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Identification and analysis of the meandering of a fin-tip vortex using Proper Orthogonal Decomposition (POD)

Yunpeng Xue¹,²,³, Chetan Kumar¹, Soon-Kong Lee¹, Matteo Giacobello¹, and Peter Manovski¹

¹ Defence Science & Technology Group, Victoria 3207, Australia
² Australian Maritime College, University of Tasmania, Tasmania 7248, Australia
³ School of Energy, Geoscience, Infrastructure and Society, Heriot-Watt University, Edinburgh, United Kingdom

Abstract

The meandering of a vortex exists in a broad range of engineering applications and can lead to flow instability and other undesirable characteristics. Compared to a static vortex, measurement of a meandering vortex can result in a ‘smeared’ mean-flow field and increased levels of turbulence at the centre of the vortex. A case study was performed here on the meandering nature of a fin-tip vortex generated by a manoeuvring submarine. From stereoscopic particle image velocimetry (SPIV) measurements, it is possible to remove the meandering by shifting each instantaneous velocity field so as to produce a common centre for the vortex. In this paper, a snapshot Proper Orthogonal Decomposition (POD) technique is used to capture the dominant large-scale coherent structures (from inspection of eigenvalue or energy distributions) and to improve vortex centre identification. The POD reconstructed velocity field using only the most energetic modes enabled the coherent structures of the flow to be clearly visualised, providing improved identification of the vortex centre and subsequent evaluation of the meandering effect on the turbulent statistics. The present findings suggest that the vortex meandering only has a small impact on the ensemble-averaged resultant velocity, while contributing up to a maximum of 28% for the fluctuating component. The meandering correction also leads to an overall decrease of turbulence intensity in the peak fluctuating region of the vortex core.

1. Introduction

Vortex flow is a classic topic in fluid dynamics. It continues to attract intense research interest because of its complexity, instability and significant impacts in a broad range of engineering applications. The terms vortex core meandering, vortex core wandering or vortex core precession are used to describe the instability of a swirling flow which is a time-dependent flow pattern, where the core of the system oscillates around the rotational axis. It appears as a mechanism for the rapid transport of fluid, momentum, and energy, hence significantly influencing the flow characteristics, the mixing process, and the overall performance of a vortex producing device [1, 2]. The instability of vortex flow in an open field gives rise to tornado-like flow [3] and vortex meandering [4].
Figure 1. A fin-tip vortex generated after a fin appendage on a generic submarine model at a sideslip angle of 10 degree.

Vortex meandering has been observed in experiments at least since the 1970s [5], however, the underlying mechanism that causes the wandering remains an active area of research. This phenomenon typically is attributed to wind tunnel freestream turbulence and considered as an experimental artifact not found in flight conditions [5-8]. Rokhsaz et al. presented data suggesting a cause other than freestream turbulence [9]. The meandering was also found raised from turbulent fluctuations originating in the shear layer as the vortex rolls up from the wing [10]. In a recent experimental investigation, it was concluded that the cause of the wandering is the non-zero radial velocity on the vortex centreline, which acts to transversely displace the trailing vortex [11]. Regardless of the source, vortex meander clearly does influence the mean measured properties of the vortex. It creates a smoothing effect that can make the vortex appear closer to the classically predicted structure, causing the vortex core to appear to have a larger diameter and reduced peak tangential and axial velocities [5, 7, 8, 12, 13]. The meandering can also result in large values of turbulent stresses [7, 8, 12].

The vortex core meandering aft of the fin-tip of a submarine during yaw or turn manoeuvre (see Figure 1) leads to undesirable flow characteristics, such as unsteady flow and high level turbulence which can generate flow noise that is detrimental to the submarines’ performance [14]. The wandering vortex core is also an important element in hydrofoil ventilation, which results in an undesirable and rapid loss of lift if it occurs on the foil, or a loss of side force and transverse stability if it occurs on the strut [15]. In many engineering applications, steady-state Reynolds-averaged Navier-Stokes (RANS) simulations are routinely used to
ascertain the performance characteristics of vehicles. Experimental wind tunnel data is often used to validate these simulations (e.g. [14]). In vortex laden flows it is important to be able to distinguish between the meandering induced fluctuations and those due to the turbulence in the flow. Particularly, as steady RANS simulations cannot account for the meandering, the simulation results may predict lower peak values of turbulence [14]. The ability to remove or isolate the effects of meandering is necessary to enable a more direct comparison of experimental data with steady based simulations.

