



Heriot-Watt University
Research Gateway

Symbol-Level Design for Joint Radar Communication Based on Directional Modulation

Citation for published version:

Li, M, Zhang, B, Zhang, B, Ding, Y, Kim, T & Zhao, X 2024, 'Symbol-Level Design for Joint Radar Communication Based on Directional Modulation', *IEEE Wireless Communications Letters*, vol. 13, no. 2, pp. 442-445. <https://doi.org/10.1109/lwc.2023.3331694>

Digital Object Identifier (DOI):

[10.1109/lwc.2023.3331694](https://doi.org/10.1109/lwc.2023.3331694)

Link:

[Link to publication record in Heriot-Watt Research Portal](#)

Document Version:

Peer reviewed version

Published In:

IEEE Wireless Communications Letters

Publisher Rights Statement:

© 2023 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.

General rights

Copyright for the publications made accessible via Heriot-Watt Research Portal is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

Take down policy

Heriot-Watt University has made every reasonable effort to ensure that the content in Heriot-Watt Research Portal complies with UK legislation. If you believe that the public display of this file breaches copyright please contact open.access@hw.ac.uk providing details, and we will remove access to the work immediately and investigate your claim.

Symbol-level Design for Joint Radar Communication Based on Directional Modulation

Maolin Li, Bo Zhang*, Baoju Zhang*, Yuan Ding, Taekon Kim, Xiaonan Zhao

Abstract—Direction modulation (DM) is a physical layer security (PLS) technology that has attracted extensive research. In this paper, a symbol-level design for radar communication integration systems is proposed based on DM. In the radar direction, the performance of target estimation is ensured by minimizing the Cramér-Rao bound (CRB) of sensing, while maintaining the desired constellation patterns in the communication directions and meeting the mismatch threshold with the random signal in the undesired directions, disrupting the constellation patterns. The numerical results demonstrate that the proposed method has an extremely low CRB in the radar direction and low bit error rate (BER) levels in the communication directions.

Index Terms—Cramér-Rao bound, directional modulation, dual-function radar communication.

I. INTRODUCTION

Traditional link security technologies belong to the category of computational security, including public key encryption and private key encryption, which are essentially a class of mathematical problems. Once an effective solution to the corresponding mathematical problem is found, or the encryption is broken using a supercomputing device with high computing power, it means that the security of this type of encryption method and device cannot be guaranteed. To overcome the drawbacks of mathematical encryption, people began to study physical layer security technologies. They are mainly divided into three categories, including keyless physical layer security transmission technology [1], [2], channel based physical layer key generation technology [3], and low probability of interception covert communication technology [4], [5]. Directional modulation (DM) technology belongs to keyless physical layer security technology, which combines artificial noise and beamforming processing at the transmitting end to distort the signal constellation diagram received in non-target directions, making it impossible to interception. At the same time, the signal received in the target direction is not affected, achieving secure transmission.

With the increasing demand for modern communication, radar communication integration technology has been widely researched and developed. This technology can effectively utilize one set of hardware for joint communication and radar,

Maolin Li, Bo Zhang, Baoju Zhang and Xiaonan Zhao are with the Tianjin Key Laboratory of Wireless Mobile Communications and Power Transmission, College of Electronic and Communication Engineering, Tianjin Normal University, Tianjin 300387, China. (e-mail: limaoлин0302@163.com; b.zhangintj@tjnu.edu.cn; wdxzyzbj@163.com; xiaonan5857@163.com).

Yuan Ding is with the School of Engineering & Physical Sciences, Heriot-Watt University, Edinburgh, Scotland, United Kingdom EH14 4AS. (e-mail: yuan.ding@hw.ac.uk).

Taekon Kim is with the Department of Electronics and Information Engineering, Korea University, Korea. (e-mail: taekonkim@korea.ac.kr).

reducing overall system costs. Dual-function radar communication (DFRC) technology combines radar target perception and communication information transmission [6]–[8], and its joint waveform design mainly includes three strategies: communication-centric, radar-centric, and joint design. In [9], by minimizing the peak sidelobe level of the integrated signal, the disturbance phase in the communication BPSK and QPSK sequences were solved, and the integrated signal design of sensing and communication was studied. In [10], the radar detection performance was improved by suppressing the local sidelobe of the ambiguity function in the low-Doppler region, and suppressing the local sidelobe modulation communication information in the high-Doppler region.

