Inter-generational cross-subsidies in the UK's first CDC pension scheme

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Abstract

The UK's first collective defined contribution pension scheme will shortly be launched. Three types of inter-generational cross-subsidies in this scheme are identified, explained and illustrated. One of them, arising from financial unfairness, means that the first generations in the scheme have highest replacement ratios, on average. Another enables pension smoothing, by the transfer of risk from the old to the young. In turn, this allows pensioners to be protected from the volatility of a higher risk investment strategy. The third cross-subsidy reduces the impact of actual economic outcomes being different to those expected.

Pension outcomes under a model of the CDC scheme are compared to those of three defined contribution-based alternatives. The comparison is done for each generation in the scheme. The results suggest that longevity risk-sharing alternatives, such as pooled annuity funds, may provide a similar or better outcome than the CDC scheme, depending on the generation studied.

1. Introduction

Collective Defined Contribution (CDC) pension schemes are arriving in the UK, after a long, uncertain beginning (Wilkinson, 2008). These are pension schemes in which the contribution rate is fixed, as a percentage of salaries, and the benefits paid out vary. The idea is that the scheme

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members share both investment and longevity risk in order to have a better outcome in retirement. The adjustment lever in a CDC pension scheme, that benefits are adjusted in order to keep the contributions unchanged, are different to those in defined benefit (DB) pension schemes, in which the contributions are adjusted in order to keep the benefits unchanged. The CDC scheme uses the same lever as a defined contribution (DC) pension scheme,

The first CDC scheme to launch in the UK, the Royal Mail CDC, was the driving force behind the introduction of the legislation enabling CDC schemes in the UK (Wilkinson, 2008). For this reason as well as the scale of the forthcoming Royal Mail CDC plan, which is anticipated to have well over 100 000 members, the Royal Mail CDC plan is very important. It is the prototype which is presented by the actuaries involved in its design and testing to other actuaries. When CDC schemes are criticised, whether positively and negatively, it is the relative importance of the advantages and disadvantages of the Royal Mail CDC scheme which is really criticised. This is despite the fact that UK CDC schemes do not need to have a similar design.

Owadally et al. (2022) discuss comprehensively CDC plans in the UK with respect to policy in the UK, as well as their risks and the ensuing advantages and disadvantages. They model the Royal Mail CDC plan, including the 'funding gate', which is not considered here, and which controls the pension increases awarded if the notional funding level falls outside a specified range. They compare the outomces for a single member of their model, mature CDC plan against typical DC-based, alternative pension schemes. They find that their CDC plan can pay higher and more stable pensions compared to the alternative pension schemes considered.

This work is an attempt to understand better the broad implications of the Royal Mail CDC plan. It is clear that the plan's design is a modification of the defined benefit (DB) pension plans which are common (but closing down) in the UK. Here the broad design is captured but not all of it, for example, death and spousal benefits are omitted. The only benefit modelled is the pension income which is to be paid in retirement to each retired, formerly contributing, member.

The collective risk-sharing in the Royal Mail CDC scheme is done through annual pension increases. Every year, a pension increase on accrued benefits is declared, and the increased pension amount becomes the accrued benefit of each member. The same annual pension increase is declared on all members' accrued pension. It is this uniform declaration which makes the scheme a collective risk-sharing one.

It is important to note several things. The design of the Royal Mail CDC scheme, a whole-life scheme, is not the only design of a CDC scheme. For example, post-retirement-only CDC schemes which operate as pooled annuity funds have gained increasing attention from the UK industry. Even within whole-life schemes, there are many different possible designs. It is also true that the 'best' plan from a purely financial point of view is probably not the 'best' plan if no-one wants it. It may not be the best plan if it is so complicated that the members do not understand what they get from it. However, this paper does not consider these crucial aspects and focuses only on the financial characteristics. In particular, the focus is on inter-generational risk-sharing.

Inter-generational risk-sharing is a fundamental part of CDC schemes. It is also the most controversial. The idea is that risks, such as volatile investment returns and uncertain future lifetimes, are shared across the generations of the pension scheme. If this can be done fairly, then all generations with the same risk characteristics will face the same risk-return trade-off.

Three types of inter-generational cross-subsidies are identified in this paper, in the modelled Royal Mail CDC scheme. The first is a cross-subsidy at each point of time, among the members who are in the scheme at that time. It enables the smoothing of pensions in the scheme and is explained and illustrated later. It is a form of risk-sharing in which the younger members bear more risk to reduce the risk borne by older members. This type of risk-sharing should benefit all members, if they stay in the scheme for the whole of their life.

This transfer of risk from the old to the young is extremely important to the CDC plan being able to out-perform other alternative pension options. Because the younger members bear the brunt of the investment volatility, the scheme can invest in a riskier investment strategy while shielding the older members in the scheme from the volatility of such a strategy. In contrast, an individual going in to income drawdown, in which they invest their pension savings in an investment product and withdraw an income from it, would experience directly the volatility and would likely choose a lower risk strategy in their retirement.

The ability of this CDC plan to take on more investment risk is explained by the risk transfer from young to old. Other CDC plans have the same feature (Bonenkamp and Westerhout, 2014; Cui et al., 2011; Gollier, 2008) and they can also out-perform alternative pension options simply due to taking on more investment risk.

The second inter-generational cross-subsidy occurs across time. It is due, for example, to actual investment returns being different to their predictions. Since the level of pensions paid out depend on future predictions of returns, poor predictions of those returns lead to the pension payments out which are, in hindsight, either too low or too high. Consequently, future generations have higher or lower pensions. This type of cross-subsidy can go either way: it may benefit earlier or later generations to join. However, it is also true that once the plan begins its life, the cost due to previous predictions deviating from their observed expected values could be calculated.

A third cross-subsidy occurs in the modelled CDC scheme due to financial fairness not holding with respect to the benefit accrued by each contribution made. It means that the first generations to join get a higher benefit than later generations, on average. Consequently, it gives rise to a certain subsidy for which the later generations bear the cost and the first generations get the benefit. It is a complicating factor in CDC structures which have both a constant rate of benefit accrual and a constant contribution rate. Even if financial fairness on an accrued basis was not implemented, perhaps due to the desire to give members more advance certainty about the level of benefit accrued by each other their contributions, an age-related benefit accrual would reduce the cost on later generations of this anomaly.

The lack of financial fairness on an accrued basis distinguishes a Royal Mail-type CDC scheme from those studied in the literature. For example, Haan et al. (2015), which shows a better performance of their studied collective scheme, is based on CDC plans which are seen in the Netherlands.

This paper contributes to the literature in several ways. First, it describes and explains three types of inter-generational cross-subsidies in the first CDC scheme to exist in the UK. In doing so, it allows a better understanding of the implications of various design choices in the Royal Mail CDC plan. In turn, it makes it easier to design a different CDC plan, with the desired inter-generational cross-subsidies present. Second, the outcomes for each and every generation to join the modelled CDC plan are simulated, instead of the outcome for a single entrant to a fully mature scheme. The numerical illustrations shows that different generations have a different distribution of outcomes. It is not the case that, uniformly, a CDC plan is better than DC-based, alternative pension options.

The model of the CDC plan is detailed in Section 2. The three types of inter-generational cross-subsidy are discussed in Section 3. Alternative pension options in which each CDC member could have invested their contributions are presented in Section 4. The results of the numerical study are presented in Section 5.

2. Model of the CDC plan

The CDC plan is presented next, detailing the membership, the benefits, the contributions and the collective risk-sharing mechanism: the calculation of the annual pension increases. Time is measured in years.

2.1. Membership

At the start of each year, a new generation joins the CDC scheme until a total of $M \geq 1$ generations have joined. The generation who join at time g consists of $N^{(g)}(g)$ members at time g who are all age $x \in \mathbb{N}$. Denote the number of members alive in generation g at time $n \in \{g, g+1, \ldots, M\}$ by $N^{(g)}(n)$. Retirement occurs exactly $T \in \mathbb{N}$ years after a member joins the scheme.

Let $\omega \in \{T+1, T+2, \ldots\}$ be the maximum lifetime of each member. In other words, no-one lives for more than ω years from the time that they join the scheme. Thus $N^{(g)}(\omega) = N^{(g)}(\omega+1) = \cdots = 0$.

It is assumed that no member dies before retirement. Thus $N^{(g)}(g) = N^{(g)}(g+1) = \cdots = N^{(g)}(g+T)$. This is to avoid valuing death-in-service benefits which won't change the message of the results but will complicate the algebra.

At retirement, the chance of a member of generation g (who join at time g) dying during the time period $(g+n, g+n+k]$, when they are between ages $x+n$ and $x+n+k$, and conditional on being alive at age $x+n$, is $q(x+n,k)$. Similarly, the probability of surviving from age $x+n$ to age $x+n+k$, conditional on being alive at age $x+n$, is $p(x+n,k) = 1 - q(x+n,k)$. As mentioned above, $q(x+n,1) = 0$ for $n \in \{0, 1, \ldots, T-1\}$.

In the mortality model used to calculate the life annuities, the corresponding probabilities of death and survival are the same. Furthermore, it is assumed that there is no systematic longevity risk in the model. Observed deviations in mortality from the true probabilities $\{q^{(g)}(x+n,k)\}$ are a consequence of their being an insufficient number of members to fully diversity random fluctuations.

The number of members alive in generation g at time $n \in \{T+1, T+2, \ldots, \omega-1\}$ is $N^{(g)}(n) \sim \text{Bin}\big(N^{(g)}(n-1), p(x+n,1)\big)$, i.e. $N^{(g)}(n)$ has the binomial distribution with mean $N^{(g)}(n-1)\, p(x+n,1)$.

2.2. Salaries

Salaries are paid annually in advance to members before their retirement and increase each year in line with a random wage index. There are no promotional salary increases, so that all members who are working at the same time will receive the same amount of salary, regardless of their age. Denote the amount of salary paid at time n by $S(n) > 0$, with $S(0)$ a known constant and $S(n)$ a random variable for $n \in \mathbb{N}$.

