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Multi-scale fracture network characterization and impact on flow:
a case study on the Latemar carbonate platform

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Abstract (248 words)
An fracture network arrangement is quantified across an isolated carbonate platform from
outcrop and aerial imagery to address its impact on fluid flow. The network is described
in terms of fracture density, orientation and length distribution parameters. Of particular
interest is the role of fracture cross-connections and abutments on the effective
permeability. Hence the flow simulations explicitly account for network topology by
adopting Discrete-Fracture-and-Matrix description. The interior of the Latemar carbonate
platform (Dolomites, Italy) is taken as outcrop analogue for subsurface reservoirs of
isolated carbonate build-ups that exhibit a fracture dominated permeability. New is our
dual strategy to describe the fracture network both as deterministic and stochastic based
input for flow simulations. The fracture geometries are captured explicitly and form a
multi-scale data set by integration of interpretations from outcrops, airborne imagery and
LiDAR. The deterministic network descriptions form the basis for descriptive rules that

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are diagnostic of the complex natural fracture arrangement. The fracture networks exhibit a variable degree of multi-tier hierarchies with smaller sized fractures abutting against larger fractures under both right and oblique angles. The influence of network topology on connectivity is quantified using Discrete-Fracture-Single phase fluid flow simulations. The simulation results show that the effective permeability for the fracture and matrix ensemble can be 50 to 400 times higher than the matrix permeability of $1.0 \cdot 10^{-14}$ m$^2$. The permeability enhancement is strongly controlled by the connectivity of the fracture network. Therefore, the degree of intersecting and abutting fractures should be captured from outcrops with accuracy to be of value as analogue.

**Keywords:** natural fractures, multi scale characterization, discrete fracture networks, network hierarchies, network connectivity, effective fracture permeability.

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**1. Introduction**

Fracture networks can exert a strong influence on fluid circulation through water aquifers and hydrocarbon reservoirs [Odling et al., 1999; Berkowitz, 2002; Gillespie et al., 2011] and influence many geologic process such rock alteration and mineral precipitation [Oliver and Bons, 2001; Carmichael et al., 2008]. In addition to controlling single- and multi-phase heat and mass transfer [Geiger and Emmanuel, 2010; Geiger et al., 2010; Nick et al., 2011], the structural arrangement of fracture networks plays an important role
on the effective flow response [de Dreuzy et al., 2002; Matthäi and Belayneh, 2004; Davy et al., 2010; Paluszny and Matthai, 2010; Latham et al., 2013; Lang et al., 2014; Lei et al., 2014], especially for rocks with low primary porosity and permeability.

Fractures can introduce highly conductive pathways in case of connected open joints or form barriers to flow in case of cemented veins or fault gauge [Nelson, 2001]. Fractures influence flow across multiple length scales from sub-meter joints to kilometer long fractures and faults [Davy et al., 2006, 2010; Gross and Eyal, 2007].

The central question of this study is in how far the network arrangement of fractures increases the permeability of tight carbonates. Beside fracture aperture, the network connectivity is one of the critical factors. Network connectivity is controlled by fracture orientation and length distributions and in turn influenced by hierarchies and abutment relationship [Barton, 1995; Odling et al., 1999; Sanderson and Nixon, 2015].

Fracture network connectivity is best addressed by models that account explicitly for the geometric arrangement as captured by a Discrete Fracture Network (DFN) [Berkowitz and Balberg, 1993; Berkowitz, 2002] and applied to better understand fluid flow patterns [Long and Witherspoon, 1985; Belayneh et al., 2009; Davy et al., 2010; de Dreuzy et al., 2012]. DFN descriptions are used as input for effective permeability calculations based on analytical and flow-based upscaling methods [Long and Witherspoon, 1985; Geiger et al., 2010 and references therein]. Boro et al. [2014] presented effective permeabilities for outcrop-scale DFN models (20x20 m) of the Latemar based on the Oda method [Oda, 1985], an analytical approach that assumes flow to occur only through the fracture
network. We pursue more rigorous flow-based simulations to quantify the role of fractures by using Discrete Fracture and Matrix (DFM) models. DFM account for fluid flow in both fractures and matrix. Simulations that account for flow through the rock matrix at bottlenecks in the fracture network due to dis-joint, two nearby dead-ending fracture may still channelize flow as the flow path continues through the matrix \cite{Matthäi and Belayneh, 2004; Paluszny et al., 2007; Lang et al., 2014}.

While fracture networks impact subsurface fluid flow, seismic data cannot distinguish discrete fracture geometries and borehole data lack information on the spatial arrangement \cite{Bourbiaux et al., 2005}. Hence, the network arrangement is most satisfactory studied from outcrops \cite{Rotevatn et al., 2009; Gillespie et al., 2011}. Outcrops can deliver explicit descriptions of natural fracture network over a length scale of $10^{-1}-10^{4}$ m and with sufficient resolution to examine the connectivity and abutment between fractures \cite{Odling et al., 1999; Rotevatn et al., 2009; Wilson et al., 2011a}.

This study combines the explicit account of fractures in flow simulations with a systematic description of fracture network input models. Descriptions source directly from the Latemar carbonate platform in the Dolomites of northern Italy. The Latemar is one of the most studied platforms for its sedimentary architecture and dolomitization features \cite{Egenhoff et al., 1999; Carmichael et al., 2008} and as reservoir analogue \cite{Boro et al., 2014; Whitaker et al., 2014; Jacquemyn et al., 2015}.

A multi-scale fracture data set is collected, unique in its size range from meter to km scale as we integrated data from airborne LiDAR (Light Detection and Ranging), satellite
imagery and outcrop studies. The data is used for building deterministic fracture network models and for deriving rules as basis for stochastic network models. A series of numerical fluid flow simulations are performed both on deterministic and stochastic network models. The implications of the topology rules are examined by comparing the differences in effective permeability of the stochastic realizations against the deterministic based models and in reference to the critical features as observed in the natural fracture network from the outcrops.

2. Model Context and Design

2.1 Model design considerations

In this study we use a workflow that has similarities to Enge et al. [2007], Rotevatn et al. [2009] and Wilson et al. [2011] in that Discrete Fracture Networks (DFN) are built from digitized fracture geometries and used as input for flow simulations.

Our main focus is on how natural fracture network arrangements impact fluid flow. Statistical fracture properties such as orientation and length distributions may not be treated as independent properties when network connectivity is concerned. For instance, the cross-connectivity of an orthogonal fracture network [e.g. Gross, 1993; Sanderson and Nixon, 2015] is controlled by large parallel trending fractures and smaller 2\textsuperscript{nd} order cross-joints; an arrangement in which length, orientation and topology are interdependent properties. To this end, we capture the geometric arrangement of the fracture network explicitly and fracture size, orientation and spatial distributions are obtained to signify the
network topology. This collective data handling serves our two-fold objective to construct both deterministic DFNs [Gillespie et al., 2011] and feed stochastic simulations based on fracture statistics and distribution functions [Dershowitz and Einstein, 1988; Bonnet et al., 2001; Bour et al., 2002]. Our flow simulations start from deterministic network descriptions and following up with stochastic arrangements to address changes in effective permeability due to changes in orientation sets, their length and cross-connectivity.

We concentrate on the geometric arrangement of the fracture network which is broadly regarded as critical for understanding hydraulic properties such as effective permeability [de Dreuzy et al., 2002; Matthäi and Belayneh, 2004; Lang et al., 2014]. Variations in fracture aperture, including their morphology, asperities and cementation, certainly have a relevant effect on fracture permeability. Models that account for non-parallel fracture walls show in 3D how flow is localized due to local heterogeneities in aperture and fracture intersection streaks and controlled by mechanical closure [Brown, 1987; Dijk et al., 1999; Nick et al., 2011; de Dreuzy et al., 2012; Ebrahimi et al., 2014]. At the field scale, parallel-plate fracture apertures may be assigned as function of length [Bonnet et al., 2001; Ortega et al., 2006], which may still represent the average volumetric flow through a rough-walled fracture reasonably well [Dijk et al., 1999; Nick et al., 2011]. However, aperture is to a large extent controlled by in-situ stresses and are best resolved in refined geomechanical models [Bai et al., 2000; Olsson and Barton, 2001; Paluszny and Matthai, 2010; Lei et al., 2014].
Instead, the excellent exposure of the Latemar platform provides a legitimate basis for geometric descriptions to focus our work on quantifying network arrangement and their effect on flow. Our efforts thus concentrate on the parameterization of complex natural fracture networks as a whole. As coupling of aperture to in-situ stresses falls beyond scope of this study, we chose for uniform apertures to illustrate most clearly the role of natural fracture network geometry at a $10^1$-$10^3$ meter devoid of other relevant factors. Such meso-scale network estimates of the effective permeability can change up to one order magnitude because of aperture heterogeneities alone [Matthäi and Belayneh, 2004; de Dreuzy et al., 2012; Ebrahimi et al., 2014], although the impact of different fracture apertures should be addressed in future research.

The question how fluid flow is influenced by variations in fracture network arrangement can best be addressed by accounting for the fractures and matrix explicitly [Matthäi and Belayneh, 2004; Matthai et al., 2007]. We refer in this paper to DFN as the deterministic or stochastic network input description which is combined with a permeable rock matrix.