In [11], explicit optimization of estimation performance metrics was systematically studied in the context of DFRC design. However, symbol-level security design has not been considered. In this paper, a Cramér-Rao bound (CRB) expression related to the weight vector of a DM transmitter is derived for a multi-input multi-output (MIMO) DFRC system, considering multiple desired users. Then, the radar target estimation performance is optimized by minimizing CRB, designing constellation points in desired directions, and meeting the mismatch threshold in undesired directions to ensure low bit error rate (BER) levels in communication directions.

This paper is organized as follows. The system model is presented in Sec. II. The derivation of CRB and the optimization of radar communication integrated waveform are proposed in Sec. III. Simulation results are provided in Sec. IV, followed by conclusions in Sec. V.

Notations: $[\]^T$ and $[\]^H$ denote transpose and conjugate transpose, respectively; s , \mathbf{s} , and \mathbf{S} denote scalar, vector, and matrix, respectively; $\| \cdot \|_2$ denotes the l_2 norm; $| \cdot |$ denotes the absolute value operation; $\text{tr}(\cdot)$ denotes the trace.

II. SYSTEM MODEL

A MIMO DFRC system to be studied in this paper is shown in Fig. 1. A uniform linear array (ULA) used for transmitting and receiving signals is divided into T_x and R_x . There are R desired users and a single radar target located in different directions, and E potential eavesdroppers' locations are unknown. The MIMO DFRC system detects the target while transmitting confidential information to the users. The transmit antenna array T_x and receive antenna array R_x are equipped with N_t and N_r antennas, respectively, and $R \leq N_t \leq N_r$ is set to ensure the design feasibility. For clarity, set $n_t = 0, 1, \dots, N_t - 1$, $n_r = 0, 1, \dots, N_r - 1$, $r = 0, 1, \dots, R - 1$, and $e = 0, 1, \dots, E - 1$.

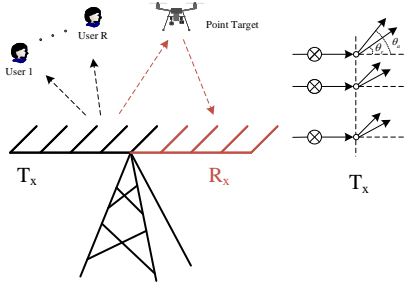


Fig. 1. DFRC System.

For a ULA, the steering vectors $\mathbf{h}_r(\theta_r) \in \mathbb{C}^{N_t \times 1}$ and $\mathbf{h}_e(\theta_e) \in \mathbb{C}^{N_t \times 1}$ corresponding to the user and the potential eavesdropper can be written as

$$\begin{aligned} \mathbf{h}_r(\theta_r) &= [1, e^{j2\pi f \frac{d_1 \cos \theta_r}{c}}, \dots, e^{j2\pi f \frac{d_{N_t-1} \cos \theta_r}{c}}]^T, \\ \mathbf{h}_e(\theta_e) &= [1, e^{j2\pi f \frac{d_1 \cos \theta_e}{c}}, \dots, e^{j2\pi f \frac{d_{N_t-1} \cos \theta_e}{c}}]^T, \end{aligned} \quad (1)$$

where f , c , and d_{n_t} are the frequency, the speed of propagation, and the distance from the zeroth antenna to the n_t -th antenna, and the transmission angles associated with the user and the eavesdropper are θ_r and θ_e , respectively. For multiple users and eavesdroppers, we define matrices $\mathbf{H}_r = [\mathbf{h}_r(\theta_0), \mathbf{h}_r(\theta_1), \dots, \mathbf{h}_r(\theta_{R-1})] \in \mathbb{C}^{N_t \times R}$ and $\mathbf{H}_e = [\mathbf{h}_e(\theta_0), \mathbf{h}_e(\theta_1), \dots, \mathbf{h}_e(\theta_{E-1})] \in \mathbb{C}^{N_t \times E}$. The steering vectors $\mathbf{h}_0(\theta_a) \in \mathbb{C}^{N_t \times 1}$ and $\mathbf{h}_1(\theta_a) \in \mathbb{C}^{N_r \times 1}$ of the transmit and receive antennas for the radar target can be written as