2.3. Contributions

Each CDC scheme member pays a series of regular contributions in exchange for receiving a regular benefit payment at retirement. Each contribution relates to the accrual of a specified percentage of the target benefit. The contributions are expressed as a fixed percentage of salary.

In general, a scheme member who is not retired at integer time n has to pay a contribution of amount $C(n) = \alpha S(n)$. Contributions cease once a member retires. Thus their last contribution is one year before retirement, and they receive their first retirement benefit immediately upon retirement.

2.4. Benefits

Each contribution funds a specified amount of benefit payable annually during retirement. In the studied CDC scheme, the benefit is accrued at a constant fraction $1/\beta$ of salary. Specifically, in respect of a contribution of amount $C(n)$ made at time $n \in \mathbb{N}_0$, the initial benefit $B(n) = 1/\beta \times S(n)$ is accrued. The benefit amount represents the amount paid annually in retirement, before future pension increases have been added to it. The benefit stops being paid once the member dies.

Once accrued, the benefit amount $B(n)$ attracts annual pension increases during the remaining working life and into the retirement of the member who paid the contribution. The first pension increase on $B(n)$ occurs exactly one year after it is accrued, being calculated and applied at time $n+1$.

The calculation of the annual pension increases is the expression of risk-sharing in the CDC scheme. The annual pension increase $h(n)$ awarded at time n is known only at time n, and not before. Thus the amount of pension paid out at retirement is not known with certainty until the time of payment.

Since there are no promotional salary increases, the cumulative benefit accrued by generation g at time n, just after the payment of any contribution at time n, is

$$B^{(g),\mathrm{cum}}(n) = \begin{cases} B(g) & \text{if } n = g, \\ \sum_{\ell=g}^{n-1} B(\ell) \prod_{m=\ell+1}^{n} (1 + h(m)) + B(n) & \text{if } n = g+1, g+2, \dots, g+T-1, \\ \sum_{\ell=g}^{g+T-1} B(\ell) \prod_{m=\ell+1}^{n} (1 + h(m)) & \text{if } n = g+T, g+T+1, \dots, g+\omega-1. \end{cases}$$

The following recursion relations hold. For $n = g, g+1, \dots, g+T-2$, i.e. up to retirement,

$$B^{(g),\mathrm{cum}}(n+1) = B^{(g),\mathrm{cum}}(n) \, (1 + h(n+1)) + B(n+1),$$

and for $n = g+T-1, g+T, \dots, g+\omega-2$, i.e. once a member in generation g retires,

$$B^{(g),\mathrm{cum}}(n+1) = B^{(g),\mathrm{cum}}(n) \, (1 + h(n+1)) \, .$$

2.5. Contribution rate

The contribution rate, $\alpha > 0$, is chosen so that: (i) the discounted value of the lifetime contributions paid by Generation 0 equals the discounted value of the lifetime benefits anticipated to be paid to Generation 0; and (ii) the contribution rate α is a constant.

In this calculation, salaries are expected to increase at the constant annual rate $g(0)$ for the foreseeable future. Consequently, the contribution at time n is expected to be $\alpha S(0)(1+g(0))^n$ and the benefit accrued by that contribution is expected to be $1/\beta \times S(0)(1+g(0))^n$. Furthermore, annual pension increases are expected to be $h(0)$.

Thus the benefit at retirement time $k = T, T+1, \dots, \omega-1$ for a surviving member of Generation 0 is expected to be

$$\sum_{\ell=0}^{T-1} 1/\beta \times S(0)(1+g(0))^{\ell} \prod_{m=\ell+1}^{k} (1 + h(0)) \, .$$

Let $P(n,m) = P_{\mathrm{CDC}}(n,m)$ represent the value at time $n \geq 0$ of a payment of 1 unit at time $m \geq n$, with $P(n,n) = 1$. The value represents the expected return on assets from time n to time m, allowing for any anticipated changes in the investment strategy.

Recall that members join the scheme when they are age x years and are assumed to survive until their retirement, during which time they can die. Then α is the solution to

$$\sum_{n=0}^{T-1} \alpha S(0)(1+g(0))^n P(0,n) = \sum_{\ell=0}^{T-1} 1/\beta \times S(0)(1+g(0))^{\ell} \sum_{k=T}^{\omega-1} (1+h(0))^{k-\ell} p(x,k)P(0,k). \quad (1)$$

2.6. Financial fairness

In this CDC scheme, lifetime financial fairness occurs only for Generation 0. For subsequent generations, the expectations of future pension increases and salary increases will vary over time. If the value of α was to be calculated afresh for these generations, it would vary. However, the contribution rate will be fixed in the proposed Royal Mail CDC scheme and so α is fixed here too, being the solution to (1).

In addition, there is no financial fairness linking each contribution to the benefit accrued by that contribution, for any generation. This lack of financial fairness has implications in the CDC scheme, for the value of the pension increases awarded at each integer time, which is illustrated later.

2.7. Dynamics of the CDC scheme

The evolution of the assets and accrued benefits of each member of each generation is described next. Additionally, the calculation of the annual pension increases are detailed.

The pension increases in the CDC scheme are the expression of the risk-sharing in the scheme. Every scheme member earns an annual pension increase on their accrued benefit every year, with the pension increase awarded once the benefit has been accrued for at least one year. The pension increases are calculated so that the total asset value in the scheme equals the total liability value.

2.7.1. Starting phase of the CDC scheme

In the starting phase of the CDC scheme, no-one has yet retired. The asset value of the scheme just before anyone joins the scheme is $A(0_-) = 0$. Let the effective investment return over the time period $(k-1, k]$ be represented by the random variable $R(k)$, for integer $k \geq 1$. Then the asset value at integer time $k \in \{1, 2, \ldots, T-1\}$, just after the contributions paid at time k have been received, is

$$A(k) = A(k_-) + \sum_{g=0}^{k} N^{(g)}(k)\, C(k).$$

For example, the asset value at time 0 just after the first contributions paid by Generation 0 are received is $A(0) = N^{(0)}(0)C(0)$. The benefit accrued at time 0 by each member of Generation 0 upon receipt of their contributions is $B^{(0),\mathrm{cum}}(0) = B(0)$.

No matter what phase the CDC scheme is in, the asset value at time $k \in \{1, 2, \ldots\}$, just before the contributions paid at time k have been received (and, later in the middle phase onwards, just before any benefits have been paid out), is

$$A\left(k_-\right) = A(k-1)(1+R(k-1)).$$

For example, the asset value at time 1 just before the contributions paid by Generations 0 and 1 are received is $A(1_-) = A(0)(1+R(1))$.

The first pension increase, $h(1)$, is applied at time 1 to the benefit accrued at time 0 by Generation 0. To determine $h(1)$, equate the asset value at time 1 with the discounted value at time 1 of the projected benefits payable at retirement. The asset value is the value just before the new contributions received at time 1, which will be received from both Generation 0 and Generation 1. The projected benefits are the benefits accrued at time 0 and projected to retirement at the pension increase rate $h(1)$. Thus $h(1)$ is the solution to

$$A(1_{-}) = N^{(0)}(1)\, B^{(0),\mathrm{cum}}(0) \sum_{\ell=T}^{\omega-1} (1+h(1))^{\ell} \cdot P(1,\ell) \cdot p(x+1,\ell-1).$$

The amount of accrued benefits of a member of Generation 0 is then $(1+h(1))B^{(0),\mathrm{cum}}(0) = (1+h(1))B(0)$, just before Generation 0 accrues more benefits by the payment of contributions at time 1.

Once $h(1)$ has been calculated, the new contributions paid by Generations 0 and 1 are added to the scheme's assets. The cumulative benefits accrued to each member of Generation 0 is then $B^{(0),\mathrm{cum}}(1) = (1+h(1))B^{(0),\mathrm{cum}}(0) + B(1)$ and those to each member of Generation 1 is $B^{(1),\mathrm{cum}}(1) = B(1)$. The scheme's asset value increases to $A(1) = A(1_{-}) + N^{(1)}(1)C(1) + N^{(0)}(1)C(1)$.

In general, in this starting phase, the annual pension increase $h(k)$ granted on the amount of accrued benefits at time $k-1$ is determined as the annual pension increase which makes the asset value equal to the value of the discounted, projected accrued pensions paid in retirement, i.e. at time $k \in \{1, 2, \ldots, T\}$

$$A(k_{-}) = \sum_{g=0}^{k-1} N^{(g)}(k)\, B^{(g),\mathrm{cum}}(k-1) \sum_{\ell=g+T}^{g+\omega-1} (1+h(k))^{\ell-(k-1)} \cdot P(k,\ell) \cdot p(x+k-g,\ell-k). \quad (2)$$

2.7.2. Middle phase of the CDC scheme

In the middle phase of the CDC scheme, some members have reached their retirement period while new members continue to join. The middle phase begins at time T, when Generation 0 retires. Note that annual pension increases continue to be granted while a pension is in payment.

The pension increase $h(k)$ granted at time $k \in \{T, T+1, \ldots, M-1\}$ is the solution to

$$\begin{aligned} A(k_{-}) = & \sum_{g=\max\{0,k+1-\omega\}}^{k-1} N^{(g)}(k)\, B^{(g),\mathrm{cum}}(k-1) \\ & \times \sum_{\ell=\max\{k,g+T\}}^{g+\omega-1} (1+h(k))^{\ell-(k-1)} \cdot P(k,\ell) \cdot p(x+k-g,\ell-g). \end{aligned}$$

As before, once $h(k)$ is calculated at time k, the accrued benefits with pension increases declared at time k are $B^{(g)}(k) = (1+h(k))B^{(g)}(k-1)$. At time k, the $(k-T+1)$th generation retires, with an annuity benefit payment of initial amount $B^{(k-T),\mathrm{cum}}(k)$. Thus the benefits paid out at time k have just received an annual pension increase $h(k)$.

The asset value just after the new contributions received and the benefits paid out at time $k \in \{T, T+1, \ldots, M-1\}$ is

$$A(k) = A(k_{-}) + \sum_{g=k+1-T}^{k} N^{(g)}(k)\, C(k) - \sum_{g=\max\{0,k+1-\omega\}}^{k-T} N^{(g)}(k) B^{(g),\mathrm{cum}}(k).$$

2.7.3. End phase of the CDC scheme

In the end phase of the CDC scheme, which occurs from a fixed time $M > T$ onwards, no-one joins the scheme. The last generation to join the scheme was Generation $M-1$ at time $M-1$.