Instantaneous PIV measurements allow the possibility of shifting the image sequence to a common vortex core centre, thereby eliminating the meandering effect from the ensemble average and the flow statistics [4, 12, 16, 17]. Deconvolution has been used to correct the vortex wandering in the velocity measurements of wing-tip vortices [18]. However, the accuracy is limited by several factors, including the approximation of the noise to signal ratio, the estimation of probability density function (PDF) of the vortex centre and the high sensitivity to the measurement noise. Indeed, the measurement noise and the general presence of small-scale turbulence in the PIV measurement can lead to inaccurate identification of the vortex centre, which has not been considered in previous studies [4, 12, 16].

In the investigation of the vortex meandering, accurate identification of the vortex centre is an essential requirement. There have been many different methods used to identify the vortex core or vortex centre based on the analysis of the velocity-gradient tensor, vorticity threshold, pressure minima threshold and other principles. Summaries and analysis of these different methods can be found in [19-21]. However, there has not been a conclusion of the most accurate criterion for the identification of the vortex centre directly from PIV measurement results. Furthermore, Proper Orthogonal Decomposition (POD) technique has been found effective in the analysis of turbulent flow and vortex flow [11, 22, 23]. The POD analysis can extract coherent flow structures based on the relative contribution of eigenmodes to the total turbulent kinetic energy. Reconstruction of the flow with only the most energetic modes enables the study of the dominant coherent flow structures from instantaneous turbulent flow fields.

This paper shows the results of POD analysis on a fin-tip vortex to reveal the most dominant modes. It also reports the effectiveness of several widely used local vortex-identification criteria when applied to raw PIV data as well as when applied to POD reconstructed flow field data. The meandering of the fin-tip vortex is then analysed and corrected. Comparison of the results before and after the correction provides a clear evaluation of the meandering and allows characterisation of the vortex core region without the smearing effect induced by the meandering.

2. Experiment apparatus

A model scale generic conventional submarine (also known as BB2) was built to allow stereoscopic particle image velocimetry (SPIV) testing in the Low-Speed Wind Tunnel at Defence Science and Technology (DST) Group [24]. The submarine consists of an axisymmetric body with a casing, a fin, two hydroplanes and control surfaces (shown in
Figure 1). The full model has a length ($L$) of 2 m and a length-to-diameter ratio of 7.3. The model in this experiment is truncated at 95.0%$L$ for sting mounting. On the casing, the shape of the fin is that of NACA-0022 with a height of 8%$L$ (160 mm) and a chord length of 15.7%$L$ (314 mm). A nominal freestream velocity of 29 m/s and 61 m/s (maximum operational airspeed) was set, resulting in a model-length Reynolds number of 4 $\times$ 10$^6$ and 8 $\times$ 10$^6$, respectively. The results discussed below are with the freestream velocity of 29 m/s, except stated otherwise. The chord-length Reynolds numbers are 0.63 $\times$ 10$^6$ and 1.26 $\times$ 10$^6$. SPIV measurements have been conducted to obtain the detailed velocity characteristics at three selected measurement planes along with the model at 10° yaw (51.1%$L$, 65.0%$L$ and 81.5%$L$); see Figure 2. The three measurement planes were selected to capture the main flow characteristics at the trailing edge of the fin, middle-body section and the casing/tail junction, respectively. The SPIV parameters are summarized in Table 1. For further details on the submarine geometry, wind tunnel configuration, SPIV setup and parameters, data processing, and uncertainty analysis see [24, 25].

![SPIV setup](image)

**Figure 2.** The BB2 generic conventional submarine model at 10° yaw and the SPIV measurement planes.

<table>
<thead>
<tr>
<th>Table 1. Key parameters used in the SPIV measurement</th>
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<td><strong>Measurement area</strong></td>
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<td><strong>Recording lens</strong></td>
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<td><strong>Nominal object distance</strong></td>
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### 3. Analysis methodology