$$\begin{aligned} \mathbf{h}_0(\theta_a) &= [1, e^{j2\pi f \frac{d_1 \cos \theta_a}{c}}, \dots, e^{j2\pi f \frac{d_{N_t-1} \cos \theta_a}{c}}]^T, \\ \mathbf{h}_1(\theta_a) &= [1, e^{j2\pi f \frac{d_1 \cos \theta_a}{c}}, \dots, e^{j2\pi f \frac{d_{N_r-1} \cos \theta_a}{c}}]^T, \end{aligned} \quad (2)$$

where d_{n_r} and θ_a are the distance from the zeroth to the n_r -th antenna at R_x and the transmission angle corresponding to the radar target, respectively. At the T_x , symbols are synthesized using weight coefficients $\mathbf{w}_b \in \mathbb{C}^{N_t \times 1}$, which can be represented as

$$\mathbf{w}_b = [w_{0,b}, w_{1,b}, \dots, w_{N_t-1,b}]^T, \quad (3)$$

where $w_{n_t,b}$ corresponds to the weight of the n_t -th antenna, and $b = 0, 1, \dots, B-1$ is the b -th constellation point. Then, the received symbols at users and eavesdroppers can be represented as

$$\begin{aligned} \mathbf{y}_{r,b} &= \mathbf{H}_r^H \mathbf{w}_b + \mathbf{z}_{r,b}, \\ \mathbf{y}_{e,b} &= \mathbf{H}_e^H \mathbf{w}_b + \mathbf{z}_{e,b}, \end{aligned} \quad (4)$$

where $\mathbf{z}_{r,b} \in \mathbb{C}^{R \times 1}$ and $\mathbf{z}_{e,b} \in \mathbb{C}^{E \times 1}$ represent additive white Gaussian noise (AWGN) with variances δ_r and δ_e , $\mathbf{y}_{r,b} \in \mathbb{C}^{R \times 1}$, $\mathbf{y}_{e,b} \in \mathbb{C}^{E \times 1}$, respectively. For monostatic radar, the echo signal $\hat{y}_{a,b}$ received at R_x can be represented as

$$\hat{y}_{a,b} = \alpha \mathbf{h}_1 \mathbf{h}_0^H \mathbf{w}_b + z_{a,b} = \alpha \mathbf{h}_1 \tau_{a,b} + z_{a,b}, \quad (5)$$

where α , $z_{a,b}$, and $\tau_{a,b}$ are the reflection coefficient, an AWGN with variances δ_a and the b -th symbol transmitted at T_x , $\mathbf{h}_0 \triangleq \mathbf{h}_0(\theta_a)$, $\mathbf{h}_1 \triangleq \mathbf{h}_1(\theta_a)$.

III. PROPOSED DESIGN FOR RADAR COMMUNICATION INTEGRATION

Firstly, consider a classic DM design problem. Under the constraint of maintaining the constellation patterns along the user directions, while minimizing the matching error with desired responses for the eavesdropping directions, this optimization problem can be formulated as [12]

$$\min_{\mathbf{w}_b} \|\mathbf{H}_e^H \mathbf{w}_b - \boldsymbol{\tau}_{e,b}\|_2 \quad \text{s.t.} \quad \mathbf{H}_r^H \mathbf{w}_b = \boldsymbol{\tau}_{r,b}, \quad (6)$$

where $\boldsymbol{\tau}_{r,b} \in \mathbb{C}^{R \times 1}$ and $\boldsymbol{\tau}_{e,b} \in \mathbb{C}^{E \times 1}$ represent the desired responses corresponding to the b -th symbol in the user directions and the eavesdropping directions, respectively. Then, the lower bound on the variance of unbiased estimators, i.e., CRB, is used as radar performance metrics [11]. According to the derivation in [13], CRB_b corresponding to the b -th symbol is given as