During the end phase, non-retired members continue to make contributions until their retirement. However, at time $M-1+T$, the last generation to join begins their retirement and no further contributions are made.

The pension increase $h(k)$ granted at time $k \in \{M, M+1, \ldots, M-2+\omega\}$ is the solution to

$$A(k_-) = \sum_{g=\max\{0,k+1-\omega\}}^{M-1} N^{(g)}(k)\, B^{(g),\mathrm{cum}}(k-1) \sum_{\ell=\max\{k,g+T\}}^{g+\omega-1} (1+h(k))^{\ell-(k-1)} \cdot P(k,\ell) \cdot p(x+k-g, \ell-g).$$

The last contribution is paid at time $M+T-2$ by each member of Generation $M-1$. Let the function $\chi_{[k\leq M+T-2]}$ take the value 1 if $k \leq M+T-2$ and value zero otherwise. Then the asset value just after the benefits paid out and any contributions received at time $k = M, M+1, \ldots, M+\omega-2$ is

$$A(k) = A(k_-) + \chi_{[k\leq M+T-2]} \cdot \sum_{g=k+1-T}^{M-1} N^{(g)}(k)\, C(k) - \sum_{g=\max\{0,k+1-\omega\}}^{\min\{M-1,k-T\}} N^{(g)}(k) B^{(g),\mathrm{cum}}(k).$$

At some point at the end of the scheme's life, there will be only a few members alive, which means that there is little longevity pooling. This matters because the valuation of benefits assumes that there is longevity pooling and no idiosyncratic longevity risk. Furthermore, it may very well be the case that the assets are insufficient to pay out a lifetime benefit to the few remaining members. To eliminate longevity risk when the scheme is at the end of its life the scheme managers could, for example, either buy in life annuities or transfer out its residual liabilities to an insurance company.

Since the end of life of the CDC is not the focus of this paper, and its consequences will not be studied, the scheme is allowed to continue until all members have died, as described above. If all members have died before time $M+\omega-2$, the last time at which a member of generation $M-1$ could be alive to receive a benefit payment, then the scheme ceases to exist. There may be some surplus asset value left at the time of the last death in the scheme, but the discussion of who owns this and how it should be dealt with is not addressed here. Once all members have died, the scheme ceases to exist.

If there is at least one member is alive at time $M+\omega-2$, then the last pension increase, $h(M-2+\omega)$ is calculated as

$$h(M-2+\omega) = \frac{A((M-2+\omega)_-)}{N^{(M-1)}(M-2+\omega)\, B^{(M-1),\mathrm{cum}}(M-3+\omega)} - 1.$$

Once the final benefit is paid out, of total amount $A((M-2+\omega)_-) = N^{(M-1)}(M-2+\omega)\, B^{(M-1),\mathrm{cum}}(M-3+\omega)$, the asset value falls to zero, i.e. $A(M-2+\omega) = 0$, and the scheme ceases to exist.

3. Inter-generational cross-subsidies

Three types of inter-generational cross-subsidies are identified:

- Transfer of risk from the old to the young, at each point in time. This cross-subsidy gives rise to pension smoothing, which in turn allows the CDC plan to take more investment risk on behalf of its retirees;
- Transfer of cost from the first generations to the later generations. As a consequence of a constant benefit accrual rate and a constant contribution rate, the first generations obtain, on average, a significantly higher pension than later generations.

- Inter-generational cross-subsidies across time. These may be cross-subsidies due to investment returns not turning out as predicted or due to fewer people dying than expected.

These cross-subsidies are discussed next, in turn.

3.1. Cross-subsidies leading to pension smoothing

One of the industry-cited benefits of CDC schemes which are structured like the proposed Royal Mail one, is their ability to smooth members' pensions (Aon, 2020; Wilkinson, 2008; Willis Towers Watson, 2020). In other words, the pension accrued by a member over time exhibits a low volatility (Aon, 2020, Chart 5, page 31).

A smooth development of a retirement pension is ideal for individuals. Research by Lusardi and Mitchell 2011 shows that individuals who plan for retirement have better retirement outcomes. Pension smoothing assists in retirement planning, by allowing individuals to predict better the ultimate amount of pension they will have in retirement.

How pensions are smoothed

How does the modelled CDC scheme achieve pension smoothing? It does this by transferring risk from the older members of the scheme to the younger members at each point in time. This risk transfer is enabled by the expression of risk-sharing in the scheme via the annual pension increases. A simplified example can illustrate what is going on.

Suppose that the discount rate is 8% per annum. There are 40 generations in the CDC scheme, with the member of each generation joining at age 25, contributing every year for 40 years and then retiring at age 65. It is assumed that no-one dies before retirement. From retirement at age 65, the mortality of every individual follows the Continuous Mortality Investigation life table S1PMA Continuous Mortality Investigation (2008), calculated from UK male pensioner mortality data.

In this simple example, all members have a constant salary S per annum at each time; there is no increase in the salaries over time. Contributions are paid at the start of each year and the annual retirement benefit accrued by each contribution is calculated at the start of each year, too, as a fraction 1/80 of the salary. Assume that Generation n joins at time n, for $n = 0, 1, \ldots, 39$. The retirement benefits accrued at time 39 by each generation, just after the contributions paid at time 39, are shown in Table 3.1.1.

Generation	0	1	$\cdots$	g	$\cdots$	38	39
Accrued benefit at time 39	$\frac{40}{80} \times S$	$\frac{39}{80} \times S$	$\cdots$	$\frac{40-g}{80} \times S$	$\cdots$	$\frac{2}{80} \times S$	$\frac{1}{80} \times S$

Table 3.1.1: Accrued benefits, for a selection of generations, at time 39.

Let us assume that, one year later at time 40, future annual pension increases are calculated to be 3% per annum. Under these future pension increases, the asset value equals the current discounted value of the accrued pensions so that, analogous to equation (2),

$$\text{Asset value at time } 40 = \sum_{g=0}^{39} \frac{40-g}{80} \times S \times 1.03 \times \left[\frac{1.03}{1.08}\right]^g \ddot{a}_{65}^{1.08/1.03-1}.$$

Now suppose that, just after the calculation was done, the asset value suddenly falls by 10%. Expectations of the future discount rate of 8% per annum remain unchanged. Future annual pension increases are re-calculated immediately to be 2.40% so that the discounted value of the accrued benefits also falls by 10%. The change in the discounted value of the benefits for generation g is

$$\left(\frac{40-g}{80} \times S \times 1.0240 \left[\frac{1.0240}{1.08}\right]^g \ddot{a}_{65}^{1.08/1.0240-1}\right) / \left(\frac{40-g}{80} \times S \times 1.03 \left[\frac{1.03}{1.08}\right]^g \ddot{a}_{65}^{1.08/1.03-1}\right) - 1.$$

These changes in value are shown in Table 3.1.2, for a selection of the generations.

Generation	0	1	2	⋯	20	⋯	37	38	39
Asset value down by 10%	−5.1%	−5.7%	−6.2%	⋯	−15.5%	⋯	−23.5%	−23.9%	−24.4%
Asset value up by 10%	+4.8%	+5.4%	+5.9%	⋯	+16.1%	⋯	+26.7%	+27.3%	+28.0%

Table 3.1.2: Changes in the discounted value of accrued benefits, for a selection of generations, upon a sudden change in the asset value at time 40. All members are awarded the same annual pension increase of either 2.40% or 3.53%, depending if the asset value went down or up.

The total scheme asset value has fallen by 10% and hence the total discounted value of accrued benefits must also fall by 10%. However, the oldest generation, Generation 0 (who will receive their first retirement payment at time 40), experience a fall of much less than 10% in their discounted value of accrued benefits, at 5.1%. In contrast, the youngest generation, Generation 39, experiences a fall much greater than 10%, at 24.4%. The results if the total scheme asset value had, instead, jumped up by 10% are shown in Table 3.1.2. Clearly, the discounted benefit value of the younger generations is more sensitive to changes in the asset value, compared to the older generations.

The imbalance in the sensitivity means that the older generations have transferred risk to the young generations. This is the essence of the pension smoothing. As the CDC scheme members age, their accrued CDC pension amount becomes less volatile.

Reduction in income volatility for older members

If the members of Generation 0 had the fair value of their contributions invested instead in a DC plan, rather than in the CDC plan, how would they deal with a fall in 10% in their asset value at their retirement? In a DC plan's projection of their annual pension at retirement, the anticipated future pension increases could remain unchanged. This would be fairly standard if the targeted pension increases were tracking a price inflation index. In that case, the amount of annual pension paid from the DC plan would be cut by 10%. Alternatively, they could reduce their future anticipated annual pension increases so that the discounted value of their anticipated annual pension was reduced by 10%. Whatever they chose to do, they would have to cut the value of their benefits in the DC plan by 10%. In contrast, in the CDC plan, the value of their benefits is cut by only 5.1%.

These changes are effectively hidden from the membership. From any member's perspective, their pension amount appears to have grown by 2.40% since last year. They are not necessarily aware of any fall in the asset value. While the value of the current accrued pension between generations has changed by radically different proportions, this is not obvious to the members. Ignorance is not necessarily a bad thing either; Merton (2014) makes a lucid and compelling argument for the focus to be on a member's income in retirement and not on their underlying value.

The risk transfer occurs due to the the higher sensitivity of the younger generations' discounted values, to changes in the annual pension increase rate. This is simply because they are further from their retirement. Compounding the annual pension increase up to retirement means that their projected-to-retirement annual benefit will be more volatile than the older generations' projected-to-retirement annual benefit.

Highly leveraged returns for young members

Turning to an investment standpoint, suppose that the members are individually assigned an investment strategy in the CDC scheme. The individual investment strategy serves no purpose other than to determine the collective investment strategy.

Suppose that all members are assigned the same investment strategy, for example, 100% in risky assets. The implication of the increased sensitivity of the younger members' individual liability values is that younger members are effectively invested much more than 100% in risky assets than older members. The older members are, in practice, invested much less than 100% in the same risky assets, despite both the individual and collective strategies being 100% in risky assets.