#### 3.1. Selection of the velocity field

Instead of the analysis being applied to the whole velocity field of each measurement plane, the fin-tip vortex region (which includes the vortex and some wake component) and the vortex core region are selected. The centre of the fin-tip vortex has been identified by the ensemble-averaged minimum local vorticity with the normalised centre coordinates given in [24]. As presented in Figure 3, the fin-tip vortex and the vortex core are defined by the swirl velocity distribution ($U_y$, the velocity component in ‘y’ direction along the ‘z’ centreline of the vortex in this case). A threshold of 30% of the maximum swirl velocity ($U_{y,\text{max}}$) is selected to separate the fin-tip vortex ($D_{\text{fin-tip vortex}}$) from the whole velocity field, particular the fin wake, which will affect the POD analysis. The forced vortex within the central region is defined as the vortex core, $D_{\text{vortex core}}$. The diameter of the vortex core is about 0.01$L$, which is in a good agreement with the vortex core diameter as defined by a turbulent intensity threshold [24]. Figure 4 shows an ensemble-averaged resultant velocity field at 65.0%$L$ with a freestream velocity 29 m/s, in which the fin-tip vortex is clearly the most dominant flow feature. Using the proposed criterion, the fin-tip vortex and the vortex core regions are shown.
Figure 3. Swirl velocity distribution across the fin-tip vortex and the selected regions for the current analysis.

Figure 4. Selected fin-tip vortex and vortex core sub-regions from the whole velocity field (contours of the ensemble-averaged resultant velocity field at 65.0%L, 29 m/s, 10° yaw).
3.2. Vortex identification and Proper orthogonal decomposition

Five widely used local vortex identification criteria are selected in the current analysis, they include the out-of-plane velocity ($U_x$), the local out-of-plane vorticity ($\omega_x$), the Q criterion, Lambda-2 criterion ($\lambda_2$) and Gamma-1 criterion ($\Gamma_1$). In the current work, the minimum local out-of-plane vorticity ($\omega_x$) obtained using a second-order central finite difference scheme was used to identify the vortex centre. This scheme has been shown to provide robust performance compared to other velocity derivative methods [26]. The Q-criterion and $\lambda_2$ criterion use the balance of the strain rate tensor ($S$) and rotation rate tensor ($\Omega$) to identify the existence of a vortex and location of the vortex centre.

The accuracy of gradient-based vortex detection operators can suffer from experimental noise amplification and spatial resolution. Therefore, $\Gamma_1$ was developed to characterise the locations of the vortex centre and only considers the topology of the velocity field, not its magnitude [27]. Moreover, $\Gamma_1$ filters the small-scale turbulent intermittency and is sufficiently robust to process large data sets consisting of several thousands of velocity fields. The application to PIV measurements, in which the velocity field is sampled at discrete spatial locations, is defined as:

$$
\Gamma_1(P) = \frac{1}{N} \sum_A \frac{(PM \wedge U_M) \cdot z}{\|PM\| \cdot \|U_M\|} = \frac{1}{N} \sum_A \sin(\theta_M)
$$

Where, $A$ is defined as a rectangular domain of fixed size centred on point P, point M lies in $A$ and $z$ is the unit vector normal to the measurement plane, $N$ is the number of points M inside $A$, $\theta_M$ represents the angle between the velocity vector $U_M$ and the radius vector $PM$. Regardless of the dimension of the selected rectangular domain [27], the vortex centre is determined by local maximum detection. Figure 5 shows the vortex centre identification using the $\Gamma_1$ criterion and further correction of the vortex meandering by shifting each vector field to be coincident with the identified vortex centre.

For further detailed information about these vortex identification criteria and their advantages and limitations, please refer to [19-21, 27-31].
Figure 5. $f_1$ criterion and meandering correction by shifting each vector field to be coincident with the vortex centre[32].

To enable a more robust and accurate identification of the vortex centre in the instantaneous vector field and to identify the most dominant coherent structures, a snapshot Proper Orthogonal Decomposition (POD) of the velocity vector field was performed. POD has been found to be effective in decomposing complex turbulent flow into a set of modes that represents the dominating flow structures. The elementary idea of POD is to define a set of orthogonal functions (eigenmodes) with coefficients representing the flow field based on an energy-weighted calculation (eigenvalue or energy level of the total turbulent kinetic energy, TKE). It is subsequently possible to identify the dominating large-scale coherent structures that contribute the most energy to the flow. Reconstruction of the velocity vector field using only the most energetic modes enables the dominant structures of the flow to be more clearly captured. More information on snapshot POD method and its mathematical process can be found in [11, 22, 23, 27, 33-36].