The meshing requirements of the discretized fractures challenge large domain sizes and the number of fractures that can be handled. We chose for repeated simulations on 2D representative domains of a few hundred meters across to address the role of network topology and connectivity on the effective permeability, $\kappa_{\text{eff}}$. The effective permeability of fractured porous media can be straight-forwardly computed for each spatial direction using flow-based upscaling [Durlofsky, 1991; Renard and de Marsily, 1997; Matthäi and Belayneh, 2004]. Here we use the Complex System Modelling Platform (CSMP)
[Matthäi et al., 2007] to perform the numerical simulation and flow-based upscaling calculations. The simulation domains in this study are rectangular shaped cuts from our larger structural interpretation domains. The number of fractures that need to be accounted for remains limited to less than 100-300 fractures for domain sizes of $0.1 \cdot 10^6 - 0.3 \cdot 10^6$ m$^2$ and for fracture densities in the range of $\sim0.05-0.15$ m$^{-1}$ (Table 1).

2.2 The Latemar platform architecture

The Latemar carbonate platform crops out in the Dolomites of Northern Italy between the Eggental and Adige valleys (Figure 1a). The Dolomites comprise a series of isolated carbonate platforms (including the Latemar) formed between Late Anisian and Late Ladinian times (Middle Triassic) on top of a Hercynian metamorphic basement and Permian to Lower Triassic sediments [Bosellini et al., 2003].

The platform margin is only exposed in a few areas of the Latemar, forming a narrow discontinuous band that comprises mainly reefal boundstones in which bedding is poorly developed [Egenhoff et al., 1999; Emmerich et al., 2005]. The reefal facies is massive and bedding is poorly developed. The slope of the Latemar platform is characterized by massive and steeply dipping (30-40°) foreslope strata and contain materials derived from the platform margin or interior [Emmerich et al., 2005].

The platform interior of the Latemar is formed by a well-exposed and well-stratified lagoonal facies. The interior consists of a more than 700 m thick cyclic succession with predominant wacke and packstone alternations [Egenhoff et al., 1999; Preto et al., 2011].
The succession is interrupted by three tepee intervals associated with episodes of regional sea-level drop [Egenhoff et al., 1999; Christ et al., 2012] and which act as baffles that vertically subdivide first order flow units [Whitaker et al., 2014]. The isolated platform geometry of the Latemar provides a well-confined case for addressing fluid flow. We focus on the platform interior because of its low matrix permeability, which was originally probably in the range of 1-50 mD based on forward modelling of the platform-wide diagenesis and dolomitisation [Whitaker et al., 2014].

2.3 Structural configuration

The Dolomites were subject to southward thrusting during the Tertiary Alpine collision [Doglioni, 1987] and forms a large scale pop-up (Figure 1b) [Doglioni, 1987; Castellarin et al., 2006]. Despite the deformation, the original geometry the platform and horizontal stratification of its interior are essentially preserved [Preto et al., 2011]. Substantial parts of the platform interior, of the slope and remnants of the intervening margin are well exposed (Figures 2d-g).

The structural fabric of the platform is well depicted from the first-order features that are visible as prominent morphological trends in satellite imagery and LiDAR scans and with smaller sized fractures visible in outcrops. The Latemar is cut by a few NNW-SSE trending strike-slip and normal faults and numerous NNW-SSE striking volcanic and neptunian dikes (Figure 1c) [Bosellini et al., 2003; Preto et al., 2011].
We refer to fractures as all veins, joints or shear fractures, and larger faults or clastic/magmatic dikes, inclusive of all length scales within our observation range of $10^1$ to $10^4$ meter. Given their different nature, the exposed fractures can be open and weathered, closed, brecciated or filled with cement or intruded by igneous or clastic rocks. At the very least, fractures in this loose definition all form structural discontinuity surfaces that appear in a 2D slice as curvilinear geometries which can be digitized from outcrop or satellite imagery or LiDAR data.

Most of the NNW-SSE fractures perceivably formed during early burial in association with the similarly aligned dikes [Boro et al., 2013]. The linear trend and continuity of the dikes suggest that the causative stress field had a regional origin and was not affected by the Predazzo intrusion. Fractures are often subdivided into orientation sets to distinguish successive stages in fracturing evolution. For the purpose of this study, all fractures formed preceding the Alpine exhumation history are relevant in that they form the evolved finite network arrangement as analogue for a still buried platform.

3. Data acquisition and processing

3.1. Methodological outline

Fracture geometries are digitized from outcrop photos that were interpreted in the field, from satellite imagery and from 3D textured surfaces that are derived from airborne LiDAR data. The digitizing of fracture geometries concentrated on interpretation domains ($D_i$) for which a representative collection of fractures were captured.
While the fluid flow simulations will be based on 2D satellite image interpretations, the network properties are first assessed for an as large as possible length scale range and also in 3D, including both the outcrops and LiDAR data sets.

Despite the diversity of data sources, a similar, systematic workflow for acquisition and analyses was followed using the DigiFract software and data model [Hardebol and Bertotti, 2013]. Each fracture belongs to an interpretation domain and contains attribute information such as orientation measurements when taken from the outcrop. Fracture size is intrinsic information to the digitized fracture geometries. The geometry sizes represent height information in case of (sub-)vertical outcrop surfaces and fracture lengths for horizontal outcrop surfaces or for satellite imagery. Fracture geometries digitized from LiDAR provides information of the 3D plane, its exterior limits, height and length as well as orientation [Wilson et al., 2011b]. Extracting trace maps from the 3D LiDAR based interpretations would require extrusion of the planes and intersection with an average topography reference plane. Imagery from satellite have the advantage of minimum topographic distortion and shadowing effects because of their remote bird eye view.

3.2. Small length scale field constraints

The protocol and the main findings of fracture field data acquisition are given in detail by Boro et al. [2013]. That study focused on 5-20 meters sized interpretation domains of the Latemar and forms the lower range in the $10^{-1}$-$10^4$ m length distribution of the data set in this study. Boro et al. [2013] delivered over 1500 fractures from 33 mostly sub-vertical
outcrop surfaces that were located in the interior, margin and slope of the carbonate platform. Fracture geometries and attributes like orientation were collected with the DigiFract acquisition software [Hardebol and Bertotti, 2013]. The dimensions of the outcrop surfaces were large enough to have a representative number of stratigraphic layers and to include a sufficient number of fractures to limit potential sampling biases (see Boro et al. [2013] for further detail).

It should be noted that a ground-based study of individual outcrops, while providing highly detailed and specific observations, carry strong orientation biases [Terzaghi, 1965]. Fractures with orientations highly oblique angle to the outcrop surface have strong distorted intersection lines and can only be satisfactorily captured in case of an adjacent outcrop of semi-perpendicular orientation. The outcrop surfaces have been carefully selected such that all fracture orientations were reasonably well sampled when combining the different outcrop locations and orientations [Boro et al., 2013].

3.3. High resolution satellite image interpretation

Satellite imagery of 1 m² resolution was used to capture large scale lineaments of the Latemar platform (Figure 2). In total over thousand fractures were digitized across the six interpretation domains (Table 1) at a few tens of meters to kilometer length scale. A fracture lineament is understood to be continuous as long as it can be traced as a straight to little curved trace. The ability to denote a lineament in the imagery depends both on its length and width as well as on pixel contrasts that can be enhanced or blurred by
topographic shadowing effects. We experienced that a fracture should at least extend 20 pixels to be recognizable as continues lineament (~20 m for a pixel resolution of 1 m²). In addition, the lower and upper limit of fracture lineament lengths are constrained by the interpretation domain size. For instance, a circular domain size of 5 km diameter would allow to capture fractures from a length of 5 km down to 30 m.

Representative sampling down to the lineament resolution limit is not feasible if only for the immense number of fractures. Tracing fractures with (semi-)automatic procedures [Gillespie et al., 2011; Kudelski et al., 2011] certainly helps future work, but require substantial filtering and editing by a geological trained ‘eye’ especially to determine fracture termination (i.e. abutments) and intersection relationships which are critical for subsequent evaluation of fluid flow or mechanical response.

Instead, we chose for manual digitizing to assure the collection not only of density and length information, but especially of intersection and termination relationships. The digitizing of lineaments for a given interpretation domain is done for a zoom level that gives a representative coverage. As a practical rule, the detection of a lineament is done at the given zoom level and further zoom-in is mainly done to determine the continuation, termination or intersections of lineaments. Segments of lineaments may be divided by gaps and still considered as one continues trace when the gap is small (< 10%) compared to the length of largest adjacent segment and the azimuth direction of the segments is similar.
Figure 2 shows the lineament interpretations for the six domains with a Lat-West domain containing the largest order lineaments and various high detail interpretations for smaller domains (e.g. domain C). Domains B and C provide the most detailed picture of the fracture network arrangement. The top view of the satellite image allows for higher quality observations of (dis-)connectivities and abutment relationships in the horizontal plane and least topographic blurring effects.

The interpretation domains are listed in Table 1 and have aerial sizes in the range of $0.09 \times 10^6$ to $0.7 \times 10^6$ m² compared to $11 \times 10^6$ m² for the ‘whole Latemar’ study area.

3.4. 3D LiDAR data interpretation

An airborne LiDAR data set, acquired in the frame of the (FC)² industry–academic research alliance, provided a high resolution 3D point cloud and large collection of photographs. For methodology see [McCaffrey et al., 2005; Buckley et al., 2008] and for a specific application to the Latemar see Jacquemyn et al. [2015]. Geologic interpretation of such raw point cloud data textured with imagery is computational challenging unless efficiently processed and rendered with software such as LIME [Buckley et al., 2013].