$$CRB_b(\theta_a) = \frac{\delta_a^2 \text{tr}(\mathbf{h}_a^H \mathbf{h}_a \mathbf{w}_b \mathbf{w}_b^H)}{2|\alpha|^2 (\text{tr}(\bar{\mathbf{h}}_a^H \bar{\mathbf{h}}_a \mathbf{w}_b \mathbf{w}_b^H) \text{tr}(\mathbf{h}_a^H \mathbf{h}_a \mathbf{w}_b \mathbf{w}_b^H) - |\text{tr}(\bar{\mathbf{h}}_a^H \mathbf{h}_a \mathbf{w}_b \mathbf{w}_b^H)|^2)}, \quad (7)$$

where $\bar{\mathbf{h}}_a \in \mathbb{C}^{N_r \times N_t}$ is the derivative of $\mathbf{h}_a \triangleq \mathbf{h}_1 \mathbf{h}_0^H$, given as

$$\begin{aligned} \bar{\mathbf{h}}_0 &= [0, j2\pi f \frac{d_1 \cos \theta_a}{c} e^{j2\pi f \frac{d_1 \cos \theta_a}{c}}, \\ &\dots, j2\pi f \frac{d_{N_t-1} \cos \theta_a}{c} e^{j2\pi f \frac{d_{N_t-1} \cos \theta_a}{c}}]^T, \\ \bar{\mathbf{h}}_1 &= [0, j2\pi f \frac{d_1 \cos \theta_a}{c} e^{j2\pi f \frac{d_1 \cos \theta_a}{c}}, \\ &\dots, j2\pi f \frac{d_{N_r-1} \cos \theta_a}{c} e^{j2\pi f \frac{d_{N_r-1} \cos \theta_a}{c}}]^T. \end{aligned} \quad (8)$$

Then, we have

$$\begin{aligned} \text{tr}(\mathbf{h}_a^H \mathbf{h}_a \mathbf{w}_b \mathbf{w}_b^H) &= \text{tr}(\mathbf{h}_1 \mathbf{h}_0^H \mathbf{w}_b \mathbf{w}_b^H \mathbf{h}_0 \mathbf{h}_1^H) \\ &= \|\mathbf{h}_1\|^2 |\mathbf{h}_0^H \mathbf{w}_b|^2, \\ \text{tr}(\bar{\mathbf{h}}_a^H \mathbf{h}_a \mathbf{w}_b \mathbf{w}_b^H) &= \text{tr}(\mathbf{h}_1 \mathbf{h}_0^H \mathbf{w}_b \mathbf{w}_b^H (\mathbf{h}_0 \bar{\mathbf{h}}_1^H + \bar{\mathbf{h}}_0 \mathbf{h}_1^H)) \\ &= \mathbf{h}_1^H(\theta_a) \bar{\mathbf{h}}_1(\theta_a) |\mathbf{h}_0^H \mathbf{w}_b|^2 + \|\mathbf{h}_1\|^2 \mathbf{h}_0^H \mathbf{w}_b \mathbf{w}_b^H \bar{\mathbf{h}}_0, \\ \text{tr}(\bar{\mathbf{h}}_a^H \bar{\mathbf{h}}_a \mathbf{w}_b \mathbf{w}_b^H) &= \text{tr}((\bar{\mathbf{h}}_1 \mathbf{h}_0^H + \mathbf{h}_1 \bar{\mathbf{h}}_0^H) \mathbf{w}_b \mathbf{w}_b^H (\mathbf{h}_0 \bar{\mathbf{h}}_1^H + \bar{\mathbf{h}}_0 \mathbf{h}_1^H)) \\ &= \|\bar{\mathbf{h}}_1\|^2 |\mathbf{h}_0^H \mathbf{w}_b|^2 + \|\mathbf{h}_1\|^2 |\bar{\mathbf{h}}_0^H \mathbf{w}_b|^2 \\ &\quad + \mathbf{h}_1^H(\theta_a) \bar{\mathbf{h}}_1(\theta_a) \bar{\mathbf{h}}_0^H \mathbf{w}_b \mathbf{w}_b^H \mathbf{h}_0 \\ &\quad + \mathbf{h}_1^H(\theta_a) \bar{\mathbf{h}}_1(\theta_a) \mathbf{h}_0^H \mathbf{w}_b \mathbf{w}_b^H \bar{\mathbf{h}}_0. \end{aligned} \quad (9)$$