4. POD analysis of the fin-tip vortex

To perform the POD analysis efficiently and reduce the computational cost, 1000 snapshots out of the available 3000 instantaneous velocity fields were used. Figure 6 presents the contributions from the POD modes to the total energy for three configurations, i.e., 1000 snapshots of the fin-tip vortex, 3000 snapshots of the fin-tip vortex and 1000 snapshots of the vortex core. It can be seen from the figure that the lower modes contain most of the energy, while the contributions from the higher modes ($\geq 3$) are much lower. There is a maximum difference of 3% in the eigenvalues between the 1000 and the 3000 snapshots cases for the fin-tip vortex. This confirms the accuracy and acceptance of the POD analysis using 1000 snapshots of the vector fields instead of the 3000 vector fields. Furthermore, the eigenvalues of the first three modes of the fin-tip vortex, i.e., 9.5%, 6.8%, and 5.1% (percentage of the total TKE), respectively, indicate the existence of dominant large-scale coherent structures
compared to the remaining eigenmodes. The larger eigenvalues of the vortex core (30.3%, 21.1% and 3.96%) imply a more dominant large-scale coherent structure with the first three modes containing about 55.4% of the total TKE. The difference of the eigenvalue distribution between the fin-tip vortex and vortex core is due to the different regions selected for the POD analysis. For the selected fin-tip vortex region, part of the fin wake is also included in the analysis, while it is a pure vortex flow in the selected vortex core region. This difference is clearly shown in Figure 7 and 8.

Figure 6. Contributions from the POD eigenmodes to the total turbulent kinetic energy for different cases with a particular focus on the first ten modes (1000 snapshots of the fin-tip vortex, 3000 snapshots of the fin-tip vortex and 1000 snapshots of the vortex core).

An instantaneous velocity field of the vortex core at 65.0% L, the first three POD modes and the reconstruction of the flow field based on the first three modes are presented in Figure 7. The two most energetic modes mainly consist of two counter-rotating vortex pair and owing to the symmetry along the axis of the mean flow; mode 2 is obtained from mode 1 by a 90-degree rotation. This is in good agreement with the POD analysis of vortex flow in a cylinder duct [27] and that of a wing-tip vortex in an open field [37]. Based on the three most energetic modes (representing 55.4% of the total TKE), the velocity field can be reconstructed as shown in Figure 7 and shows more clearly the vortex core.

Figure 8 (left) shows the reconstruction of the same instantaneous velocity field as Figure 8 (right) of the fin-tip vortex at 65.0%L based on the first three POD modes (21.5% of the total TKE). Comparing with the original image, the small-scale fluctuations have been removed providing a much cleaner view of the vector field, which will enable accurate identification of the dominating flow structure and location of the vortex centre. The lower magnitude observed in Figure 8 (red region in the bottom left of the view) is the fin wake included in the POD analysis mentioned above and leads to the lower value of the energy level in Figure 6.
Figure 7. Contours of out-of-plane streamwise velocity ($U_x$) for the vortex core with every 8th vector shown; Top: the reconstructed flow field based on the first three modes and the original instantaneous velocity field of the vortex core. Bottom: the first three POD modes with their eigenvalues/energy level.

Figure 8. Contours of out-of-plane streamwise velocity ($U_x$) for the fin-tip vortex with every 8th vector shown; Left: reconstruction of an instantaneous velocity field based on the first three modes from the POD analysis. Right: the same raw instantaneous velocity field of the fin-tip vortex at 65.0% of the model length.
Further analysis of the decomposition also reveals the development of the dominant turbulent structures along the model length. The eigenvalue distributions of the fin-tip vortex and the vortex core at the three measurement planes along the model are presented in Figure 9. In both cases, the eigenvalues of the first two modes increase with increasing streamwise distance and indicate the growth of the turbulent kinetic energy contained by the dominating turbulent structures as presented in Figure 8. The growth of the dominating coherent structure and the dissipation of the small-scale turbulent structures imply the recovery of the flow from the fin-induced disturbance. This agrees well with previous POD analysis of a trailing vortex \[33\]. It should be noted that this increase of the energy level only relates to the concentration of the turbulent kinetic energy of the dominating structure or strengthening of the dominating turbulent structure, but does not imply any change of the total turbulent energy of the selected vortex. Figure 10 shows the overall resultant fluctuating velocity \((U_{rms})\) normalised by the freestream velocity (29 m/s) of the selected fin-tip vortex at different planes, from which no significant change with axial distance downstream can be observed.