The rugged topography of the Latemar turns airborne LiDAR data in a valuable source for collecting 3D fracture information with both good lateral and vertical coverage. The disadvantage is strong shadowing effects from sharp incisions and ridges. The shadowing in the LiDAR somewhat limits the ability to the precisely capture the topological relationships and is better resolved from high resolution satellite imagery.

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We digitized 3D polylines that formed intersections of fracture planes with the LiDAR surface description of the Latemar topography. Polyline segments that were interpreted to belong to one continuous fracture plane were either merged to one line or labelled as distinctive parts of the same structural feature. Geometric elements were further processed with the Rhinoceros© 3D software and with custom python scripts, involving the extrusion of line traces and surface fitting operations.

Figure 2e displays the 3D interpretation for domain C. The orientation, density and continuity of the first order features in the interpretation can be clearly distinguished. The NNW-SSE trending features typically form first-order steep mountain slope surfaces and sharp trenches, whereas the WSW-ENE features form less pronounced linear undulations on the mountain slopes. Figure 2e and other 3D interpretations show that most fractures extend vertically across the multiple lithostratigraphic intervals of the platform interior [Egenhoff et al., 1999; Preto et al., 2011; Jacquemyn et al., 2015]. As the 3D fracture planes larger than 10-20 m in length are dominantly subvertical in orientation, their structural alignments can be sufficiently captured by 2D trace line interpretations from satellite imagery with only minor geometric distortion. Furthermore, as the 3D interpretations from the LiDAR data set show vertically continuity of most features within the indicated stratigraphic intervals of interest, the interpretations can be condensed into a 2D horizontal plane. In this case, also the network connectivity in 2D is similar to 3D, which differs from stereological rules based on networks made of elliptic discs [Dershowitz and Einstein, 1988; Darcel et al., 2003].
4. Results on fracture network characteristics

4.1. Fracture length scales and sampling biases

One of the most prominent aspects of fractures is their dominant presence, not just at the outcrop scale, but at a wide range of length scales from the microscopic scale up to hundreds of kilometers, continent scale lineaments [Geiser and Sansone, 1981; Davy et al., 2010]. In case that the fracture network can be described by means of scaling laws, analytical descriptions can be adopted for calculating effective permeabilities by means of percolation theory [Berkowitz and Balberg, 1993; Bour and Davy, 1997; Davy et al., 2006]. One main concern is whether fracture properties like size are best described by scale-limited lognormal or exponential distribution laws or by scale-invariant power law distributions with fractal patterns [Baecher, 1983; Dershowitz and Einstein, 1988; Odling et al., 1999; Bonnet et al., 2001; Bour et al., 2002].

However, the multi-scale description of fractures is challenged by the restricted length scale ranges of individual data sets [Nicol et al., 1996; Odling, 1997] and the truncation and censoring biases that are involved [Bonnet et al., 2001]. The fracture geometry descriptions from outcrops, aerial or satellite photos each cover a limited length scale, of 2-3 orders of magnitude at best, and require careful integration to determine appropriate distribution function descriptions [Ouillon et al., 1996]. It should also be noted that fractures that appear as single, continuous traces on an aerial map often consist of a series of smaller fracture traces when observed at outcrop scale [Odling, 1997; Berkowitz, 2002].
The digitized fracture traces in this study constitute at the outcrop scale of both fractured, brecciated and magmatically intruded zones of several meters in width [Boro et al., 2013; Jacquemyn et al., 2015]. The continuity of individual fracture traces and the degree of disjoint relationships between distinct fractures are important features for understanding fluid flow paths.

The size of the interpretation domain ($A_D$) sets a fundamental upper limit and an effective lower limit to the size of fractures that can be captured. In case of a circular 2D interpretation domain, the upper limit to the fracture size that can be contained ($S_{ul}$) equals its diameter as given by $2\sqrt{A_D}/\pi$. The lower limit ($S_{ll}$) to the fracture size that can be captured the digitizing of fracture geometries is fundamentally confined by the image resolution. However, for a circular interpretation domain with a domain size of $10^6$ m$^2$ and image resolution of 0.5 m, the $S_{ll}$ is effectively not defined by the minimum number of pixels times the resolution. Instead, minimum observation length above which ‘all’ fractures can be reasonably digitized across a given interpretation domain. The practical limit of $S_{ll}$ depends on number of fractures that do occur for that given size range (safely assuming that the number of fractures increases for successively smaller lengths). Consequently, one can reasonably digitize manually only a limited length scale range.

For this study, we propose a length scale order of magnitudes ($r$) in the range of 1.0-2.5. For a domain area size of $A_D = 10^6$ m$^2$ this implies an upper limit of $S_{ul} = 1128$ m and for a chosen range of $r = 1.3$ the lower limit is then set to $S_{ll} = S_{ul} \cdot 10^{-r} = 36$ m.
4.2. Fracture size distribution findings

In this study, fracture size distributions ($S$) are fetched from the digitized fractures. Figure 3 display histograms with distributions of respectively fracture heights from sub-vertical outcrops (Figure 3a) and horizontal lengths for two of the six satellite image interpretation domains (Figures 2b and 2c).

The fracture size distribution of lineaments of, for instance, domain C (Figure 3b) has an upper size limit of 223 m and lower size limit of 11 m, following our $r = 1.3$ observational scale range. Its size distribution may appear as log-normal distribution when the acquisition bias of the smaller fractures (10-20 m) are not properly accounted for. Outcrop-based observations provide a better coverage of the smaller length scale and suggests a negative exponential distribution (Figure 3a).

The histogram plot show sampling bias threshold limits that are applied to avoid misrepresentation of frequencies for the smallest fracture sizes. No other straightforward quantification exists of potential under-sampling of the smallest fractures other than minimizing its effect with such an artificial threshold limit [Ortega et al., 2006].

The fracture size frequencies are plotted of individual domains to log-log axes (Figure 3d) and to lin-log axes (Figure 3e). The degree to which these curves follow linear trends determines in how far the distributions follow power law or negative exponential distribution functions. A log-normal distribution for length, apparent from the histogram plots (Figures 3a-c), may be omitted when the reduced frequencies for the smaller...
fractures are artefacts from a truncation bias at the lower threshold limit [Bonnet et al., 2001]. Over a length scale range of 40 to 200-300 m the size distributions for domains A, B, C, E align between the power law distribution functions \( f(S) = 10^3 S^{-2.0} \) and \( f(S) = 10^2 S^{-2.5} \) (Figure 3d). For power law length distributions with exponents between 2-3, the network connectivity is ruled by both small and large fractures [Davy et al., 2006]. However, because of strong frequency fluctuations at higher length scales, a linear trend in log-log space (Figure 3e) beyond two length scale orders cannot be established. Neither a power law nor a negative exponential trend can be confirmed, despite the comprehensive acquisition of our fracture geometries across several length scale orders. Size populations may fail to follow a power-law distribution with a consistent exponent not only because of sampling bias, but may also result from spatial clustering and separation of scale effects [Korvin, 1989; Nicol et al., 1996]. We continue with examining the orientation distribution of the network with particular focus on possible scaling effects.

4.3. Fracture orientation distributions and densities

The occurrence of two prominent orientation sets are depicted from the outcrop data (Figures 4a and 4b) [Boro et al., 2013] and are visible across the different domains of the platform, insensitive to the presence of first-order lithofacies boundaries. The first set has a NNW-SSE strike and the second set a WSW-ENE trending strike. Figures 4a and 4b
show only one of the two sets per outcrop because only fractures oriented with large
intersection angle can be feasibly digitized and measured.

A combined frequency distribution of the two dominant fracture orientations can
therefore not be obtained directly from outcrops (i.e. < 20 m). No notable difference in
fracture densities between the two dominant fracture sets could be observed. The fracture
density as the number of intersections along 1D automated scanlines (P10) [Hardebol
and Bertotti, 2013] vary between values of 1.0-4.0 m\(^{-1}\) [Boro et al., 2013], i.e. spacings
between 0.25-1.0 m

However, domains A-E, having diameters of 150-500 m (Table 1), sample the fracture
population at larger length scales (10-1000 m) compared to the outcrop studies. The
LiDAR interpretation of domain C shows the WSW-ENE trending fractures having a
smaller spacing of 15-20 m compared to the 25-40 m spacing for the NNW-SSE trending
fractures (Figure 2e). This offers a first indication of how the orientation frequency
distribution and fracture spacing are influenced by the observation length scale.

Detailed lineament interpretation of the domains in the satellite image (Figure 2) provide
the basis for Figure 4 in which orientation distributions are plotted for different length
scale subsets. This helps to determine in how far the relative frequencies of the two
dominant orientations vary with the length scale. In combination with fracture abutment
relations, length-scale dependent orientation subsets can be defined that better
parametrizes the inter-connected arrangement of the network.
Four orientation distributions for fracture size subsets are present; one for fractures larger than 500 m, for fractures between 500-200 and 200-100 m in length, and one for fractures smaller than 100 m. Fractures smaller than 50 m are disregarded from this analyses to avoid under-sampling artefacts of the satellite photo based lineament interpretation.

The two dominant fracture orientations can be depicted from Figure 4, although more subsets with changing frequencies can be distinguished at different length scales. Only NNW-SSE trending lineaments occur for fractures with lengths over 500 m. Instead, WSW-ENE striking fractures, absent at lengths over 200 m, become more prominent for lengths between 100-200 m.