The exploration of a CDC scheme which does not use pension increases to express the risk-sharing mechanism in Barajas-Paz and Donnelly (2022), but rather applies the same change to the value of all accrued pensions, did not find evidence of pension smoothing. This means that using annual pension increases to enable the risk-sharing is essential for pension smoothing.

Disadvantages of the pension smoothing mechanism

As members age, and assuming they do indeed stay in the CDC scheme for the whole of their life, they should benefit from pension smoothing, on average. However, members do not have to stay in the scheme their whole lives. For example, a member who joins when they are a few years from retirement will experience the pension smoothing. A member who leaves at a fairly young age will not benefit from the lower exposure to investment risk at older ages.

There are risks in the transfer of risk from the old to the young. Suppose there is a sustained period of negative investment returns. As they are leveraged, the younger members experience a much larger drop in the value of their pensions than the returns suggest. Suppose that returns do bounce back after some time. However, by that time, the younger members are less young, and hence less leveraged within the CDC scheme. They may not be able to recover the loss in value. The opposite also holds true as well, if there is a sustained period of highly positive investment returns.

3.2. Cross-subsidies caused by financial unfairness on an accrued basis

Financial fairness is an idea which underpins many products which are managed by actuaries. The cost of a benefit should equal its expected value, if cost, fees and profits are excluded. The calculation of the expected value should use current predictions of, for example, investment returns and future lifetimes.

In the existing literature on CDC plans, the first generations in a CDC plan do less well than later generations (for example, Døskeland and Nordahl 2008). The reason is that some of the first generations' contributions are used to build up a buffer, whether it is implicit or explicit, rather than providing benefits. One the buffer is established, the middle generations benefit from it as a imaginary storehouse from which money can be withdrawn when investment returns are lower than predicted, and deposited when returns are higher than predicted.

Importantly, the plans studied in the literature assume that each contribution is sufficient to fund, in isolation, the pension accrued by that contribution (Cui et al., 2011). This financial fairness requirement means that the discounted value of the benefit accrued by each contribution should equal that contribution.

In the modelled CDC scheme, financial fairness occurs only on a lifetime basis for the very first generation to join. That is, when the first generation joins, the expected value of the future contributions they will make is equal to the expected value of the future pension they will receive. The calculation basis uses predictions of investment returns and future lifetimes expected at that time of joining. Economic predictions will have changed since the scheme started, and the contributions paid by future generations may be either insufficient or too high for the desired target pension level.

It also means that financial fairness does not hold for the benefit accrued by each contribution. It means that a younger member pays too much for the pension accrued by a contribution, and an older member too little. If a younger member leaves the scheme early, then the value of what they transfer out is likely to be small relative to the contributions they have made.

Moreover, an anomaly arises in the pension increase calculation, due to the lack of financial fairness on an accrued basis. The calculation of the annual pension increase compares the accumulated value of the contributions made to the accumulated value of the benefits accrued by those contributions. When the first generations join, their contributions are large relative to the benefits they have accrued. To compensate, and make the value of the accrued benefits equal the accumulated contributions, the annual pension increase is relatively large. This anomaly persists, as is seen in the numerical illustrations below, and its cost is borne by later generations.

Barajas-Paz and Donnelly (2022) observe the same inconsistency for a similar CDC scheme, but one in which all members receive the same change to their projected-to-retirement pension. As is shown here, the inconsistency holds for the CDC scheme studied in this paper.

To reduce the level of financial unfairness, benefits could be accrued on an age-related basis. This means that, if two members pay the same amount of contribution then the younger member accrues a higher level of pension than the older one. Perfect financial fairness would mean that the level of pension accrued would vary over time, as predictions of future investment returns and lifetimes changed. However, even a fixed age-related basis would reduce the financial unfairness and hence reduce the level of the pension increases awarded to the first generations in the scheme.

3.3. Cross-subsidies due to other factors

The need to pay and value a regular income to CDC scheme members means that economic predictions of the future must be made. If these predictions deviate from what is experienced, then notional gains and losses are made in the scheme. While these notional gains and losses are crystallised once pensions are paid out, the share of their cost among members is allocated by the annual pension increases. These gains or losses must benefit or be paid for by future generations, which gives risk to this third form of inter-generational cross-subsidies.

At the commencement of the CDC scheme, it is not known who will gain or lose from economic reality deviating from its predictions. However, once the CDC scheme begins operating, then the historical gains and losses could be calculated. This means that there each new generation to join the scheme will bear the known cost of the past. If this cost becomes too great, new generations may be deterred from joining the scheme and the inter-generational contract is broken.

The level of cross-subsidies borne by each member will also change as the number of members joining changes. The more the members, the lower the cost per member but the higher the absolute cost which may be passed onto a smaller number of later joining members.

4. Alternative retirement choices

An alternative option for pension saving is for an individual to accumulate their contributions in a DC arrangement. The contributions are invested in a specified way. At retirement, they convert their accumulated pension savings into an income using one of three options:

- Income drawdown (denoted by 'DWD' in the mathematical notation), in which the pension savings remain invested and an income is withdrawn for a fixed number of years with the goal that it increases in line with inflation;
- Purchase of a life annuity ('ANN'), in which the pension savings are paid over to a life insurance company in exchange for an inflation-linked, lifetime income; or
- Entry into a pooled annuity fund ('PAF'), to which the individual's pension savings are transferred. The pension savings remain invested, but earn longevity credits – a share of the money of those who have died – as well as investment returns. A lifetime income is withdrawn from the individual's pension savings, again with the goal that the income increases in line with inflation.

To develop the mathematical notation, consider a member of Generation g, for $g = 0, 1, \dots, M-1$. Upon retirement, the individual chooses one of the three options above. Let X denote which of the three options above was selected, with $X \in \{\mathrm{DWD}, \mathrm{ANN}, \mathrm{PAF}\}$. Let $R_X^{(g)}(n)$ denote the random investment return earned on the accumulated value of the member's pension savings, from time $n-1$ to time n, for $n = g+1, g+2, \dots, g+\omega$. Denote by $A_X^{(g)}(n)$ the accumulated value of their contributions at time $n = g, g+1, \dots, g+\omega$. Let the annual income paid to the individual be $B_X^{(g)}(n)$ units at time $n = g+T, g+T+1, \dots, g+\omega$.

Then, for all three options, the accumulated pension savings at time n of the member of generation g is calculated as

$$A_X^{(g)}(n) = \begin{cases} C(g) & \text{if } n = g, \\ A_X^{(g)}(n-1)\left(1 + R_X^{(g)}(n)\right) + C(n) & \text{if } n = g+1, g+2, \dots, g+T-1, \\ A_X^{(g)}(n-1)\left(1 + R_X^{(g)}(n)\right) - B_X^{(g)}(n) & \text{if } n = g+T, g+T+1, \dots, g+\omega-1. \end{cases}$$

Assume that every individual starts saving into the DC plan when they are age x, and they retire at age $x+T$. The calculation of the annual income is specified below, according to which option is chosen.

4.1. Income drawdown during retirement

In the DC plan followed by income drawdown in retirement, each member of generation g keeps their accumulated savings invested at retirement. At the start of every year, they withdraw an income from their savings. Each member assumes that they will live, at best, to age $x_{5\%}$, the age at which they have a 5% chance of surviving to, from age $x+T$, under a specified mortality distribution.

The income is calculated so that it should be constant until age $x_{5\%}$, if their future expected investment returns had no volatility. At time $n \geq g$, a member of generation g is age $x+n-g$. Allowing for price inflation, this means that the annual, inflation-linked payment calculated during retirement at time $n = g+T, g+T+1, \dots, g+x_{5\%}-x-1$ is

$$B_{\mathrm{DWD}}^{(g)}(n) = \frac{A_{\mathrm{DWD}}^{(g)}(n-1)\left(1 + R_{\mathrm{DWD}}^{(g)}(n)\right)}{\ddot{a}_{\overline{x_{5\%}-(x+n-g)}|}}.$$

The present value of the term annuity, $\ddot{a}_{\overline{x_{5\%}-(x+n-g)}|}$, is calculated assuming that the annuity payments increases at the rate of price inflation each year and the discount rate reflects the investment returns expected for a stated investment strategy.

Once the value of $B^{(g)}_{\text{DWD}}(n)$ is known, the amount $B^{(g)}_{\text{DWD}}(n)$ units is paid out and their accumulated pension savings falls to $A^{(g)}_{\text{DWD}}(n-1)\left(1+R^{(g)}_{\text{DWD}}(n)\right)-B^{(g)}_{\text{DWD}}(n)$.

Before retirement, a prediction of the annuity payment can also be calculated. Abusing notation, let the value of the predicted annuity payment be $B^{(g)}_{\text{DWD}}(n)$ at time $n=g,g+1,\ldots,g+T$. Let $P_{\text{DWD}}(n,m)$ denote the return expected from time n to time m, calculated using the information available at time n. For $n=g,g+1,\ldots,g+T-1$, the predicted drawdown payment payable from time $g+T$ is

$$B^{(g)}_{\text{DWD}}(n)=\frac{A^{(g)}_{\text{DWD}}(n)}{(1+\text{infl}(n))^{g+T-n}\,P_{\text{DWD}}(n,g+T)\,\ddot{a}_{\overline{x_{5\%}-(x+T)}|}},$$

in which infl(n) is the annual price inflation rate expected from time n and the present value of the term annuity, $\ddot{a}_{\overline{x_{5\%}-(x+T)}|}$, is calculated using the same approach as the term annuity presented above.

4.2. Life annuity contract at retirement

In this option, the individual invests in a DC plan before retirement and then purchases a life annuity contract at retirement. It is assumed that they buy a price inflation-linked life annuity.

Let the constant real number $\alpha>0$ reflect the cost of guaranteeing the life annuity payment. The cost distinguishes it from the pooled annuity fund option. In the model considered here, the cost reflects only the investment guarantee. It is assumed that there is no longevity risk for the insurance backer of the life annuity contract. If there was, then a higher cost would be charged.