Figure 9. Eigenvalue distribution of the 1000 snapshots of the fin-tip vortex (left) and the vortex core (right) at three measurement planes along with the model at 29 m/s.

Figure 10. The turbulence intensity of the selected fin-tip vortex through the centre of the vortex at three measurement planes (normalised by the diameter of the vortex core).
Figure 11 presents the normalised resultant velocity and fluctuating components (fin-tip vortex) averaged using the original velocity field, the approximated velocity field based on the first 50 modes of the POD analysis and the approximation based on the first three modes. As expected, the reconstructed mean velocity field has no change from the original value without any regards to the modes used in the approximation. While the significant impact of the POD approximation on smoothing the velocity field, removing the small scale turbulent structure and measurement noise was shown in Figure 7 and Figure 8. Of course, the original velocity field has 100% of the turbulent energy, and the approximations based on the first 50 and 3 modes contain 56% and 21.5% of the total turbulent kinetic energy, respectively. The effects on the turbulence intensity are shown in Figure 11 (bottom), and with a decrease of the modes used in the reconstruction, the turbulence intensity drops significantly. The more modes used, the more turbulent energy will be kept and the more accurate approximation will be obtained, but the identification of the dominating flow structure will be more difficult. The characteristics of POD is that it does not change the mean flow field but only impacts the turbulent components and therefore makes it a good pre-processing technique for further vortex identification. The same profile shape and magnitudes were observed for the vortex core sub-region, thus to reduce the computational burden, the POD reconstructed velocity fields of the vortex core region were used for the meandering correction analysis in the
following section. From herein the POD reconstructed flow fields refer to the reconstruction with only the first three modes as they were the most dominant modes (representing 55.4% of the total TKE of the vortex core). The TKE contributions of the remaining modes are shown to be less than 3% each.

5. Vortex identification using different criteria

The vortex centres of both the raw and the reconstructed vortex core velocity fields are identified by the different criteria in this section. Figure 12 shows the examples of different criteria used for the same instantaneous velocity field of the vortex core region at 65.0%L and a freestream velocity of 29 m/s. The results from the POD reconstructed velocity field and the original field are normalised by the local extremums and presented in the left and right columns, respectively, to enable a clear comparison. The location of the instantaneous vortex centre from the ensemble-averaged vortex centre is normalised by the vortex core diameter and presented in the figure, (Y, Z)_{core}.
Figure 12. Different criteria used for the vortex centre identification at 65.0%L and a freestream velocity of 29 m/s. Normalised by the local extremums, the results from POD reconstructed and raw velocity vector fields are presented in the left and right columns respectively. The location of the instantaneous vortex centre from the ensemble-averaged vortex centre is normalised by the vortex core diameter and indicated by \((Y, Z)_{\text{core}}\). The same axis and colorbar were used for all cases.

The results based on the POD reconstruction are cleaner than those from the raw data, as the small scale turbulence and measurement noise have been removed in the reconstruction. Except for the out-of-plane velocity criterion, the vortex centres identified by the local extremums (the red peak with a white cross marked in the first case) approximately coincide with each other. However, this does not mean the identification criteria are accurate for all of the instantaneous cases. In PIV measurement of a gaseous vortex flow, the low seeding density in the vortex core and subsequent ability to produce accurate measurement is a common issue [38-41], and was also observed in the current study. Due to the swirling nature and centrifugal forces, the small aerosol particles used as tracers are moved outwards from the centre. The low particle density in the vortex core region affects the accuracy of the PIV method leading to spurious measurement points, which are removed based on validating criteria and can be seen as holes in the velocity field. Similar to other filtering algorithms [17], the valid vectors in this study are also determined by a global range filter based on scatter plots of particle-image displacement, and the universal outlier median test with 2 px tolerance [24]. If the first PIV correlation peak yields a spurious velocity vector, then it is replaced by the second, third or fourth highest correlation peak, whichever first satisfies global range and the universal outlier median test. If these conditions are not satisfied and the neighbouring points have valid data, interpolation is used to fill the missing data point (interpolation is only done in one pass and not iterated). However, if more than one neighbouring vectors do not satisfy the criteria the vector is rejected entirely and a ‘hole’ results.
Figure 13. (a) a normalised instantaneous raw out-plane velocity field ($U_x$) at 65.0%L and freestream velocity of 29 m/s, (b) & (c) vorticity ($\omega_x$) and $\Gamma_1$ criteria ($\Gamma_1$) of the same instantaneous velocity field used in (a), (d) a normalised instantaneous raw out-plane velocity field ($U_x$) at 65.0%L and freestream velocity of 61 m/s. The axis are the same as used in Figure 12.