The intricate change in orientation distribution at distinct length scale orders is most evident when zooming in below the 200 m (Figures 4a and 4b). The frequencies of NNW-SSE striking fractures are much less prominent and shows a much wider spread with a more oblique NW-SE trending mean (i.e. ~125°, set A1) and below the 50 m also a N-S trend (set A2) appears. Moreover, a new orientation that stands semi-perpendicular to set A appears with a mean strike of ~040° (set C1).

4.5. Fracture hierarchies and termination relationships

The NNW-SSE trending fractures occur at the largest length scale (> 500 m) as through-going discontinuity surfaces that cut systematically over large distance across the Latemar Platform (Figure 4d). The wider spread about the same NNW-SSE strike for fractures at 200-500 m (Figure 4c) and the distinctive oblique sets A1 and A2 at 100-200
m length scale (Figure 4b) form 2nd order branches. This topological relationship has been observed in different interpretation domains. Figures 4b and 4d show examples of 1st order NNW-SSE fractures from which 8-out-of-10 smaller, oblique NW-SE trending fractures branch off.

These smaller fractures have lengths in the order of 100-200 m and occur under multiple prominent strike orientations (Figure 4b). Many of the < 200 m fractures branch off from or abut on the > 500 m fractures, depending on their relative orientation to the 1st order fractures. Detailed observations of smaller sized fractures show a repetition of these topological relationships at smaller length scales.

Based on these observations of length-scale dependent orientation distributions and topology, we propose a fracture hierarchical subdivision. The fracture hierarchical sets are defined firstly by topology and secondly by length scale (Table 2). Figure 5a illustrates how this topological arrangement may be conceived across the Latemar platform. For instance, a 2nd order fracture of ~300 m terminates at or in proximity of a 1st order fracture of 500 m, while a 3rd order fracture with 0° strike terminates against or in proximity of a 2nd order fracture (Figure 5c).

We apply this hierarchy as a non-stringent description in that the degree of abutment has a certain probability. This is because some fractures clearly defer from the proposed topological arrangement. Moreover, the satellite imaging and LiDAR data also leave interpretation room as to whether and within what threshold distance fractures terminate at other fractures.
4.6. Representative size and separation of scales

The Representative Element Area (REA) is the minimum area size for which repeated sampling of a given property averaged over the area across a larger observation domain will be reasonably consistent [Schultz, 1996; Esmaieli et al., 2010]. The REA defines the minimum size that is large enough to contain a sufficient number of heterogeneities to give a representative value of a property such as fracture density. The 2D area density is defined as total sum of fracture lengths divided over the target domain area size (P21).

The REA can be used to determine whether a rock mass can be treated as an equivalent continuum, i.e. whether it is justifiable to use an average effective property value for the fractured rock [Long and Witherspoon, 1985; Esmaieli et al., 2010]. It may work for network properties that are subject to scale separation and typically lead to some break of power law descriptions [Bonnet et al., 2001]. It may apply to P21 as geometric properties when the network exhibits a nested arrangement, while the permeability distribution exhibiting no such REA [Geiger et al., 2010; Lang et al., 2014].

The domain boundaries of our interpretations are restricted to omit areas with insufficient coverage for lineament interpretation. The main concern of the REA assessment is in how far the domains provide a representative sample of network.

We determine REA values from our detailed satellite image interpretations by calculating fracture area density (P21) values for successively larger circular windows; the smallest area size for which any larger area gives similar P21 values is considered the representative element area (Figure 6a). P21 are calculated for 25 circular windows of a...
given diameter that are randomly placed within the interpretation domain boundaries. The P21 calculations are repeated for growing window diameters and plotted to denote the smallest window size at which the P21 remains constant (Figure 6a).

The results of these analyses are plotted and REA sizes and P21 values are estimated (Figure 6; Table 1). For domain B, the P21 starts converging to 0.17 m$^{-1}$ for sampling circles with radius > 100 m (Figure 6b). As this radius is 2/3 of the largest circular sampling window that can fit the domain, the number of independent samples becomes limited. For domain A, the P21 for circular windows of 130 and 140 m radius approach similar values of 0.16 m$^{-1}$ (Table 1), while for Domain C no constant P21 value is obtained for growing window radius of 160 m and 170 m.

For the three domains, a possible REA falls near or beyond the domain size. A REA the order of $\sim 45 \cdot 10^3 - 200 \cdot 10^3$ m$^2$ (r = 120-250 m) would be similarly to Schultz [1996], which is 5–10 times larger than the mean fracture spacing which is 15-30 m spacing for fractures larger than 200 m (Figure 2e). Larger interpretation domains would be needed for sufficient numbers of independent sampling circles to establish a possible REA size; current domain sizes samples about three to four of the largest length scale order of fractures. Furthermore, this REA assessment considers the fracture area density (P21) only and does not address whether a representative area exists for other properties such as connectivity or permeability [Geiger et al., 2010; Lang et al., 2014]. However, our assessment at least indicates that our domain sizes are at or near to the representative element area size for the density of the network.
4.7 Implications for network connectivity and flow

Our above analyses on explicit natural fracture networks from the Latemar carbonate platform confirm how fracture orientation distributions are dependent on length scales. This required us to not only define fracture subsets based on orientation, but to formulate a hierarchical subdivision that combines orientation and length scale and which also takes intersection and abutment relationships into account. We assigned fractures in the network to hierarchical subgroups with corresponding topological rules. A given fracture belonging to a certain length scale order and orientation (Table 2) has a probability to intersect or terminate at a higher order fracture (Figure 5). Our parameterization of the fracture arrangement helps distinguishing relevant spatial variations in the deterministic network arrangements. Discerning how 2\textsuperscript{nd} order 130° and 80° striking fractures occur oblique and orthogonally to the 1\textsuperscript{st} order fracture corridors, the latter having high probability of abutment, is significant for understanding variations in network cross-connectivity across the Latemar platform. The hierarchical subdivision also provides a topological scheme as basis for stochastic network simulations. Both the deterministic and stochastic descriptions serve as input in our subsequent fluid flow simulations for domain sizes of 0.1\cdot10^6 -1.1\cdot10^6 m\textsuperscript{2} that qualify as representative element area.

5. Discrete fracture network modelling

The 2D lineament and 3D surface interpretations (Figure 2) provide the geometric data source for Discrete Fracture Network (DFN) descriptions. For deterministic DFN models
the geometric interpretations are used as direct input [Bourbiaux et al., 2005; Belayneh et al., 2009; Gillespie et al., 2011; Wilson et al., 2011b] and adjusted only to meet subsequent meshing and numerical modelling requirements.

In addition to the deterministic models, stochastic models are generated based on the established network rules. Numerical fluid flow simulations are performed both on deterministic and stochastic network models and differences in flow response due to the network arrangements will be assessed.

5.1. Deterministic network representations

Figure 5d shows the fracture network from the domains B prepared as input for subsequent flow simulations, involving rectangular cropping and meshing that comprises both the matrix and fracture line segments. For the domain B size of 330 by 255 m (Figure 5d-i), triangle cell sizes are between 1-3 m and proper aspect ratios are ascertained, while the nature of the observed network is sufficiently maintained.

At the acquisition stage, fractures were digitized with seamless connections when supported by the observations and in consideration of the resolution and shadowing effects of the imagery. Fractures that are clearly disconnected were given endpoints at a sufficient threshold distance. However, for numerous fractures the (dis-)connectivities cannot be ascertained. Endpoints of digitized fracture traces are repositioned in case the dis-joint distance between adjacent fractures falls within a threshold distance that is twice the element size of the mesh. These adjustments fall within the spatial uncertainty during
digitizing. Two deterministic input model representations are constructed with a respectively higher and lesser degree of network connectivity.

The deterministic descriptions of domain B (Figure 5d) and domain C consider two different representations each. The Deterministic-Well-Connected (DWC) representation has all dis-connected fractures healed which have their endpoints within the threshold distance from an adjacent fracture (Figure 5d-iii). In contrast, the Deterministic-Less-Connected representation (DLC; panels iv) has all end-points of dis-joint fractures cropped to twice the mesh element size. As result, the average amount of connections respectively vary from 154 and 118 between the two deterministic model descriptions and for the $85 \cdot 10^3 \ m^2$ size of domain B. Likewise for domain C where the well-connected representation holds 174 connections against 85 connections for the well-connected versus less-connected representations of a similar sized cropped rectangular area.

This variation falls within the range of uncertainty, taking into account the image resolution, and sampling biases caused by shadowing effects and erosional expression. One of the questions of this study is to determine to what extent a small degree of separation between dis-joint fractures affects the fluid flow behavior.

5.2 Stochastic network modelling approach

The effect of variability in network parameters on connectivity is addressed with some stochastic network modeling using a flexible in-house developed stochastic simulator. The simulation places fractures of a certain hierarchical level, length and orientation in a
2D domain according to certain placement rules until it reaches the prescribed fracture area density (P21). The described hierarchical subsets (Table 2) define the order in which the placement algorithm builds the network. Fractures are placed according to placement and topological rules of the corresponding subset and follow probability density functions (PDFs) for length and orientation. The simulation starts by adding fractures of the highest order to the domain by positioning their centroid according to a Poisson process. This implies placement according to a uniform distribution and independent to the centroids of already placed first order fractures [Baecher, 1983; Dershowitz and Einstein, 1988]. Our stochastic algorithm is limited to 2D, unlike the Enhanced Baecher Model [Dershowitz and Einstein, 1988] that handles polygon shaped disc approximations in 3D. However, we can customize the placement of 2D fractures by better accounting for network topology [Sanderson and Nixon, 2015] and more closely resemble our observed deterministic network descriptions. As placement rule an equal-level non-intersection condition can be applied so that a fracture will not be added to the network if it intersects or runs too close to another fracture of equal hierarchical level.