Substituting (9) into (7), $CRB_b(\theta_a)$ can be written as

$$CRB_b(\theta_a) = \frac{\delta_a^2 \|\mathbf{h}_1\|^2}{(2|\alpha|^2 \|\bar{\mathbf{h}}_1\|^2 \|\mathbf{h}_1\|^2 - |\mathbf{h}_1^H(\theta_a) \bar{\mathbf{h}}_1(\theta_a)|^2) |\mathbf{h}_0^H \mathbf{w}_b|^2}. \quad (10)$$

To ensure the performance of radar target detection, combined with the DM design in (6), the integrated waveform of radar communication is designed by minimizing CRB, which is described as

$$\begin{aligned} \min_{\mathbf{w}_b} CRB_b(\theta_a) \\ \text{s.t.} \quad & \left\| \mathbf{H}_e^H \mathbf{w}_b - \boldsymbol{\tau}_{e,b} \right\|_2 \leq \beta, \\ & \mathbf{H}_r^H \mathbf{w}_b = \boldsymbol{\tau}_{r,b}, \mathbf{h}_0^H \mathbf{w}_b = \tau_{a,b}, \\ & \|\mathbf{w}_b\|_2^2 \leq P_0, \end{aligned} \quad (11)$$

where β , $\tau_{a,b}$, and P_0 are the pre-defined mismatch threshold with desired responses, desired response in the radar target direction, and the power budget, respectively. Note that under the same level of noise power, equality constraint $\mathbf{H}_r^H \mathbf{w}_b = \tau_{r,b}$ ensures stable signal-to-noise ratio (SNR) for user directions. It can be known from (10) that minimizing $CRB_b(\theta_a)$ is equivalent to maximizing $|\mathbf{h}_0^H \mathbf{w}_b|$. However, equation constraint $\mathbf{h}_0^H \mathbf{w}_b = \tau_{a,b}$ makes $|\mathbf{h}_0^H \mathbf{w}_b| = |\tau_{a,b}|$, which prevents $CRB_b(\theta_a)$ from being optimized. Therefore, by introducing a variable t to maximize the amplitude of $|\mathbf{h}_0^H \mathbf{w}_b|$, the non-convex optimization problem in formulation (11) is changed into a convex, and is described as

$$\begin{aligned} \min_{\mathbf{w}_b} \quad & -t \\ \text{s.t.} \quad & \left\| \mathbf{H}_e^H \mathbf{w}_b - \tau_{e,b} \right\|_2 \leq \beta, \\ & \mathbf{H}_r^H \mathbf{w}_b = \tau_{r,b}, \quad \mathbf{h}_0^H \mathbf{w}_b = t\tau_{a,b}, \\ & \|\mathbf{w}_b\|_2^2 \leq P_0, \quad t > 0. \end{aligned} \quad (12)$$

Formulation (12) can be solved by CVX toolbox [14]. Note that $|\mathbf{h}_0^H \mathbf{w}_b| = t|\tau_{a,b}|$, and $|\tau_{a,b}| = 1$ is usually pre-set.

IV. SIMULATION RESULTS

In this section, numerical results are presented. For QPSK, the four symbols ‘00’, ‘01’, ‘11’, and ‘10’ are designed in the radar and user directions, respectively. For radar target tracking, the presence of targets can be detected through BER [15]. For clarity, assuming the phase of the desired response in radar direction is 45° , the same symbol is transmitted to all users during a symbol time.