Figure 13 (a) presents a normalised instantaneous raw out-plane velocity ($U_x$) at 65.0%L and freestream velocity of 29 m/s, which has a ‘hole’, i.e., no data available, in the vortex centre due to the PIV resultant not satisfying the validation criteria. This hole is carried through to the derived quantities, $\omega_x$ (shown in Figure 13 (b)), Q and $\lambda_2$ criteria, respectively. The existence of the ‘hole’ or ‘holes’ will most likely result in a false vortex centre identified by the local extremums. While, the $\Gamma_1$ criterion can not only remove the small-scale turbulent intermittency but also overcome the impact of the missing data in the vortex centre as a full field is obtained, see Figure 13 (c). The low-quality data or ‘holes’ in the vortex core region increases when the vortex moves downstream or higher swirl velocity is experienced [41]. To overcome this, neutrally buoyant tracer, such as helium-filled soap bubble has been used for accurate PIV measurement of a gaseous vortex flow [41], although they are generally limited to lower speeds of less than 20 m/s. Figure 13 (d) is a normalised instantaneous raw out-plane velocity ($U_x$) field at 65.0%L and a freestream velocity of 61 m/s, from which more holes are observed. The missing data at 81.5%L and a freestream velocity of 61 m/s reaches a maximum of 47.3% in the instantaneous velocity fields. The maximum number of detected holes in the vortex core is presented in Table 2. A high percentage of missing data increases the likelihood of false vortex centre identification, therefore, highlighting the difficulty in direct identification and tracking of gaseous vortical structures from PIV measure velocity fields. The current work focuses on the lower Reynolds number case due to the better quality data and much fewer detected ‘holes’ (see Table 2).
Figure 14. Probability density function (PDF) of the 1000 instantaneous vortex centre of the POD-reconstructed (left column) and raw (right column) velocity vectors identified by different criteria at 65.0\%L and 29 m/s. The red circles represent the selected vortex core (diameter~0.01L).

The 1000 snapshots of the vortex core defined in Figure 4 are used to evaluate the vortex identification criteria and the meandering of the fin-tip vortex. A probability density function
(PDF) of the instantaneous vortex centre of the 1000 selected POD-reconstructed and raw velocity fields are plotted in Figure 14. The relative location of the instantaneous vortex centre can be seen from the figure and its probability is indicated by the colour bar. The selected vortex core region is presented by the red circle in the figure to show the meandering area relative to the selected field of view. The impact of the different criteria and the POD reconstruction are evident from this figure. The clusters of concentrated vortex centres found in the POD approximated velocity fields indicate the meandering area. For all the different criteria tested the results from the POD reconstructed velocity fields are much cleaner and result in a significant decrease in the vortex meandering area. The vortex centres identified using the raw data located outside of the main cluster should be considered as false results caused by small scale turbulence, measurement noise and the existence of ‘holes’ in the instantaneous velocity vector fields. Hence, the POD reconstruction of the velocity field can significantly improve the robustness of the vortex centre identification, especially with the PIV measurement result that contains measurement noise, small scale turbulence, and missing data in the core region. Comparisons among the different criteria reveal that, for both POD reconstructed and raw data, the out-of-plane velocity gives a larger meandering area than all the other methods, while the vorticity, Q and $\lambda_2$ criteria result in the similar meandering area. This is mainly because the maximum out-of-plane velocity is only a single component measure and is more susceptible to small scale turbulence and measurement noise, while other criteria are all based on velocity gradient methods. Therefore, the out-of-plane velocity is not recommended for vortex identification and will not be included in further analysis in the current work. For the raw velocity field, the $\Gamma_1$ criterion provided the most robust performance, it produced one main cluster with the smallest meandering area, as such it was not severely impacted by small scale turbulence and was not as susceptible to missing data.
Figure 15: Probability density function (PDF) of the instantaneous vortex centre of the POD-reconstructed and raw velocity vectors identified by the vorticity ($\omega_x$) at three axial locations (51.1%L, 65%L, 81.5%L) along with the model at both freestream velocity. The meandering area is indicated for each case.
The out-of-plane vorticity identified vortex centre for all the three measurement planes and two freestream velocities are presented in Figure 15. The vortex centre meandering area is estimated by the average distance between the instantaneous vortex centre and the average vortex centre. There is a slight decrease in the meandering area at 65.0%L from 51.1%L, which requires further investigation, while the vortex core meandering area grows in magnitude from 65.0%L to 81.5%L. With an increase of the freestream velocity, the vortex meandering also increases in magnitude. Although, the meandering area is only estimated based on the PDF results, the tendency for the meandering area to increase with downstream distance agrees with the results in [7]. Furthermore, the significant decrease of the vortex meandering area in the POD reconstructed cases as shown in both Figure 14 and 15 indicates the benefit of the POD analysis by improving the identification of the vortex centre.