The lower order fractures are placed following the described nested topological rules such that fractures terminate at, intersect with, or have a dis-joint distance relative the higher order fractures. This stochastic procedure, in answer to our observation of network nesting, imitates the mechanical role of large fractures in stopping the growth of smaller ones similarly to the procedure outlined by Davy et al.[2010, 2013]. The frequency of (dis-)connectivities is set by a probability of abutment (pAbt) value and the distance in
case of disconnectivity is set by probability density functions as described in Table 3. The stochastic placement of new fractures alternates between sets of the same hierarchical level until the fracture density reaches the defined limit as conditioned by calculated values of Table 1. The stochastic simulation proceeds with the placement of fractures at successively lower hierarchical levels.

5.3. Stochastic input models

Figure 7 presents the results of 2D stochastic simulations for domain B. Models Dfn0.1-0.3 as reference models have a non-hierarchical definition of the fracture sets with a Poisson point process for the placement of fracture centroids uniformly in space. The output of two runs of the three models Dfn1.1-1.3 are shown in Figure 7b. They present perfectly connected fracture networks with all lower order fractures of set L3-C1 connecting at both ends to the higher order L2-A1 fractures as the pAbt value is set to 1.0. The variations indicate which aspects of the network remain constant and which vary for different runs with the same stochastic parameters.

For models Dfn2.1-2.3, the L2-A1 fracture set follows the same rules whereas for L3-C1 the probability that a fracture abuts to an higher order fracture is set to 0.7. Fractures in this model have a minimum size of 200 m and extend according to a power law, unless conditioned by abutments. Despite that 30% of the fractures have dis-joint terminations, the network is still well-connected as result of the many large L3-C1 fractures that each intersect with multiple L2-A1 fractures. Instead, for models Dfn3.1-3.3 the L3-C1
fractures have limited lengths set to a power law distribution within a limited range of 20-50 m. Also the $p_{\text{Abt}}$ is set to a lower value of 0.3. As result, the stochastically generated networks of Mdl3 have L3-C1 fractures that typically intersect with only one of the L2-A1 fractures and therefore do not substantially enhance the connectivity of the network. As indication, the number of connections per unit area for selected domains vary between 250-290 intersections for the perfectly connected models $Dfn1.1-1.3$ and 89-115 intersections for the least connected models $Dfn3.1-3.3$. The number of intersections vary between 150-170 for models $Dfn2.1-2.3$ and $Dfn4.1-4.2$.

The stochastic network models $Dfn5.1-5.3$ have a definition that most closely resembles the distinctive orientation sets of the interpreted network. The interpreted fracture network of domain B exhibits two 3$^{\text{rd}}$ order fracture sets that respectively trend NNE-SSW and N-S. Accordingly, the stochastic model definition contains the $L3-C1$ and $L3-C2$ fracture sets (Table 3) with P21 values of 0.035 for the first and 0.015 for the latter and the same abutment relationship and length scale distributions. The extra orientation subset and larger upper limit to the fracture length distribution lead to a network that has 160-165 connections which closely resembles the number of connectivities of the well-connected deterministic description.

Figure 8 presents stochastic model realizations for domain C with three-tier hierarchies. Models $Dfn6.1-6.3$ form a perfectly connected network with $L2-B$ fractures abutting against the higher order $L1-A$ fractures (Figure 8a) and the $L4-A$ fractures against the $L2-B$ fractures. Models $Dfn7.1-7.3$ and $Dfn8.1-8.3$ have the same three fracture sets and
hierarchy with the addition of a sparse distribution of \( L2-A1 \) fractures at the 2\(^{nd} \) level in the hierarchy. The P21 per set are the same for the \( Dfn6 \) and \( Dfn7 \) models and amount to a total of 0.070 m\(^{-1} \), which is similar to domain B (Table 3). The \( Dfn8 \) models have a higher P21 value of 0.15 m\(^{-1} \) with particularly the 2\(^{nd} \) order (\( L2 \)) and 4\(^{th} \) order (\( L4 \)) fractures increase to densities of respectively 0.06 m\(^{-1} \) and 0.07 m\(^{-1} \) (Table 3).

6. Fracture network impact on flow

6.1 Model design

6.1.1. Set-up and scenarios

Using the DFM technique implemented in CSMP [Matthäi et al., 2007] and steady state single phase flow (water flooding) simulations to perform flow-based upscaling [Durlofsky, 1991; Renard and de Marsily, 1997], we compute the effective permeabilities (\( \kappa_{\text{eff}} \)) and quantify the respective contributions of both the rock matrix and discrete fracture network in the x and y direction of the model geometry [Belayneh et al., 2009; Matthäi and Nick, 2009; Lang et al., 2014].

Table 4 lists the different simulation configurations. Simulation \( Stoflow01 \) refers to the first out of three fluid flow simulations of model 0 that is based on the stochastic network model \( Dfn0.1 \). Reference to \( MdlX \) addresses the three simulations \( StoflowX.1-X.3 \) based on the stochastic network descriptions \( DfnX.1-DfnX.3 \). While stochastic modelling is easily repeated to validate the consistent nature of network topology, the modelling flow
is limited to three simulation runs because of the meshing of both the fractures and matrix is complex. As the results will show, the flow simulations render similar effective permeability despite the fact that our fracture area density (P21) appear close to the percolation threshold. The repeatability of flow behavior for our stochastic network arrangements can be understood from the fact that the topological rules make the degree of fracture interconnections dependent on the topology, which differs from most percolation studies connectivity between sets is a random process [Sanderson and Nixon, 2015]. The termination relations between fractures also confirmed by the observation that fracture length depends on the orientation (Figure 4). We note that percolation theory is applied to systems with no such correlation [Davy et al., 2006].

All simulation runs have the same fracture aperture of 1.0·10^{-3} m to facilitate the comparison and address the role of different network arrangements. The fracture permeability of 8.3·10^{-8} m^2 is calculated using the cubic law [Witherspoon et al., 1980] and is assigned to the discretized fractures.

6.1.2. Matrix permeability

Platform-wide mean permeabilities of the three main stratigraphic intervals of the platform margin and interior have been estimated with forward coupled modelling of the sedimentary and diagenetic evolution [Whitaker et al., 2014]. Outcrop observations confirm the first order trends of the modelled platform architecture [Egenhoff et al., 1999; Christ et al., 2012]; sufficient for the model to deliver relevant predictions on diagenetic
overprint and matrix permeability evolution. The simulation findings of Whitaker et al. [2014] outline matrix permeabilities of the margin and interior in the order of 3-30 mD for a low-permeable end-member and 0.6-300 D for an high-permeable end-member scenario. For this study we build from the low matrix permeable scenario and address the role of the fracture arrangement on the platform effective permeability for matrix permeability values representative of the margin and interior across the platform. The simulations are based on the same matrix permeability of $1.0 \cdot 10^{-14} \text{ m}^2$ (10 mD).

6.2. Flow simulation results

6.2.1. Effective permeabilities of deterministic models

Table 4 gives the $\kappa_{\text{eff}}$ values calculated from the DFM simulations for the respective DFN models. As reference model, flow is calculated based on the stochastic $Dfn0$ reference model that honors the fracture densities and orientations but omits the complexities of deterministic network such as fracture network hierarchy. The consequent effective permeabilities ($\kappa_{\text{eff}}$) for the reference models $Stoflow0.1-0.3$ are $1.57 \pm 0.34 \cdot 10^{-12} \text{ m}^2$ and $1.73 \pm 0.25 \cdot 10^{-12} \text{ m}^2$ in the x and y direction (as average of three DFN models). Models $DetflowB_DWC$ and $_DLC$ present the flow response of the deterministic fracture network from a well and less connected interpretation of domain B. Their $\kappa_{\text{eff}}$ are $1.05 \cdot 10^{-12} \text{ m}^2$ in x and $2.14 \cdot 10^{-12} \text{ m}^2$ in y direction for the first and $4.47 \cdot 10^{-13}$ by $9.74 \cdot 10^{-13}$ for the latter, less connected deterministic model. Compared to the reference models, the $\kappa_{\text{eff}}$ of the $DetflowB_DLC$ is 1.5-3 times smaller, whereas the well-connected $DetflowB_DWC$
has a permeability that is 1.5 times smaller in the x direction and 1.3 times larger in the y direction. The difference in $\kappa_{\text{eff}}$ between the two deterministic interpretations give an indication of the variation in effective permeability due to the uncertainty involved in the digitizing of (dis-)joint fracture endpoints of a network. Further comparison can be made from Figure 9 with $\kappa_{\text{eff}}$ in x and y direction plotted for all simulation runs of this study.

The well-connected deterministic network model of domain C (i.e. DetflowC_DWC) results in a similar effective permeability compared to DetflowB_DWC. DetflowC_DWC has a two times higher permeability in the y direction because of longer fractures that cut to greater extent through the entire domain (Figure 9). In contrast, the DetflowC_DLC (less-connected equivalent) exhibits a much stronger drop in effective permeability in the x direction (with a factor ten relative to _DWC compared to a drop with factor two for DetflowB_DLC). This is due to the interconnected network which is much less dense at the eastern edge of domain C and pathways are more easily broken.