Firstly, consider a classic scenario, $N_t = 21$, $N_r = 24$, the desired user direction and the radar direction are assumed to be $\theta_r = 120^\circ$ and $\theta_a = 60^\circ$, respectively. Potential eavesdroppers are located in undesired directions, i.e., $\theta_e \in [0^\circ, 115^\circ] \cup [125^\circ, 180^\circ]$, sampling every 1° . The radiation patterns corresponding to four symbols are shown in Figs. 2 and 3, with the mismatch threshold and the power budget are $\beta = 2$ and $P_0 = 0$ dB, respectively. It can be seen that both the radar direction and the user direction have accurate directionality, and the constellation points are scrambled in undesired directions. Figs. 4 and 5 show the relationship between the user SNR and the power budget with CRB, respectively, where the SNR is obtained through $|\tau_{r,b}|^2/\delta_r^2$. Although the CRB becomes higher as the user SNR increases, fortunately, it can be suppressed by increasing P_0 . When the SNR of the user is below 0 dB, the power P_0 is above 0 dB, and the mismatch threshold is above 3.5, the CRB remains at a low level. Note that for constraint $\|\mathbf{w}_b\|_2^2 \leq P_0$, there is a situation where $\|\mathbf{w}_b\|_2^2 < P_0$. If \mathbf{w}_b is proportionally amplified to satisfy $\|\mathbf{w}_b\|_2^2 = P_0$, a higher $|\mathbf{h}_0^H \mathbf{w}_b|$ can be obtained, which is beneficial for reducing CRB. To verify the effectiveness of the design, $\delta_r^2 = \delta_a^2 = \delta_e^2 = 1$ is set and the BER is computed, as shown in Fig. 6. It can be seen that the BER level in the user direction is very low. With the number of communication users increases, the CRB of radar target detection becomes higher, as shown in Fig. 7.

¹Resultant beam response refers to the signal response of an array receiver or sensor to a specific incident angle.

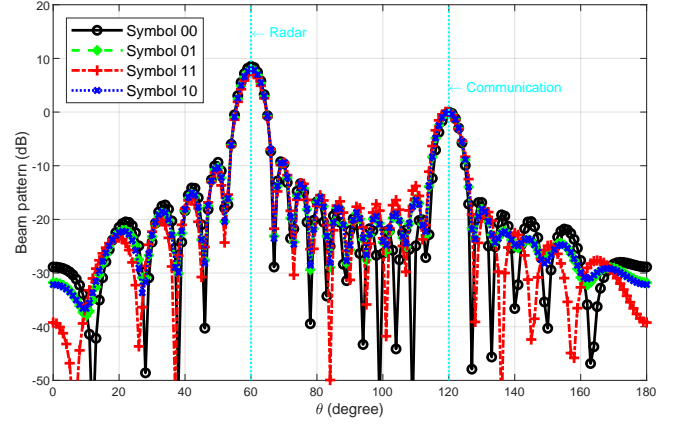


Fig. 2. Resultant beam responses¹. The user SNR, the power budget, and the mismatch threshold are 0 dB, 0 dB, and 2, respectively.

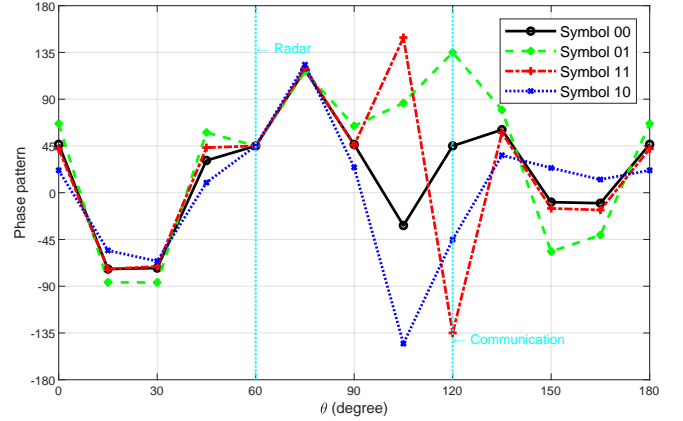


Fig. 3. Resultant phase patterns². The user SNR, the power budget, and the mismatch threshold are 0 dB, 0 dB, and 2, respectively.

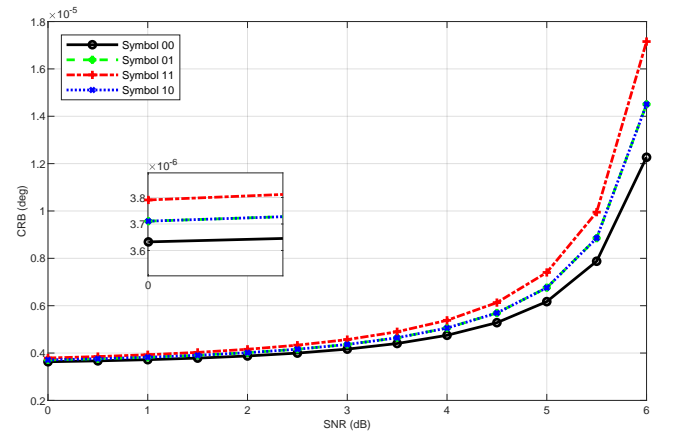


Fig. 4. Trade-off between the user SNR and the radar target detection performance CRB. The power budget and the mismatch threshold are 0 dB, and 2, respectively.