6. The fin-tip vortex meandering correction

6.1. The effectiveness of the POD reconstruction

With the vorticity ($\omega_x$) identified centre of the vortex core, the instantaneous velocity vectors are shifted to be coincident with the vortex centre in each vector field. In such a way, the effects of the vortex meandering on the flow field are removed and the meandering corrected velocity vectors are compared with the original results. The ensemble-averaged resultant velocity ($U_{xyz}$) and fluctuation components ($\langle U_{rms} \rangle$) at 65.0%L and 29 m/s are presented in Figure 16. The ‘Raw uncorrected’ and ‘POD (3 modes) uncorrected’ are the raw data and the POD reconstruction using the first three modes. The ‘Raw corrected using $\omega_x$, RAW’ refers to the raw data with the meandering corrected based on the vortex centre identified using the vorticity from the raw velocity fields. The ‘POD corrected using $\omega_x$, POD’ refers to the POD data with the meandering corrected based on the vortex centre identified using the vorticity from the POD velocity fields. The ‘Raw corrected using $\omega_x$, POD’ is the raw data processed in the following steps:

1. Use the vorticity to identify the vortex centre of the POD reconstructed velocity fields (first three modes);
2. Shift the raw velocity vector fields using these centre coordinates obtained in (1) to remove the meandering effect;
3. Obtain the normalised ensemble-averaged velocity and turbulence intensity of the meandering corrected raw data.

In Figure 16 (top) the three meandering corrected resultant velocity results show an increased maximum velocity (~1%) at about $|r/L|=1.556\times10^{-3}$ and decreased velocity at the centre of the vortex (~3.7%). This sharpening and narrowing effect of the meandering corrected resultant velocities compared to the raw and POD uncorrected cases is a result of the elimination of the meandering effect.

In Figure 16 (bottom) the raw data, the normalised turbulence intensity, $\langle U_{rms} \rangle/U_\infty$ of the ‘POD (3 modes) uncorrected’ shows an even drop across the vortex core, which is caused by removing the small-scale incoherent turbulent structure and measurement noise. Further removal of the meandering effect from the POD approximation (‘POD corrected using $\omega_x$, POD’) shows a slight decrease in the meandering area at 65.0%L from 51.1%L, which requires further investigation, while the vortex core meandering area grows in magnitude from 65.0%L to 81.5%L. With an increase of the freestream velocity, the vortex meandering also increases in magnitude. Although, the meandering area is only estimated based on the PDF results, the tendency for the meandering area to increase with downstream distance agrees with the results in [7]. Furthermore, the significant decrease of the vortex meandering area in the POD reconstructed cases as shown in both Figure 14 and 15 indicates the benefit of the POD analysis by improving the identification of the vortex centre.
POD results in a significant weakening of the fluctuation. In this case, the maximum fluctuation is about 0.1\(U_\infty\) and only consists of the coherent fluctuation from the first three dominating modes. The reduction of the flow fluctuation from the raw to the POD uncorrected and then to the ‘POD corrected using \(\omega_{x,\text{POD}}\)’ data imply the turbulence of the original vortex core mainly consists of three components, i.e., the coherent turbulence, incoherent turbulence and the fluctuation induced by the meandering. In the ‘Raw corrected using \(\omega_{x,\text{POD}}\)’ case, after the removal of the meandering effect the fluctuating component has a significant drop (16.1\%) in the central region compared to the raw data. This decrease of the fluctuation reduces in the outer region of the vortex core. It can be concluded that the meandering has a significant impact only on the central flow of the fin-tip vortex, particularly the fluctuating component. The ‘peak fluctuating core region’ is defined as \(\left\langle U_{\text{rms}}/U_\infty \right\rangle \leq 0.1\). A significant decrease of this region is observed after the correction of meandering, i.e., from 0.44 \(D_{\text{vortex core}}\) to 0.26 \(D_{\text{vortex core}}\). While, the ‘Raw corrected using \(\omega_{x,\text{RAW}}\)’ method shows a higher fluctuation in most part of the vortex core as well as a bimodal peak in the central region, which is likely a result of the false centres due to influence of small scale turbulence, measurement noise and ‘holes’ in the data.
6.2. The effectiveness of the different vortex identification criteria