Figure 10 shows pressure field distributions for the two deterministic representations of domain C which further underlines the flow response due to reduction in connected fracture pathways between the DWC and DLC representations. The effect is particularly well depicted in Figure 10d where pressured gradients are plotted in the x direction along three indicated transects, both for the well-connect model (Figure 10a) and the less-connected model (Figure 10e). The three pressure gradients of the well-connected model exhibit an almost linear decline in pressure between the left and right boundaries that are conditioned by 3 and 0 MPa. Instead, the pressure gradients of the less-connected model
exhibit strong variations along the transects, particularly toward the right side of the domain where the reduction in connected network is strongest. This response in pressure gradient is much less prominent in the y direction because of longer through going fractures that control the flow paths (Figure 10g).

6.2.2. Effective permeabilities of stochastic models

The models Stoflow1.1-1.3 of domain B, that have the stochastic hierarchical networks Dfn1.1-1.3 as input, give a $\kappa_{\text{eff}}$ of $2.52\pm0.27\cdot10^{-12}$ m$^2$ and $2.17\pm0.09\cdot10^{-12}$ m$^2$ in the x and y direction (i.e. 1.3-1.6 times that of the reference model) (Table 4). The models Stoflow2.1-2.3 give a $\kappa_{\text{eff}}$ in the range of $1.98\pm0.10\cdot10^{-12}$ m$^2$ and $1.86\pm0.31\cdot10^{-12}$ m$^2$ (i.e. 1.1-1.3 times that of the reference model). The $\kappa_{\text{eff}}$ of models Stoflow3.1-3.3 are substantially smaller with $0.047\pm0.003\cdot10^{-12}$ m$^2$ in the x direction and of $0.63\pm0.33\cdot10^{-12}$ m$^2$ in the y direction; i.e. the permeability in x direction is reduced by 0.03 times that of the reference model while in the y direction the reduction is only 0.4 times.

Comparison shows that the $\kappa_{\text{eff}}$ values in the x direction of Stoflow3.1-3.3 have the lowest values of all simulations and are only five times the matrix permeability (Figure 9). The reason is that only 30% of the $L3$-$C1$ fractures are connected to the higher order $L2$-$A1$ fractures. Moreover, the $L3$-$C1$ fractures have short lengths in the Dfn3 stochastic network description such that no L3 fractures bridge between two $L2$-$A1$ fractures. In the y direction, the larger fractures are much less sensitivity to cross-connectivity to provide
flow pathways. Therefore $\kappa_{\text{eff}}$ in $y$ direction for all models are around and above $1.0 \cdot 10^{-12}$ m$^2$.

The Stoflow4.1-4.3 models, based on the network models that exhibit larger variance in the orientation distributions of the subsets, result in $\kappa_{\text{eff}}$ of $0.44 \pm 0.04 \cdot 10^{-12}$ m$^2$ and $0.99 \pm 0.60 \cdot 10^{-12}$ m$^2$. These values fall well in between the Stoflow1 and Stoflow3 models; the first with a perfectly connected network due to the $p_{\text{Abt}}$ of 1.0 for L3 fracture and the latter with a $p_{\text{Abt}}$ of 0.3 and restricted upper limit of 50 m in length distribution. The $\kappa_{\text{eff}}$ of models Stoflow5.1-5.3 are $1.66 \pm 0.17 \cdot 10^{-12}$ m$^2$ in $x$ direction and $2.07 \pm 0.31 \cdot 10^{-12}$ m$^2$ in the $y$ direction. Its DFN input model is fairly similar to Stoflow2 with both a $p_{\text{Abt}}$ of 0.7, except that Stoflow5 has an additional L3 fracture set $L3-C1b$. Our results show that higher variability in orientation leads to no increase in $\kappa_{\text{eff}}$. This is due to our stochastic model setup in which fractures of the same orientation set are not to intersect. In that case, increase in orientation variance within one set leads to no in higher network connectivity.

The impact of fracture network arrangements is further addressed with flow simulations for the fracture network models of domain C. The $\kappa_{\text{eff}}$ of the nine stochastic realizations for domain C are plotted in Figure 9, three for each of three different network definitions of Stoflow6, Stoflow7 and Stoflow8. The calculated $\kappa_{\text{eff}}$ values of Stoflow7.1-7.3 are 1.5 times smaller than for Stoflow6. Stoflow7 has a network arrangement with 30% chance of dis-connected fracture endpoints for the lower order fracture sets, compared to Stoflow6 that has a perfectly connected network Table 3). The three realizations of Stoflow8 have
the same network definition as Stoflow7, except of a two-fold increase of the P21 (Figure 9). This increase in P21 leads to a two-fold increase in $\kappa_{\text{eff}}$ and compensates for the 30% chance of dis-connected fractures. However, the two-fold increase in P21 Stoflow8 results only in a slightly larger $\kappa_{\text{eff}}$ compared with the perfectly connected network Stoflow6. This shows the significance of network inter-connections for the flow response.

6.2.3. Directional dependence

The sensitivity of the stochastic model scenarios on resulting permeability in x direction is stronger than in y direction. This is because connectivity across the model domain is easily achieved in the y direction because of the alignment of the L2 fractures and their corresponding length distribution. Since the longer L2 fractures more easily achieve cross-connectivity across the domain, $\kappa_{\text{eff}}$ in y direction vary substantially less between different DFN input models. In the x direction, cross-connectivity is much more dependent on the length distribution and placement and abutment rules. They signify the flow behavior that preferentially occurs through the fracture network and through the matrix where network connectivity is limited.

For better understanding the effective permeability values, we look in detail into the effect of fracture inter-connectivity on the pressure field across the model domains for three of our stochastic models (Figure 12). Areas with large changes in pressure gradient mark places where little fracture network connectivity exists. Areas with a well-connected network of fractures exhibit lower, more constant pressure gradients.
This effect of network connectivity on the pressure field and gradient is clearly illustrated with the well-connected and less-connected deterministic network descriptions of domain C (Figure 10). The less-connected model DetflowC_DLC shows along the horizontal transects 3, 4 and 8 stark steps in the pressure gradient (Figures 10d and 10g). For comparison, the pressure distribution through the well-connected DetflowC_DWC exhibits a much more continuous decline as pressure steps along transects 1, 2, 5 and 6 are minor.

This response of the pressure field is further illustrated by flow simulation based on the stochastic network models. The two fracture sets in Stoflow3.1 are fully disconnected. The pressure field in the x direction exhibits stark drops along transects 1 and 2 (Figure 12b) at places where flow through the matrix occurs to bridge between disconnected fractures. Both Stoflow1.1 and Stoflow5.1 exhibit well-connected fracture networks despite their strong difference in network topology. As a result, the pressure declines along the transects show a smaller degree of undulations. However, the pressure distribution of Stoflow5.1 (Figure 12c) spatially in 2D is much more irregular compared to the almost linear pressure gradient of Stoflow1.1 (Figure 12a).

7. Discussion

7.1 Network hierarchy rules and stochastic DFN models

Fracture hierarchies have often been described at outcrop scale as a two-tier hierarchy in terms of 1st order systematic joints and 2nd order cross-joints [Gross, 1993; Odling et al.,
True multi-scale hierarchical network descriptions have been documented in a few studies [Ouillon et al., 1996], requiring an extensive geometry collection across three or more length-scale orders. New is our description of a fracture network hierarchy with three to four levels of nesting that we were able to obtain from a multi-scale satellite image and LiDAR interpretation down to the outcrop scale. It is shown that the arrangement of master fractures that abut semi-perpendicular fractures in a two-tier system at one length scale [Gross and Eyal, 2007; Ghosh and Mitra, 2009] repeats itself for the lower hierarchical levels.

Hierarchical placement rules were formulated in this study to populate the stochastic DFN models. Our stochastic algorithm was customized to take account of the observed nested network topology. Hence the generated networks preserve network features from the deterministic description that concern the connectivity. The results show that network arrangements exhibit fairly similar $\kappa_{\text{eff}}$ values for the multiple DFN realizations that have the same stochastic parameters and are notably different from models based on different stochastic parameters (Figure 9). This confirms that we parameterized relevant network discriminators that impact the effective permeability of fractured reservoirs.

The degree to which fracture hierarchy applies determines in how far fracture subsets should be defined not just as orientation sets, but according to an integrated subdivision based on orientation, length scale orders and topology. The length scale based fracture sets act as distinctive hierarchical levels at which lower order fractures abut. Such hierarchical subdivision may also provide a geometric basis to separate length scales.
[Bourbiaux et al., 2005; Geiger et al., 2010; Lang et al., 2014], features belong to hierarchical orders below a given threshold can be upscaled while higher order features remain explicit in the model representation.

7.2 Network connectivity and flow response

Our simulations of flow through both matrix and network of discrete fractures give insight in the degree to which a path of connected fractures can channelize flow and how the matrix contributes where fractures have dead-endings. In addition, notable directional dependency in effective permeability occur from variations in network topological as shown by the simulation results for the different domains across the platform (Table 4). Domain B in the western part of the Latemar platform has a topology of L3-C1 fractures that abut at the higher order L2-A1 fractures (Figure 5) and which consequently impact the $\kappa_{\text{eff}}$ in the $x$ and $y$ direction of our flow simulations. Comparison of StoFlow2 with StoFlow4, the first having a 0.7 probability of L3-C1 fracture terminations being connected to L2-A1 and the latter having a 0.3 probability of connectivity results in strong increase in directional dependency of the permeability as $\kappa_x / \kappa_y$ ratios drop from 1.0-1.2 down to between 0.1-0.2.