²Resultant phase pattern refers to a pattern in array signal processing that achieves beamforming or signal processing by adjusting the phase of the signal.

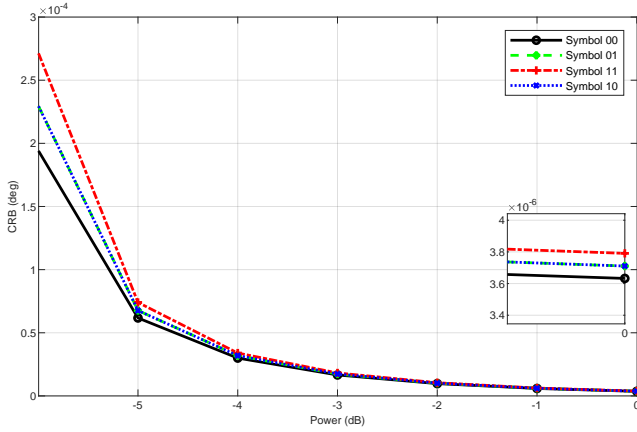


Fig. 5. Trade-off between the power budget P_0 and the radar target detection performance CRB. The user SNR and the mismatch threshold are 0 dB and 2, respectively.

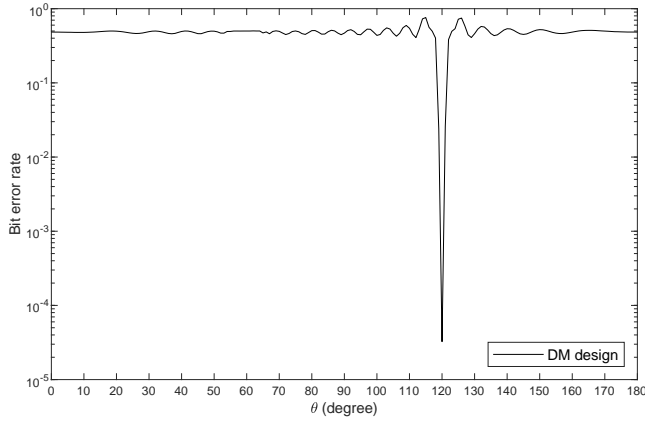


Fig. 6. Computed BER with SNR of 9 dB.

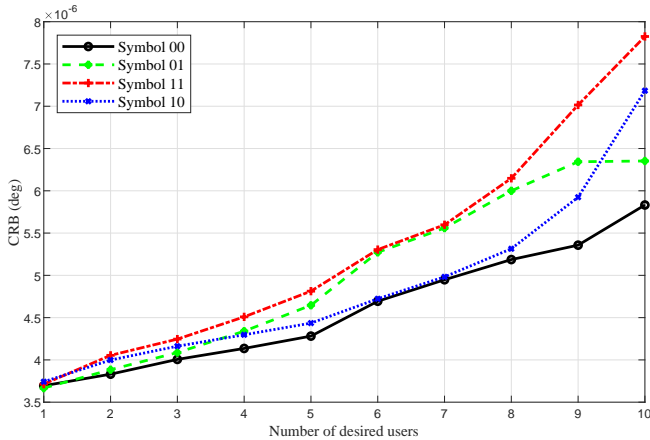


Fig. 7. The CRB with an increasing user number. The user SNR, the power budget, and the mismatch threshold are 0 dB, 0 dB, and 5, respectively.

V. CONCLUSIONS

In this paper, a symbol-level design was proposed based on DM for dual-function radar communication scenarios. Under the constraints of classical DM design, the radar communication integrated waveform was designed by minimizing CRB.

Numerical results showed that the proposed method could achieve low CRB of radar target estimation, low BER levels in the desired directions and high BER in other directions, and can achieve an angular resolution of 1° in the radar and communication directions.