This means that the annual annuity payment, which increases each year in retirement in line with a price inflation index, bought at retirement is

$$B^{(g)}_{\text{ANN}}(g+T)=\frac{A^{(g)}_{\text{ANN}}(g+T)}{(1+\alpha)\ddot{a}_{x+T}}.$$

The present value of the life annuity, $\ddot{a}_{x+T}$, is calculated assuming that the annuity payments increases at the rate of price inflation each year and the discount rate reflects the long-term bond yield prevailing at the retirement time $g+T$.

Before retirement, a prediction of the annuity payment can also be calculated. Abusing notation, let the value of the predicted annuity payment be $B^{(g)}_{\text{ANN}}(n)$ at time $n=g,g+1,\ldots,g+T-1$. Let $P_{\text{ANN}}(n,m)$ denote the return expected from time n to time m, calculated using the information available at time n. For $n=g+1,\ldots,g+T-1$, the predicted life annuity payment payable from time $g+T$ is

$$B^{(g)}_{\text{ANN}}(n)=\frac{A^{(g)}_{\text{ANN}}(n)}{e^{(g+T-n)\delta_q(n)}P_{\text{ANN}}(n,g+T)\,(1+\alpha)\,\ddot{a}_{x+T}},$$

in which the present value of the life annuity, $\ddot{a}_{x+T}$, is calculated using the same approach as the life annuity presented above in this section, and the force of inflation, $\delta_q(n)$, at time n is used as the predictor of future inflation.

4.3. Pooled annuity fund during retirement

The individual may choose instead to invest in the DC plan up to retirement, and then join a pooled annuity fund. Their accumulated savings are pooled with other participants of the pooled annuity fund, in order to get a higher expected income than in the other two options. The income is expected to be higher than that offered under a life annuity contract, as there are no guarantees in the considered pooled annuity fund. The income should also be higher than under income drawdown, since survivors in the pooled annuity fund get a share of the funds of those who have died, called a longevity credit. Various pooled annuity funds have been proposed and studied in the literature (for example, Piggott et al. 2005; Stamos 2008; Sabin 2010; Qiao and Sherris 2013; Donnelly et al. 2014; Milevsky and Salisbury 2016).

Participants of the pooled annuity fund earn both an investment return and a longevity credit on their pension savings, while they are alive. The longevity credit is always positive. At worst, it is zero, corresponding to no-one else dying over the last year. The participants withdraw an income to live on, and this income is higher than income drawdown due to the positive longevity credits.

In this simple model considered, there is only investment risk borne by the participants in the pooled annuity fund. The investment risk distinguishes the pooled annuity fund from the life annuity contract. From the perspective of the purchaser of the life annuity contract, there is no investment risk. By locking into an inflation-linked annuity payment at retirement, the investment risk inherent in providing such a payment has been transferred to the annuitant provider. The provider will charge the annuitant for bearing the risk. In contrast, the participants in the pooled annuity fund are not guaranteed an inflation-linked annuity payment. Instead, their payment will fluctuate with fluctuations in the investment returns. This approach is cheaper but riskier.

In the simple model considered here, there is no longevity risk in the pooled annuity fund. It is assumed that the pooled annuity fund has a sufficient number of members to fully diversify its idiosyncratic longevity risk at all times. While the assumption is a simplification, it is likely that a pooled annuity fund would attract many members from across many companies or industries. As shown in Donnelly (2022), around 100 members joining the fund each year would almost fully diversify the random fluctuations risk. In contrast, a CDC plan is more likely to have a more restricted membership, perhaps to a single company. This is entirely possible, particularly in the UK where very small DB pension plans of fewer than 100 members have been and are still prevalent (Pension Protection Fund, 2021, Figure 2.1). Systematic longevity risk has been ignored throughout the paper but should be studied in detail elsewhere.

For the pooled annuity fund option, each member of generation g retires at time $g+T$ and immediately transfers their accumulated savings to a pooled annuity fund. At the start of every year in retirement, they withdraw an income from their savings. It is assumed that participants withdraw an inflation-linked income.

The annual calculation of the income withdrawn from the pooled annuity fund takes account of the future investment returns expected from the time of calculation onwards. At time $n \geq g$, a member of generation g is age $n+x-g$. Allowing for price inflation, this means that the annual payment calculated (during retirement) at time $n = g+T, g+T+1, \ldots, g+x+T$ is

$$B^{(g)}_{\text{PAF}}(n) = \frac{A^{(g)}_{\text{PAF}}(n-1)\left(1+R^{(g)}_{\text{PAF}}(n)\right)}{\ddot{a}_{n+x-g}}.$$

The present value of the life annuity, $\ddot{a}_{n+x-g}$, is calculated assuming that the annuity payments increase at the effective rate $e^{(g+T-n)\delta_q(n)}-1$ each year, and the discount rate reflects the expected investment returns during retirement.

Once the value of $B^{(g)}_{\text{PAF}}(n)$ is known, the amount $B^{(g)}_{\text{PAF}}(n)$ units is paid out and their accumulated pension savings falls to $A^{(g)}_{\text{PAF}}(n-1)\left(1+R^{(g)}_{\text{PAF}}(n)\right) - B^{(g)}_{\text{PAF}}(n)$.

Before retirement, a prediction of the annuity payment can also be calculated. Abusing notation, let the value of the predicted annuity payment be $B^{(g)}_{\text{PAF}}(n)$ at time $n = g, g+1, \ldots, g+T$. Let $P_{\text{PAF}}(n,m)$ denote the return expected from time n to time m, calculated using the information available at time n. For $n = g+1, \ldots, g+T-1$, the predicted life annuity payment payable from time $g+T$ is

$$B^{(g)}_{\text{PAF}}(n) = \frac{A^{(g)}_{\text{PAF}}(n)}{e^{(g+T-n)\delta_q(n)} P_{\text{PAF}}(n, g+T)\, \ddot{a}_{x+T}},$$

in which the present value of the life annuity, $\ddot{a}_{x+T}$, is calculated using the same approach as the life annuity presented above.

5. Numerical study

To illustrate and understand better the CDC plan, and compare it to the three alternative options, they are studied numerically. The models detailed above were programmed into the statistical software package *R*. Two economic models were considered: a deterministic model to understand broadly the CDC plan and a stochastic model to understand the level of uncertainty in retirement income paid out in each pension option.

To compare the retirement options, it is assumed that the investment strategy for all pension options is to invest all assets in a risky asset, in which dividends are re-invested.

The valuation of accrued benefits requires an assumption about future investment returns. In line with the approach typically taken by UK pension actuaries, the future annual return on the risky asset is the sum of the current long-term bond yield and a constant equity risk premium. Let this sum at integer time $n \geq 1$ be denoted i_n. Thus for $m \geq n$, the annual investment return expected to be earned on the risky asset over the time period $(m, m+1]$, conditional on the information available at time n, is assumed to be i_n. In particular, for integers $m \geq n$, the discounting factor $P_{\text{X}}(n,m) = (1+i_n)^{m-n}$ in the valuation of the notional liabilities in each pension option $X \in \{\text{CDC}, \text{DWD}, \text{ANN}, \text{PAF}\}$.

5.1. Membership characteristics

Members join the scheme at age 25 and retire at age 65. There are a total of 150 generations in the scheme, with Generation g joining at time g, for $g = 0, 1, \ldots, 150$. In the notation introduced above,

$$x = 25, \qquad T = 40, \qquad M = 150.$$

All individuals have independent future lifetime random variables. Everyone survives to age 65 and then, from age 65, their mortality follows a UK life table, S1PMA (Continuous Mortality Investigation, 2008). The last integer age at which anyone is alive is 119 years under the life table, so the maximum future lifetime from the age that any individual joins the scheme is $\omega = 120 - x = 95$ years.

Under this life table, there is a 5% chance that an individual who is age 65 years will survive to age 90 years. Thus $x_{5\%} = 90$.

5.2. Constant economic model

First, consider a simple, constant return model. Wage growth is set to 3% per annum, so $S(n) = 1.03^n S(0)$. Inflation is fixed at $\text{infl}(n) = 0.02$, investment returns on the risky asset are expected to be $i_n = 0.05$ per annum effective for all integer $n \geq 0$. The long-term bond yield is expected to be 0% per annum effective and the equity risk premium is fixed at 5% per annum effective. Pension increases are expected to be equal to the annual inflation rate of 2% per annum. Using these assumptions, the contribution rate is 9.5% of salaries.

It is assumed that a very large number of members, $N^{(g)}(g) = 1\,000\,000$, join in each generation g. The focus in this investigation is on the investment risk-sharing aspect rather than the longevity risk-sharing aspect. A large number of members means that the longevity risk is well diversified, except for the last living years of the last generations to join the scheme. The longevity risks in the end phase of this CDC scheme are comparable to those faced by a pooled annuity fund, which is studied in Donnelly (2022).

In the numerical results, the replacement ratio is often reported. This is the ratio of the pension income divided by the salary. At a time n which is before retirement, it is the ratio of the pension income accrued up to time n divided by the salary amount at time n. For a time n which is during retirement,it is the ratio of the pension income paid out at time n divided by the salary amount at the time of retirement. Thus, the retirement ratio before retirement grows at the difference between the pension increase rate and the salary growth rate, as well as due to new accrued benefit. The replacement ratio during retirement grows in line with pension increases only and will generally grow at a much faster rate than the pre-retirement ratio.

5.2.1. Investment returns turn out as expected

First suppose that the assumed economic model is realised; wages and inflation grow as predicted. In particular, the actual investment return over every future year is 5%, i.e. $R_n = 0.05$ for $n = 1, 2, \ldots$. As the contribution rate is calculated assuming that annual pension increases are 2% and everything turns out as expected, it might be also expected that pension increases in the CDC plan are 2% per annum. However, the numerical results show that this is not the case. This is an example of the cost transfer from the first generations to the later generations, which is described in Section 3.2.

Generation 0 joins at age 25

Figure B.1a shows the replacement ratio at retirement, for each generation, under all four pension options. In this simplified setting, the life annuity option with 0% loading is identical to the pooled annuity fund option.