Figure 11 shows flow rates in effect to the pressure field and fracture network arrangement for the deterministic models of Domain C. High flow rates of $1.0 \cdot 10^{-4}$-$9.0 \cdot 10^{-4}$ m/s occur along paths of connected fractures in both the $x$ and $y$ direction. Comparison between Figures 11a and 11c show in the $x$ direction the effect of the well-
connected *DetflowC_DWC network* and the less-connected *DetflowC_DLC* with high flow rates along multiple pathways for the first and concentrated flow through a pathway of limited connected fractures for the latter. In the y direction the difference between the two deterministic arrangements is less significant (Figures 11b and 11d) as flow occurs mainly through a few large fractures cut through the entire domain. Flow in the y direction is therefore less dependent on an intricate network of smaller fractures. Flow through the matrix becomes significant for bottleneck regions in the model where cross-connectivity in the network is limited. Fracture to matrix flow ratios ($q_f/q_m$) [Matthäi and Nick, 2009] averaged over the domain are 141 in x and 268 in y direction for the well-connected *DetflowC_DWC* model and are 9.7 and 14.3 for the less connected *DetflowC_DLC*. The matrix contribution is less than 1% for the well-connected representation (DWC) and a similar result could probably be reached with a hydraulic channel network including artificial channels for flow through the matrix between dead-ending fractures [Ebrahimi et al., 2014]. However, $q_f/q_m$ in the less-connected representation (DLC) reach ratios below 10 (> 10% matrix contribution) for the eastern portion of the domain, where despite a well-spaced series of E-W fractures, no cross-connectivity is achieved (Figure 11c).

The effect on permeability for differences in connectivity between the DWC and DLC networks does signify the degree to which flow channelize into the matrix at a disconnected fracture tip and over what distance before reaching an adjacent well oriented fracture. Such matrix channels between disconnected fractures [Ebrahimi et al.,
2014] strongly reduce the effective permeability particularly in the direction of the disconnected fracture set (i.e. roughly aligning to the x-axis). The denoted topological differences between the domains (Figure 5) stem from nuanced differences in fracture network development under influence of causative stress field and mechanical properties across the platform. In addition, the fact that two deterministic DFN descriptions are presented for each of the domains B and C, gives an indication of the uncertainty involved in the capturing of network geometry from satellite imagery. The contrasts between LWC and LDC in the eastern part of the domain C results from acquisition bias due to topographic shadowing effects. Figure 13 highlights conceptually the implicated flow response of the two deterministic representations in network geometry arrangement for domain C. The first is the impact of regions with fracture densities deviating strongly from the domain average as illustrated by the reduced P21 in zone 1 of Figure 13a. A pressure gradient along the horizontal axis will exhibit a break from the regional trend with a local smaller pressure gradient due to the smaller effective permeability of zone 1.

The weight of the $\kappa_{\text{eff}}$ to the total $\kappa_{\text{eff}}$ of the domain can for this simple configuration be approximated with the harmonic average of the respective $K_{\text{eff}}$ per zone. The range of P21 reduced values from 0.1 to 0.01 m$^{-1}$ highlight the effect that a zone of less connected fractures has as the $\kappa_{\text{eff}}$ reduces from $6.8 \cdot 10^{-12}$ m$^2$ down to $4.6 \cdot 10^{-12}$ m$^2$ (Figure 13c). The second feature of deviating network arrangement is a localized zone of width (W) where fractures are absent. The reduction in $\kappa_{\text{eff}}$ from even a narrow 1 m zone vacant of fractures is already substantial with a 25-fold drop relative to the reference model (Figure...
13c). The low $\kappa_{\text{eff}}$ reduction for the deterministic network description of domain C is due to zone in the interpretation that holds no fractures and which might be the result of an acquisition biases from shadowing effects in the satellite image interpretations. This signifies the necessity for ground truthing in particular the abutment relationships of a fracture network.

7.3 Other factors

Further variations and directional dependency in effective permeability are expected when fracture aperture varies as function of stress conditions. Aperture is kept constant at 1 mm between different fracture sets for most simulations in this study. Only for models $Stoflow1\beta$ and $Stoflow2\beta$ are L3 fractures assigned with 0.2 apertures. Based on the cubic law [Witherspoon et al., 1980], the fracture permeability drops 25-fold from $8.3 \cdot 10^{-8}$ m$^2$ to $3.3 \cdot 10^{-9}$. Comparing the $\kappa_{\text{eff}}$ of respectively $Stoflow1\beta$ with $Stoflow1$ and of $Stoflow2\beta$ with $Stoflow2$ show how the permeability in the x direction drops with a factor 30-40. Variations must be due to nuanced differences between the network arrangements. The reduction in the y direction is only 2-5 times (Table 4; Figure 9). The strong reduction in the x direction is the outcome the alignment of the lower aperture L3 fractures (Figure 5d). The 5-fold reduction fracture aperture displays the same order of magnitude impact on the $\kappa_{\text{eff}}$ in x direction as $Stoflow3$ that has a low probability of L3 fractures intersecting L2 fractures.
Finally, we expect stronger variations in effective permeability when a wider range in fracture length scale can be accounted for in the flow simulations. Spatial variation in fracture density of the L2 and L3 length scales between the different domains are much smaller and still result in notable changes in the effective permeability with a factor 50. Outcrop observations indicate strong variations in L4 order fractures densities between platform slope and interior.

Comparison with effective permeability calculations by Boro et al. (2014) show how outcrop-scale fractures across the Latemar (< 50 m; Figure 4c) can introduce additional variability and directional dependence in permeability. Their study on 20x20 m domains constrained by outcrops display fracture permeabilities over $1 \times 10^{-8}$ m$^2$ in the ENE–WSW direction and $> 3 \times 10^{-10}$ m$^2$ in the NNW–SSE direction. These fracture permeability numbers are from their base-case scenario with 5.0 mm opening for ENE-WSW and 0.1 mm for NNW-SSE aligned fractures and fracture volumetric density value of 0.15 m$^{-1}$ (i.e., total sum of fracture area over 3D volume; P32). Hence substantially more spatial variation in effective permeability can be expected when these abundant smaller length scale fractures can be accounted for and variations in aperture values are introduced.

8. Conclusions

This study demonstrates the value of refined multi-scale fracture data acquisition of an exposed carbonate platform to quantify the role of fracture networks for large-scale effective flow behavior. Fracture network descriptions where obtained more
systematically and with unification of data from outcrop, LiDAR and satellite imagery to advance the building of deterministic DFN models and the establishment of alternative stochastic rules and parameters.

NNW-SSE trending fractures dominate in the Latemar at the largest length scale (> 500 m) and form through-going, platform-scale discontinuities. Fractures smaller than 50 m. exhibit both NNW-SSE and WSW-ENE strikes. Three intermediate length scale orders are distinguished and exhibit the two dominant fracture orientations and additional sets with variable frequencies. The WSW-ENE striking set, that is dominant at small length-scales, is absent for length scales above 200 m.

The degree to which a fracture network follows topological rules like multi-tier hierarchies, abutment and intersection relationships have a distinctive bearing on the network connectivity. As indication, the number of connections per unit area for selected domains across the Latemar is in the range of $1.0 \cdot 10^{-3}$-$3.0 \cdot 10^{-3}$ m$^{-2}$.

The role of network hierarchies over several length scale orders on the effective fracture network properties is clearly demonstrated by this study. Their role can only be pursued when data sets both provide high spatial resolution and large spatial extend. Stochastic simulations can account for multi-tier network arrangements by revising the placement of centroids according to a Poisson process to the placement of endpoints according to an offset distribution relative to higher order fractures. Hence the topological rules of the deterministic networks are reproduced and their effect on the effective permeability can be accounted for.
The fracture characterization and fluid flow simulations, based on single phase steady-state simulations that account for both fractures and matrix, show that the network can enhance the matrix permeability of $1.0 \cdot 10^{-14}$ m$^2$ with 50-400 times. This order of magnitude in increase is based on a reasonable average and constant aperture of $1.0 \cdot 10^{-3}$ m and with a fracture area density of $0.07$ m$^{-1}$ for fractures larger than 50 m. These variations in network response occur for arrangements with same fracture orientations sets and fracture densities. Also without a well-connected network, fractures still enhance the flow significantly when combined with short distance flow through the matrix between adjacent fractures.

9. Acknowledgements

We are grateful for the stimulating and productive collaborative research environment fostered by ExxonMobil’s industry–academic research alliance ‘Fundamental Controls on Flow in Carbonates’, and specifically for the inspirational leadership of S. Agar. ExxonMobil Development Company also supported the development of our DigiFract fracture acquisition and processing software. S. Geiger thanks Foundation CMG for supporting his Chair. Special thanks to Ade Shita Ekadini Nasution for her exhaustive descriptions of fractures from LiDAR and Kevin Bisdom for his contribution to DigiFract development. The data for this paper are available at the 3TU.Datacenter repository under doi:10.4121/uuid:e59c6ad1-db1e-45ce-8630-c83083bf17b3.
References


**Figure Captions**

Fig. 1 Geological context of the Latemar carbonate platform, outlining the regional context and main structural and sedimentological features.