ACKNOWLEDGEMENTS

The work was supported by the Natural Science Foundation of China (62101383, 62381340168), “Chunhui Project” cooperative research project of the Ministry of Education (HZKY20220595), UK Engineering and Physical Sciences Research Council (EP/V002635/1).

REFERENCES

- [1] A. Babakhani, D. B. Rutledge, and A. Hajimiri, “Transmitter architectures based on near-field direct antenna modulation,” *IEEE Journal of Solid-State Circuits*, vol. 43, no. 12, pp. 2674–2692, 2008.
- [2] Y. Ding and V. F. Fusco, “Establishing metrics for assessing the performance of directional modulation systems,” *IEEE Transactions on Antennas and Propagation*, vol. 62, no. 5, pp. 2745–2755, may 2014.
- [3] K. Ren, H. Su, and Q. Wang, “Secret key generation exploiting channel characteristics in wireless communications,” *IEEE Wireless Communications*, vol. 18, no. 4, pp. 6–12, 2011.
- [4] A. O. Hero, “Secure space-time communication,” *IEEE Transactions on Information Theory*, vol. 49, no. 12, pp. 3235–3249, 2003.
- [5] J. Wang, W. Tang, Q. Zhu, X. Li, H. Rao, and S. Li, “Covert communication with the help of relay and channel uncertainty,” *IEEE Wireless Communications Letters*, vol. 8, no. 1, pp. 317–320, 2018.
- [6] F. Liu, C. Masouros, A. P. Petropulu, H. Griffiths, and L. Hanzo, “Joint radar and communication design: Applications, state-of-the-art, and the road ahead,” *IEEE Transactions on Communications*, vol. 68, no. 6, pp. 3834–3862, 2020.
- [7] L. Chen, Z. Wang, Y. Du, Y. Chen, and F. R. Yu, “Generalized transceiver beamforming for dfrc with mimo radar and mu-mimo communication,” *IEEE Journal on Selected Areas in Communications*, vol. 40, no. 6, pp. 1795–1808, 2022.
- [8] F. Liu, Y. Cui, C. Masouros, J. Xu, T. X. Han, Y. C. Eldar, and S. Buzzi, “Integrated sensing and communications: Toward dual-functional wireless networks for 6g and beyond,” *IEEE journal on selected areas in communications*, vol. 40, no. 6, pp. 1728–1767, 2022.
- [9] S. Zhou, X. Liang, Y. Yu, and H. Liu, “Joint radar-communications co-use waveform design using optimized phase perturbation,” *IEEE Transactions on Aerospace and Electronic Systems*, vol. 55, no. 3, pp. 1227–1240, 2019.
- [10] J. Yang, G. Cui, X. Yu, and L. Kong, “Dual-use signal design for radar and communication via ambiguity function sidelobe control,” *IEEE Transactions on Vehicular Technology*, vol. 69, no. 9, pp. 9781–9794, 2020.
- [11] F. Liu, Y.-F. Liu, A. Li, C. Masouros, and Y. C. Eldar, “Cramér-rao bound optimization for joint radar-communication beamforming,” *IEEE Transactions on Signal Processing*, vol. 70, pp. 240–253, 2021.
- [12] B. Zhang, W. Liu, Q. Li, Y. Li, X. Zhao, C. Zhang, and C. Wang, “Directional modulation design under a given symbol-independent magnitude constraint for secure iot networks,” *IEEE Internet of Things Journal*, vol. 8, no. 20, pp. 15 140–15 147, 2021.
- [13] I. Bekkerman and J. Tabrikian, “Target detection and localization using mimo radars and sonars,” *IEEE Transactions on Signal Processing*, vol. 54, no. 10, pp. 3873–3883, 2006.
- [14] C. Research, “CVX: Matlab software for disciplined convex programming, version 2.0 beta,” <http://cvxr.com/cvx>, September 2012.
- [15] G. Huang, Y. Ding, S. Ouyang, and J. M. Purushothama, “Target localization using time-modulated directional modulated transmitters,” *IEEE Sensors Journal*, vol. 22, no. 13, pp. 13 508–13 518, 2022.