In the CDC plan, the first generations to join have a higher replacement ratio than any of the other options. As discussed earlier, this is due to contributions being financially fair on a lifetime basis but not on an accrued basis. The first generations are awarded pension increases larger than 2%, which decline over time (as can be observed in Figure B.2a).

The first generations in the CDC plan do better than later generations too. These larger annual pension increases are paid for by awarding later generations pension increases of less than 2% per annum. In this simple setting, from about Generation 30 onwards, members would do better to not join the CDC plan and rather invest their contributions in a DC plan followed by either a life annuity purchase or pooled annuity fund membership. The last generations to join the CDC scheme have the lowest replacement ratio at retirement, of all the generations to join.

The income drawdown option is the worst option, in terms of replacement ratio at retirement in Figure B.1a. Without the benefit of longevity risk pooling, it cannot compete against the other options when only income is assessed.

Forty generations join at time 0

As an aside, the following example shows that the qualitative results do not change – that the first generations do better than later generations – if the membership in the CDC plan at time 0 is more dispersed in age. Suppose that at time 0, Generations $0, 1, \ldots, 39$ join the scheme. At time 0, Generation 0 is age 64, Generation 1 is age 63, and so on, with Generation 39 at age 25. From time 1 onwards, each new generation to join does so at age 25. So at time 1, Generation 40 joins at age 25, at time 2, Generation 41 joins at age 25 and so on.

The replacement ratio at retirement of this more mature membership is shown in Figure B.1b. Generations $0, 1, \ldots, 38$ have contributed for less than 40 years at the time of their retirement, which explains their significantly lower retirement ratio in Figure B.1b. However, these first generations continue to get a higher annual pension increase than later generations (as can be observed in Figure B.2b). Again, it is the later generations who bear the cost of the higher pension increases awarded to the first generations.

Pension increases under both profiles

The pension increases under these two membership profiles are shown in Figure B.2. The horizontal, solid line indicates the pension increase given under the other three options, of 2% per annum. The vertical lines indicate different phases in the CDC scheme's life.

The higher pension increases, from which the first generations benefit the most, are seen under both membership profiles. The cost of these higher pension increases is that in the middle phase of the CDC scheme, the annual pension increases are slightly less than 2% per annum. There is a further decrease in the annual pension increases when no more generations join the scheme. This is a further crystallisation of the cost of the higher pension increases given to the first generations. Once all generations have retired, the annual pension increases stabilises for a while. When few members are left alive, the pension increases become extremely volatile (identical sample paths of the members' lifetimes are used in the two plots in Figure B.2). Of course, in practice, the CDC scheme would not run until every last member is dead.

5.2.2. Investment returns are not as expected

Suppose investment returns achieved were 1% per annum lower than expected. Figure B.3a shows that the cost of the wrong predicted return is borne by the later generations. The first forty generations have a higher replacement ratio than under the other three pension options. They do not experience the full impact of the lower, actual returns, as is seen in the DC-based pension options.

However, as returns keep below their predicted values, the CDC plan's replacement ratio at retirement declines and slips beneath the annuity options. For the last generations, their replacement ratio is below that under income drawdown. This is an example of the inter-generational cross-subsidies due to wrong predictions, as outlined in Section 3.3.

Allowing the investment returns achieved to be 1% per annum higher than expected (Figure B.3b), less is paid out to the first generations of the CDC plan, compared to the annuity options. The first generations are again shielded from the full effect of the higher actual returns, and do not benefit as much as the DC-based pension options. However, as investment returns continue to exceed expectations, the later generations gradually benefit more. From about Generation 50, the CDC plan delivers a replacement ratio that is above all the other options.

These two simple examples illustrate the risks of consistently wrong predictions of future investment returns, as is discussed in Section 3.3. Investment returns being lower than expected means that the earlier generations are paid too much, with hindsight. Consequently the later generations bear the cost of these too-high pension payments. The greatest burden is borne by the last generations, when contributions stop flowing into the CDC scheme. The reverse happens when investment returns are lower than expected; the earlier generations are paid too little and the later generations benefit.

However, it is not known when the CDC scheme begins who will bear the risks of wrong predictions. It could be argued that this is the solidarity element of investment risk-sharing. This is unlike the other two types of inter-generational cross-subsidies in which it is known who is benefiting and who is paying for the risks. On the other hand, once the CDC scheme has started, it will be known at each time how accurate were past predictions. Thus the cost of historical wrong predictions will be known and this reduces solidarity: a new generation may be joining knowing that the scheme has paid out too little in the past and thus they can benefit.

5.3. Stochastic economic model

Now turn to the application of a more realistic economic model: the Wilkie model (Wilkie 1986, 1995). The model, detailed in Appendix A, is used to generate an inflation index, equity returns and long-term bond returns. The Wilkie model is a mean-reverting, discrete-time model with prices and indices reported annually. The parameter values used are the ones from fitting the model to UK market data from 1923 to 2009 and are taken from Wilkie et al. (2010).

Across 2 000 simulations of the Wilkie model, the average annual median total return on shares is 11.8% per annum and that on the long-term bond yield is 6.6% per annum. The average annual median wage growth is 6.0% per annum and that for price inflation is 4.5% per annum. The annual contribution rate as a percentage of salary is set to 4.5% per annum, based on the starting values of the Wilkie model, as detailed in the Appendix.

The return on the risky asset is the return on the Wilkie model's total return index of shares.

5.3.1. 100% investment in the risky asset

Suppose that, for all pension options, the investment in 100% in the risky asset. In particular, for the DC and life annuity option, the life annuity is purchased at retirement for its fair value less 5% loading, to pay for the removal of investment risk from the annuitant. The 5% loading is arbitrary and has not been justified on the grounds of maximization of a utility function or financial fairness. For the purposes of this paper, the annuity loading is greater than zero to show that the removal of investment risk and longevity risk (though the latter is not studied in this paper) has a cost.

Replacement ratio at retirement

First consider the replacement ratio at retirement. The median replacement ratio at retirement decreases over time (Figure C.1). Furthermore, the standard deviation of that ratio increases over time (Figure C.2a). In other words, the later that a generation joins the CDC scheme, the lower is the median of the replacement ratio but the higher its volatility. This is illustrated by the coefficient of variation of the ratio, which is the ratio of the standard deviation to the mean replacement ratio at retirement, going up the later than a generation joins the CDC scheme (Figure C.2b).

Normally, it would be expected that a lower volatility of income is associated with a lower expected income. The surprising result here is that the lower risk faced by the earlier generations is rewarded by a higher pension income than that of later generations. Additionally, this is a different result from that of different investment risk-sharing products or plans modelled in the academic literature (for example, Døskeland and Nordahl 2008). However, in contrast to the plans modelled in the academic literature, the contributions to the modelled CDC plan are never financially fair. As discussed in Section 3.2, the lack of financial fairness leads to the first generations benefiting from higher pension increases.

Comparison with alternative pension options

In contrast to the CDC plan, each of the DC-based pension options offer risks which are the roughly the same for all generations. There are no investment cross-subsidies in the DC plans, under the assumption of no longevity risk.

For roughly the first twenty generations, the CDC offers a superior median replacement ratio at retirement than the DC-based alternatives (Figure C.1). However, later generations in the CDC plan do worse than their longevity risk-sharing alternatives; the life annuity and pooled annuity fund. Although the replacement ratio at retirement of the middle generations in the CDC plan is about the same as that in the drawdown option, the latter option pays only a fixed term annuity in retirement whereas the CDC plan pays a lifetime income. The last generations in the CDC plan have the lowest median replacement ratio at retirement of all the options.

The standard deviation of the replacement ratio at retirement shows a similar story (Figure C.2a).

5.3.2. Pension smoothing

The ability of the CDC plan to smooth the accrued pension over time, up to retirement, is explained in Section 3.1. It does this by transferring risk from the older members in the fund to the younger members, at each point in time.

Pension smoothing before retirement is illustrated in Figure C.3 by comparison to the three DC-based pension options. Even when investing 100% of its assets in the risky asset, the accrued CDC pension up to retirement is in a significantly narrower range around its median value, compared to the other options. However, in line with the observations made above, the range increases for each generation.

5.3.3. More realistic investment strategies for the alternative options

Suppose that the investment strategies followed by the four pension options are the following.

- CDC scheme (CDC): 100% investment in the risky asset at all times;
- Income drawdown (DWD): 100% investment in the risky asset until age 55, after which time the investment is reduced in even-sized steps, to 70% in long-term bonds and 30% in the risky asset at age 65;
- Purchase of a life annuity (ANN): 100% investment in shares until age 55, after which time the investment is reduced in even-sized steps, to 100% in long-term bonds and nothing in the risky asset at age 65;
- Entry into a pooled annuity fund (PAF): the same investment strategy as under income drawdown.

The effect of a reduced investment in the risky asset for the DC-based options is to reduce the expected replacement ratio (Figure C.4), as well as the volatility of the ratio (Figure C.5). It increases the attractiveness of the CDC plan, if the median replacement ratio is the criterion for comparison.

For the middle generations in the CDC plan, the DC and Pooled Annuity Fund option remains a viable contender. The cost of the higher benefits to the first generations in the CDC plan reduces the benefits to these middle generations and offsets the higher expected returns of the CDC plan's 100% risky asset strategy.

Turning to the standard deviation and coefficient of variation (the risk borne by each unit of expected unit of replacement ratio), the DC and Pooled Annuity Fund option becomes more attractive for the middle generations. It offers a lower standard deviation of the replacement ratio compared to the CDC plan (Figure C.5), while having a fairly similar median value. Furthermore, the coefficient of variation of the life annuity option is similar to that of the pooled annuity fund, for the middle generations.

These results suggest that the CDC plan having a higher risk investment strategy may not provide a superior outcome on typical statistical measures like the median and standard deviation. It may give a better outcome for some generations but not necessarily for all generations.

6. Conclusion

Inter-generational fairness is a topical issue for CDC plans in the UK and elsewhere. Three types of inter-generational cross-subsidies which exist in the UK's first CDC scheme have been described, explained and illustrated. One of them, financial unfairness, leads to the first generations in the scheme getting a better outcome than later generations. It is a one-way cross-subsidy, in which the later generations who join the CDC scheme pay for the higher benefits of the first generations, through a lower pension. Its effect could be mitigated by using an age-related benefit accrual rather than a constant benefit accrual.