(a) Overview map of the Dolomites in the SE Alps, south of the Insubric line.
(b) Cross-section based on Doglioni [1987] showing the Dolomites as pop-up above the SSE-vergent Valsugana thrust and bound to the NNW by the transpressive Pusteria Fault (PF) as part of the Insubric line.
(c) Map of the Latemar study area, a 3 by 5 km domain encompassing a well-exposed isolated carbonate platform interior fringed by margin and slope facies and cut by several faults and dikes.
(d) North facing view of the Latemar platform at Cimon Latemar showing the horizontal stratification of the platform interior and large-scale fractures.
(e) Platform margin (reef) southeast of Reiterjochspitze intersected by dikes.
(f) View on the Cresta de Peniola with steep dipping strata of the slope facies.
(g) Basin sediments in the west of Cima di Valsorda and facing the slope facies.

Fig. 2 (a) Aerial overview with of the Latemar carbonate platform outlining the different target domains for which structural lineament interpretations are shown.
(b) Fracture networks of domains A and B in the westernmost slope facies of the Latemar.
(c) Fracture network of domain C at the NW edge of the platform interior transitioning northwestward to margin and slope facies.
(d) Fracture network of domain E located in the northeastern slope facies.
(e) LiDAR views of domain C with the interpretations of the fracture planes in 3D, which are in fact brecciated, magmatically intruded zones of a few m width [Preto et al., 2011; Jacqyemyn et al., 2015].
Fig. 3 Size distributions from fracture geometries, 2D satellite imagery and 3D LiDAR data. (a) Height distributions from 33 outcrops surfaces across the platform and of variable orientation (b) Histogram of length distribution from interpretation of 2D domain C (c) Histogram of length distribution from interpretation of 2D domain E. (d) Frequency distribution over length along log-log axes (e) Frequency distribution over length along lin-log axes.

Fig. 4 Rose diagrams with fracture orientation distributions subdivided for five length-scale subsets from less than 10 m for outcrop locations in the Latemar (a-b) and from > 50 m to over 500 m and for different satellite image interpretation domains (c-f).

Fig. 5 Hierarchical organization of the Latemar fracture network.
(a) Schematic map of the different fracture length scale orders and orientation subsets that make up the fracture network of the Latemar platform.
(b) Rose diagram with orientation distributions subdivided in length scale orders and orientation subsets.
(c) Topological arrangement of the hierarchical and abutment relationships between fractures of different length scale orders and orientation subsets.
(d) Deterministic DFN models of domain B with in panel (i) the discretized fracture and matrix model representation. Two discrete network representations are shown concerning two degrees of (dis-)connectivity: (ii) healed dis-connections with gaps than smaller 2 m and (iii) extended gaps of 0.5-2 m to a minimum of 2 m.

Fig. 6 Representative element area calculations of the domains. (a) outline of the method with a moving and growing scan-window with which P21 values are calculated (b)
graph with REA inference for domain B (c) idem for domain C. (d) idem for domain E.

Fig. 7 Stochastic 2D discrete fracture network populations for domain B. The five stochastic models follow hierarchical and topological arrangements as described in Table 3. Two stochastic realizations are shown for each model.

Fig. 8 Idem as Fig. 7 for domain C.

Fig. 9 Effective permeability ($\kappa_{\text{eff}}$) plots (normalized over the matrix permeability) for all simulations. $\kappa_{\text{eff}}$ values are plotted against the number of fracture intersections of the fracture network per unit area. In x direction (b) in y direction.

Fig. 10 Flow simulation results for the deterministic models of domain C. The top panels show the pressure fields for the well-connected representation (DWC) and the bottom panels show the less connected representations (DLC).
(a) Pressure field in x direction for model detflowC_DWC.
(b) Pressure field in y direction for model detflowC_DWC.
(c) Pressure gradients for transects 1-4 in y direction.
(d) Pressure gradients for transects 5-6 in x direction.
(e) Pressure field in x direction for model detflowC_DLC.
(f) Pressure field in y direction for model detflowC_DLC.
(g) Pressure gradients for transects 7-8 in x direction.

Fig. 11 Flow rates for the detflowC_DWC and detflowC_DLC models. Flow field or the well-connected detflowC_DWC model under (a) and (b) due to pressure gradient in x and y direction respectively. Similarly, flow rates for the less-connected detflowC_DLC model under (c) and (d).
Fig. 12 Pressure field in x direction for flow simulations based on three stochastic network models of domain B.
(a) Pressure field for Stoflow 11. (b) Pressure field for Stoflow 31
(c) Pressure field for Stoflow 51. (d) Histograms of fracture to matrix flux ratios.
(e) Pressure gradient for transects in x direction across model domain.

Fig. 13 A conceptual model addressing the impact of local deviations in fracture network arrangement on the effective permeability. (a) Fracture network with (i) strong local decrease in P21 in zone 1 and (ii) a band of disconnected fractures in zone 4. (b) Pressure gradients for different fracture network perturbations. (c) Effective permeability over matrix permeability for different conceptual network perturbation scenarios.

Table 1: Target domain with area sizes and upper and lower length scale limits and area and volumetric fracture density estimates.

Table 2. Fracture hierarchical subsets.

Table 3. Model definitions for Stochastic DFN simulations

Table 4. Flow simulation models. The input DFN models as listed, apertures of $1.0 \cdot 10^{-3}$ m for all fracture sets in all models (except Stoflow1β and Stoflow2β with $0.2 \cdot 10^{-5}$ m), and a matrix permeability of $1.00 \cdot 10^{-14}$ m$^2$ (~10 mD).
a) Dolomites 

b) NW SE fractures

c) Cresta de Peniola NE SW ~30m steep foreslope slope facies platform margin with massive boundstones ~15m dike NW SE ~15m NW SE basinal deposits

d) W Cimon Latemar E

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western side view
WSW-ENE
spacing 15-20 m

northern side view
WSW-ENE
NNW-SSE
25-40 m
NNW-SSE
3D LiDAR of Domain C

domain A
domain B
domain C
domain Lat-West
domain E
domain F

'whole Latemar'

oblique angle
branching
topology

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outcrop scale

(a) outcrops in platform interior
n = 1081

(b) domain C
n = 197

(c) domain E
n = 511

(d) (e)
101 m 102
between 200-500 m.
102 m
between 100-200 m.
103 m
between 50-100 m.
20-50 m.

(f)(c)

than 20 m.

(a)

1km
station
faults

(b)

1km

plat. interior
slope
margin
volcanic

sampling domains:

A
B
C
E
F
‘Lat-West’
i) network discretization
ii) well-connected realization
iii) less-connected realization
(a) Dfn0 (ref) (b) Dfn1 (c) Dfn2 (d) Dfn3

- L1-A fractures
- L3-C1 fractures

(e) Dfn4 (f) Dfn5

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(a) Dfn6
(b) Dfn7
(c) Dfn8

L1-A fractures
L2-A1 fractures
L2-B fractures
L4-A fractures

domain C
relative pressure
flow from ΔP in x-dir

well-connect (DWC)

10^{-4} 10^{-6} 10^{-8}

flow rates (m·s^{-1})

10^{-4} 10^{-6} 10^{-8}

flow from ΔP in y-dir

less-connect (DLC)

10^{-4} 10^{-6} 10^{-8}

flow rates (m·s^{-1})

10^{-4} 10^{-6} 10^{-8}
\[ \frac{1}{P_{21\text{red}}} = 0.05 \]
\[ \frac{1}{P_{21\text{red}}} = 0.03 \]
\[ \frac{1}{P_{21\text{red}}} = 0.01 \]

(a) 

(b) 

(c) 

\[ \kappa_{\text{eff}}_1, \kappa_{\text{eff}}_2, \kappa_{\text{eff}}_3, \kappa_{\text{eff}}_4, \kappa_{\text{eff}}_5, \kappa_{\text{eff}}_6, \kappa_{\text{eff\_vacant}} \]

with vacant zone \( W = 6 \)

reference model

with \( P_{21\text{red}} \)

with vacant zone \( W = 6 \)

\( P (\text{MPa}) \)

\( \kappa \)

\( \frac{1}{\kappa_{\text{matrix}}} \)

\( 10^{-12} \)

\( 10^{-11} \)

\( 10^{-10} \)

\( 10^{-9} \)

\( 10^{-8} \)

\( 10^{-7} \)

\( 10^{-6} \)

\( 10^{-5} \)

\( 10^{-4} \)

\( 10^{-3} \)

\( 10^{-2} \)

\( 10^{-1} \)

\( 10 \)

\( 100 \)

\( 1000 \)

change \( P_{21\text{red}} \)

change \( W \)

- \( P_{21\text{red}} = \text{ref} = 0.05 \)
- \( P_{21\text{red}} = 0.03 \)
- \( P_{21\text{red}} = 0.01 \)
- \( W = 1 \text{ m} \)
- \( W = 3 \text{ m} \)
- \( W = 6 \text{ m} \)
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placement types:
1: random placement of fracture centroid with uniform distribution in x,y direction (Poisson)
2: random placement with non equal level intersection constraint between same order fractures
3: hierarchical with the new fracture end points placed relative to higher order fracture

other parameters:
Probability of Abutment (pAbt) values: 1.0 ; 0.7; 0.3
Distance of Disconnectivity (dDisConn):
Length scale Power law Distribution (LPD):
in reference model: LPD of L3 fractures is 20-200m
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