated with the parameters of SGED were less compared to the other distributions. In addition, the estimated MSE and MAE were less for SGD compared to other distributions. However, the MAE for SGED and the normal distribution were generally similar.

In general, the results from the simulation experiment show that the Student’s $t$ and skewed Student’s $t$ perform poor in terms of goodness-of-fit, MSE and MAE. These results confirm the findings of Calzolari et al. [5] who argued that the Student’s $t$ is not unsuitable distributions for the GARCH model. On the other hand the results support the findings of Cerqueti et al. [6], who, in their empirical study, found the Skew Generalized Error Distribution (SGED) performed better than the other three error distributions (i.e., Student’s $t$, skewed Student’s $t$, and normal distributions). Also, Cerqueti et al. [6], also established that the GARCH-type model with skewed generalized error distribution is more reliable and accurate than the GARCH with normal distribution errors. In addition, the GARCH model with skewed generalized error distribution models produce lower MSEs and MAEs compared to the GARCH with normal distribution errors.

**Conclusion**

This study explores the performance of GARCH-type model based on five different error distribution structures. The effect of sample size on the parameter estimates in the GARCH model was also investigated. The fitted GARCH model with varying error distributions were compared using the AIC and BIC. The estimated parameters were also compared based on standard error, MSE and MAE. From the results, the AIC and BIC as well as the standard of the parameters space in general favours the skewed generalized error distribution followed by the normal distribution. When the sample size ($T = 50, 1000, 5000, 8000$) increased, the standard error of the estimated parameters, MSE and MAE decreased. Thus, the GARCH-type model with skewed GED is recommended as the optimal distribution based on goodness-of-fit, MSE and MAE. The Student’s $t$ was found to perform poorly in all scenarios considered. The results of the Student’s $t$ is always approximated by the normal distribution.

Table 5

<table>
<thead>
<tr>
<th>Sample size</th>
<th>Standard error</th>
<th>MSE</th>
<th>MAE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$</td>
<td>$\beta$</td>
<td>$\Omega$</td>
</tr>
<tr>
<td>Normal</td>
<td>50</td>
<td>0.201</td>
<td>0.471</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.038</td>
<td>0.098</td>
</tr>
<tr>
<td></td>
<td>5000</td>
<td>0.016</td>
<td>0.040</td>
</tr>
<tr>
<td></td>
<td>8000</td>
<td>0.013</td>
<td>0.032</td>
</tr>
<tr>
<td>GED</td>
<td>50</td>
<td>0.184</td>
<td>0.423</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.038</td>
<td>0.097</td>
</tr>
<tr>
<td></td>
<td>5000</td>
<td>0.016</td>
<td>0.040</td>
</tr>
<tr>
<td></td>
<td>8000</td>
<td>0.013</td>
<td>0.032</td>
</tr>
<tr>
<td>SGED</td>
<td>50</td>
<td>0.146</td>
<td>0.367</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.037</td>
<td>0.097</td>
</tr>
<tr>
<td></td>
<td>5000</td>
<td>0.016</td>
<td>0.040</td>
</tr>
<tr>
<td></td>
<td>8000</td>
<td>0.012</td>
<td>0.032</td>
</tr>
<tr>
<td>SD</td>
<td>50</td>
<td>0.377</td>
<td>0.681</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.045</td>
<td>0.110</td>
</tr>
<tr>
<td></td>
<td>5000</td>
<td>0.020</td>
<td>0.046</td>
</tr>
<tr>
<td></td>
<td>8000</td>
<td>0.016</td>
<td>0.036</td>
</tr>
<tr>
<td>SSD</td>
<td>50</td>
<td>0.587</td>
<td>0.556</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.045</td>
<td>0.109</td>
</tr>
<tr>
<td></td>
<td>5000</td>
<td>0.019</td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td>8000</td>
<td>0.015</td>
<td>0.036</td>
</tr>
</tbody>
</table>
Credit author statement

On behalf of the authors and as corresponding author, I confirm that all the authors have contributed equally to the production of the submitted original research article entitled “A comparative study of error distributions in the GARCH model through a Monte Carlo simulation approach” for consideration by the *Scientific African*.

Declaration of Competing Interest

On behalf of the authors, I declare that we have no conflicts of interest to disclose for the submitted original research article entitled “A comparative study of error distributions in the GARCH model through a Monte Carlo simulation approach” for consideration by the *Scientific African*. I confirm that this work is original and has not been published elsewhere, nor is it currently under consideration for publication elsewhere.

References


[27] M.T. Subbotin, On the law of frequency of error, Mat. Sb. 31 (1923) 296–301.