Pension smoothing is enabled in the CDC scheme through a transfer of risk from the young to the old, which is another type of cross-subsidy from the young to the old. In the cross-subsidy arising from financial unfairness, not every generation is the first to join. In this cross-subsidy, the young members can expect to be old one day, and thus benefit from the risk transfer as we age. The risk transfer allows the scheme to shield the older members from fluctuations in their income caused by a relatively high-risk investment strategy. However, members are at risk of a variation of sequencing risk: if they experience a long run of poor returns, they may not be able to recover when returns do improve as they age and are bearing too little risk in the scheme.

The third type of cross-subsidy arises mainly from economic predictions not being borne out in practice. For CDC plans to be successful, these cross-subsidies should not become too large. This is particularly important as the cost of historically poor economic predictions could be calculated for each new generation to join the scheme. Individuals with knowledge of the cost may choose not to join and thus exacerbate a poor scheme funding situation.

Additionally, the outcomes for successive generations in the presented model of CDC scheme have been modelled and compared to DC-based pension options. The results suggest that a post-retirement only CDC plan, i.e saving in a DC plan followed by investing in a pooled annuity fund, may be a reasonable alternative to a CDC plan for some generations. A further study which includes systematic longevity risk is warranted, to further investigate the results.

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A. Stochastic financial market model: the Wilkie model

The Wilkie model is in discrete time with prices and indices reported annually. The parameter values given below are the ones from fitting the model to UK market data from 1923 to 2009 and are found in Wilkie et al. (2010). The model and parameter values used are given here for completeness, with the model notation and exposition following closely that in Hardy 2003.

Price inflation is stripped out of the returns derived from the model, since the simplifying assumption in the presented CDC schemes is that cashflows are neither wage nor price inflation-linked but the main factor driving the Wilkie model is price inflation.

The stochastic process $Z_X := \{Z_X(k) : k = 1, 2, \dots\}$ is a sequence of independent random variables which each have a standard normal distribution, for each $X \in \{c, d, q, w, y\}$. Each of these stochastic processes is independent of the other.

A total of 100 000 simulated values were generated from the Wilkie model detailed below. The time unit is one year.

A.1. Inflation

For integer k, the force of inflation over the time period $[k-1, k)$ is

$$\delta_q(k) = \mu_q + a_q\left(\delta_q(k-1) - \mu_q\right) + \sigma_q Z_q(k), \qquad \delta_q(0) = \mu_q \quad \text{a.s.}$$

in which (Wilkie et al., 2010, Paragraph 2.12)

$$\mu_q = 0.043, \qquad a_q = 0.58, \qquad \sigma_q = 0.04.$$

A.2. Share prices and dividends

For integer k, the share dividend yield over the time period $[k-1, k)$ is

$$y(k) = \exp\left(w_y \delta_q(k) + \mu_y + yn(k)\right)$$

where

$$yn(k) = a_y \cdot yn(k-1) + \sigma_y Z_y(k), \qquad yn(0) = 0 \quad \text{a.s.}$$

The parameter values used are (Wilkie et al., 2010, Paragraph 4.8)

$$w_y = 1.55, \qquad \mu_y = \ln(0.0375), \qquad a_y = 0.63, \qquad \sigma_y = 0.155.$$

The force of dividend growth over the time period $[k-1, k)$ is

$$\delta_d(k) = w_d \cdot DM(t) + (1 - w_d)\delta_q(k) + d_y \sigma_y Z_y(k-1) + \mu_d + b_d \sigma_d Z_d(k-1) + \sigma_d Z_d(k)$$

where

$$DM(k) = d_d \delta_q(k) + (1 - d_d)\, DM(k-1), \qquad DM(0) = \mu_q \quad \text{a.s.}$$

The parameter values used are (Wilkie et al., 2010, Paragraph 5.7)

$$w_d = 0.43, \quad d_y = -0.22, \quad \mu_d = 0.011, \quad b_d = 0.43, \quad \sigma_d = 0.07, \quad d_d = 0.16.$$

The dividend index is calculated recursively as

$$D(k) = D(k-1) \exp\left(\delta_d(k)\right), \qquad D(0) = 1 \quad \text{a.s.}$$

As the dividend yield $y(k)$ at time k is the dividend index value $D(k)$ divided by the share price index value $P(k)$, it follows that

$$P(k) = \frac{D(k)}{y(k)}, \qquad P(0) = D(0)/y(0) \quad \text{a.s.}$$

A total return index on shares over the time period $[k-1, k)$ is derived by assuming that dividends received are re-invested in shares, e.g.

$$py(k) = py(k-1) \cdot \frac{P(k) + D(k)}{P(k-1)}, \qquad py(0) = 1 \quad \text{a.s.}$$

In the numerical simulations of the Wilkie model, it is assumed that the return on risky assets is the return on the total return index on shares.

A.3. Long-term bond yield

For simplicity, only the long-term bond yield $c(k)$ is used in the simulations. It is determined by

$$c(k) = cm(k) + cn(k),$$

in which

$$cm(t) = d_c \delta_q(k) + (1 - d_c) \cdot cm(k-1), \qquad cm(0) = \mu_q \quad \text{a.s.}$$

and

$$cn(k) = \mu_c \exp\left(a_c\, cn(k-1) + y_c \sigma_y Z_y(k) + \sigma_c Z_c(k)\right), \qquad cn(0) = \mu_c \quad \text{a.s.}$$

The parameter values used are (Wilkie et al., 2010, Paragraph 6.11)

$$d_c = 0.045, \quad \mu_c = 0.0223, \quad a_c = 0.92, \quad y_c = 0.37, \quad \sigma_c = 0.255.$$

In the numerical simulations of the Wilkie model, it is assumed that the return on risk-free assets is the long-term bond yield.

A.4. Wage growth

The force of wage inflation over time period $(k-1, k]$ is

$$\delta_w(k) = w_{w_1} \delta_q(k) + w_{w_2} \delta_q(k-1) + \mu_w + \sigma_w Z_w(k).$$

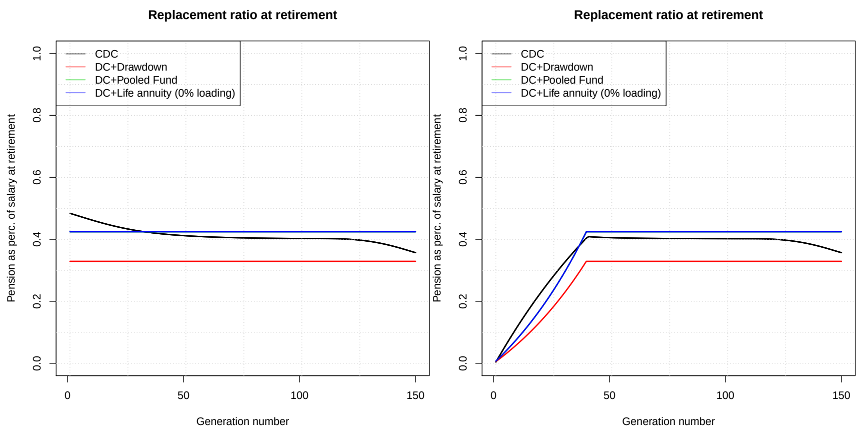
The parameter values used are (Wilkie et al., 2010, Paragraph 3.4, 3.5, 3.10)

$$w_{w_1} = 0.60, \quad w_{w_2} = 0.27, \quad \mu_w = 0.020, \quad \sigma_w = 0.0219.$$

A.5. Predicted return on assets and equity risk premium

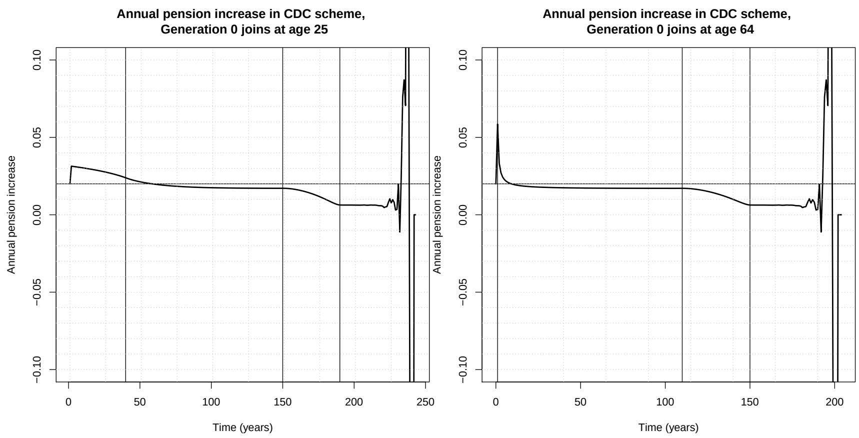
In the simulations, the predicted annual return on the long-term bond is assumed to a constant equal to the yield over the last year on the long-term bond. The predicted annual return on the risky asset is assumed to a constant equal to the return over the last year on the long-term bond plus an additional constant of 3% per annum, called the equity risk premium. The predicted annual return at each future time depends on the investment strategy in the future.

B. Constant economic model



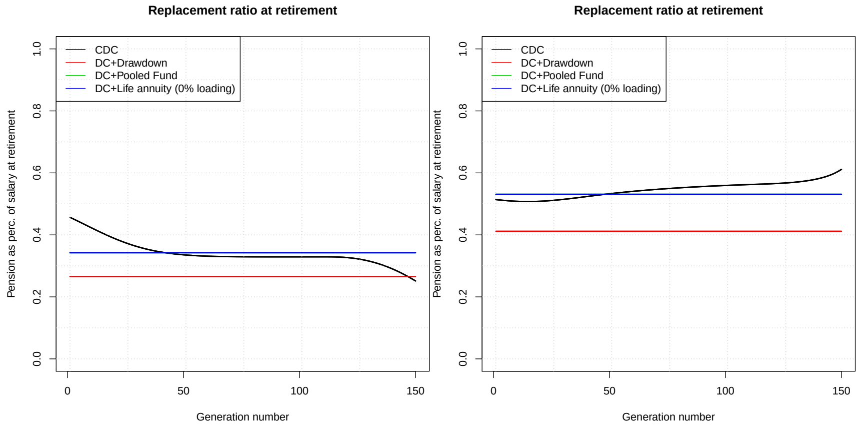
(a) Gen 0 starts at age 25 (b) Gen 0 starts at age 64

Figure B.1: Replacement ratio at retirement for all generations under each of the four possible pension options. The economic model assumes that returns turn out to be the same constant values as predicted. The replacement ratio under the DC and life annuity option is the same as the DC and pooled annuity fund, due to the perfect pooling and constant return assumptions made.



(a) Gen 0 starts at age 25 (b) Gen 0 starts at age 64

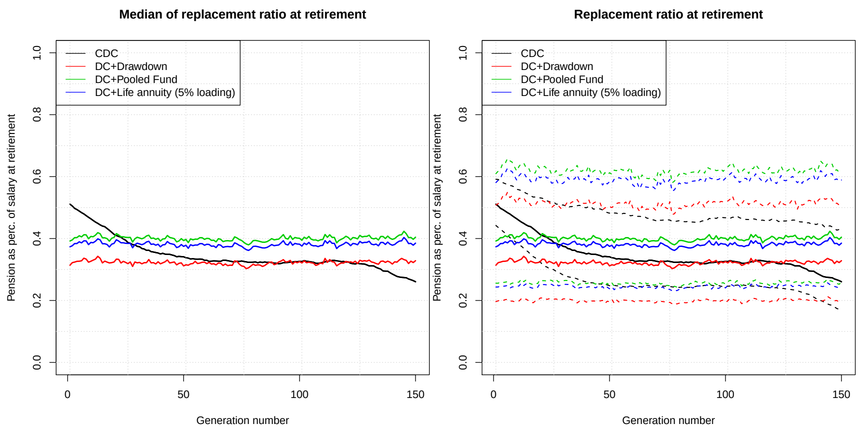
Figure B.2: Annual pension increases awarded in the CDC plan. The economic model assumes that returns turn out to be the same constant values as predicted. Reading from left to right, the first vertical line represents the time at which Generation 0 retires, the second vertical line the time at which the last generation, Generation 149, joins the scheme and the third vertical line represents the time at which Generation 149 retires. The volatility of the pension increases is high at the end of the scheme's life, when there are few members to pool longevity risk with each other.



(a) Lower by 1% per annum (b) Higher by 1% per annum

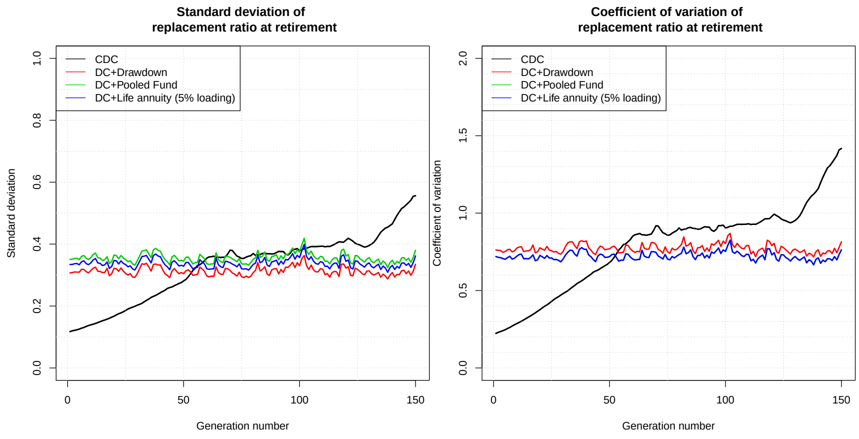
Figure B.3: Replacement ratio at retirement for all generations under each of the four possible pension options. The economic model assumes that returns are either 1% per annum lower (left-hand plot) or higher (right-hand plot) than predicted. The replacement ratio under the DC and life annuity option is the same as the DC and pooled annuity fund, due to the perfect pooling and constant return assumptions made. All generations start contributing to the chosen option at age 25.

C. Stochastic economic model



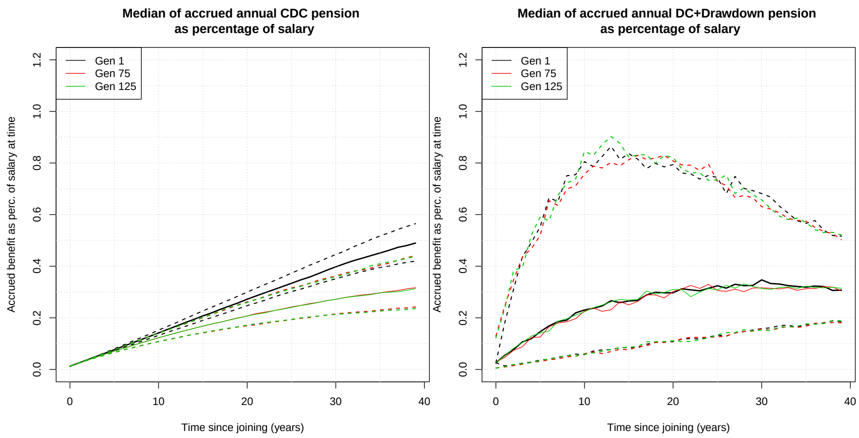
(a) Median (b) Median and quantiles

Figure C.1: Median replacement ratio at retirement (left-hand plot) and the same plot but with the 25% and 75% quantiles added (right-hand plot) for all generations under each of the four possible pension options. The economic model used is the Wilkie model and the investment strategy is 100% in the risky asset for all four pension options. A 5% loading is added to the cost of the life annuity at retirement, to account for the removal of investment risk.



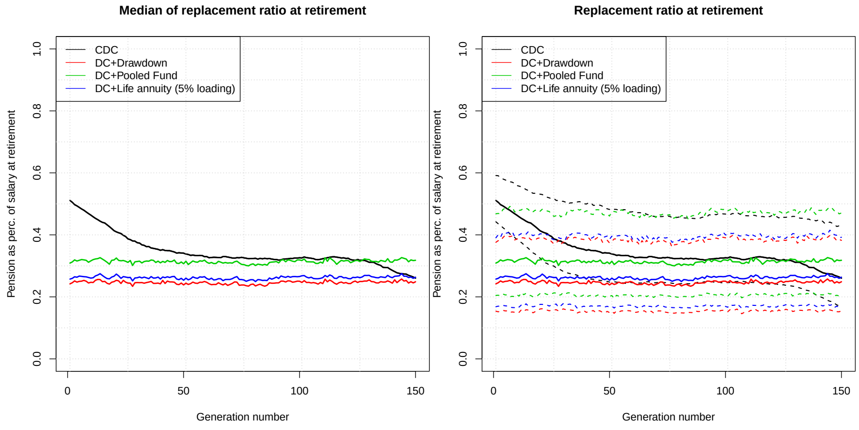
(a) Standard deviation (b) Coefficient of variation

Figure C.2: Standard deviation of the replacement ratio at retirement (left-hand plot) and the coefficient of variation (right-hand plot) for all generations under each of the four possible pension options. The economic model used is the Wilkie model and the investment strategy is 100% in the risky asset for all four pension options. A 5% loading is added to the cost of the life annuity at retirement.



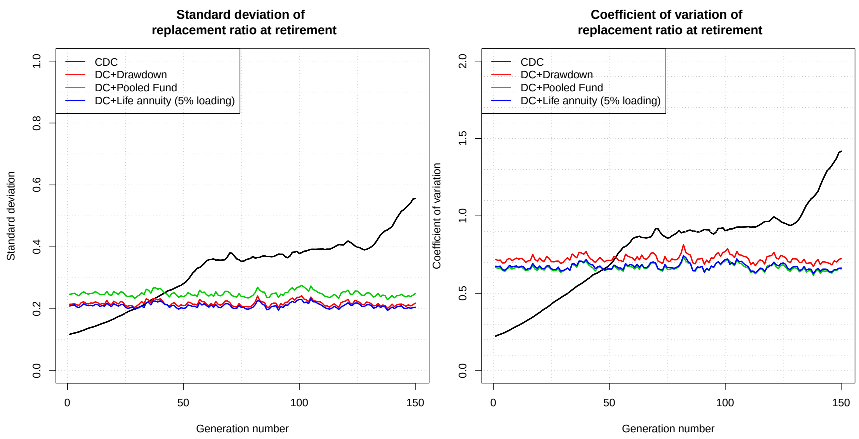
(a) CDC plan to retirement (b) DC to retirement

Figure C.3: Median and 25% and 75% quantiles of the accrued annual replacement ratio up until the age of retirement, for a selection of generations under the CDC plan and DC plan. The economic model used is the Wilkie model and the investment strategy is 100% in the risky asset for all four pension options.



(a) Median (b) Median and quantiles

Figure C.4: Median replacement ratio at retirement (left-hand plot) and the same plot but with the 25% and 75% quantiles added (right-hand plot) for all generations under each of the four possible pension options. The economic model used is the Wilkie model. The investment strategy is 100% in the risky asset for all plans up to 10 years before retirement. Then the DC & Drawdown and DC & Pooled Annuity Fund options lifestyle to 30% in the risky asset and 70% in bonds, the DC & Life Annuity option lifestyles to 100% in bonds and the CDC plan remains 100% invested in the risky asset. A 5% loading is added to the cost of the life annuity.



(a) Standard deviation (b) Coefficient of variation

Figure C.5: Standard deviation of the replacement ratio at retirement (left-hand plot) and the coefficient of variation (right-hand plot) for all generations under each of the four possible pension options. The economic model used is the Wilkie model. The investment strategy is 100% in the risky asset for all plans up to 10 years before retirement. Then the DC & Drawdown and DC & Pooled Annuity Fund options lifestyle to 30% in the risky asset and 70% in bonds, the DC & Life Annuity option lifestyles to 100% in bonds and the CDC plan remains 100% invested in the risky asset. A 5% loading is added to the cost of the life annuity.