The normalised resultant velocity and fluctuation of the raw data and meandering corrected results using different vortex identification criteria at 65.0%L are presented in Figure 17. The vortex meandering is corrected with the vortex centre identified from POD processed data in Figure 17 (top and bottom) and the raw data in Figure 17 (middle). It can be concluded from Figure 17 (top), the resultant velocity ($U_{xyz}$) of the POD processed and raw data using all criteria give similar results. In Figure 17 (middle), the fluctuating component ($\text{Raw}(U_{rms})/U_\infty$) is corrected based on the vortex centre identified in the raw data using each criteria. The meandering corrections using $\omega_{x,\text{RAW}}$, $Q_{\text{RAW}}$ and $\lambda_{2,\text{RAW}}$ have higher fluctuation in the outer region of the vortex core and a bimodal distribution is observed in the central region, which can be attributed false vortex centres identified from the raw data. Whereas, the $\Gamma_{1,\text{RAW}}$ criteria provides a narrower profile and does not exhibit a bi-modal peak, it is not as susceptible to small scale turbulence, measurement noise or holes in the data, and thus its fluctuation is much lower overall. The reduction of the turbulent intensity, in this case, is due to the removal of the meandering.

The bottom plot of Figure 17 is the meandering corrected fluctuation using different criteria with the vortex centre identified by the POD reconstructed velocity vector fields. It highlights a good collapse of the fluctuations after correction using each criteria, while the slight difference of the $\Gamma_{1,\text{POD}}$ criterion is due to the different identified vortex centres, shown in Figure 14. The decrease of the fluctuating component after correcting the meandering confirms the contribution of the vortex meandering to the total fluctuation. The results also confirm, for noisy or highly turbulent data, a POD reconstruction can significantly improve the identification of the vortex centre with all gradient-based vortex identification criteria tested here. It must be stated that a prerequisite in using this POD based correction is that the flow contains dominant or coherent flow structures allowing the flow field to be reconstructed using only the dominant POD modes.

If we compare $\Gamma_{1,\text{RAW}}$ in Figure 17 (middle) with all the POD corrected fluctuations in Figure 17 (bottom), we see that the $\Gamma_{1}$ criterion applied on the raw data ($\Gamma_{1,\text{RAW}}$) produces the lowest turbulence intensity peak and most overall reduction, while at the same its meandering area is larger than the POD corrected cases (see Figure 14). Therefore, for this case study, the $\Gamma_{1}$ criterion applied to the raw data alone provided the most robust identification of the vortex centre. This result also implies that the POD reconstruction with only the first three modes does not capture enough of the turbulent flow content to precisely identify the vortex centre but it does provide significant improvements compared to using $\omega_{x}$, Q and $\lambda_{2}$ criteria on the raw data. The POD approach is sensitive to the number of modes used in the reconstruction and a reconstruction with more modes may produce different results. As such, further investigation to assess the impact of the number of modes used in the POD reconstruction is warranted.
Figure 17. Using different vortex identification criteria at 65.0%L. The normalised resultant velocity (top). Fluctuation of raw data and meandering corrected using raw data (middle). Fluctuation of raw data and meandering corrected using POD data (bottom).
Table 2 provides a summary of the key vortex core parameters including, the maximum number of ‘holes’ in the vortex centre, the maximum difference of the resultant velocity and the fluctuation after the meandering correction (using $\omega_x$, POD) for all tested cases. The vortex meandering on the resultant velocity has only a small impact with a maximum difference ($\Delta_{xyz}$) of 3.7% after correction in all six configurations. While, the vortex meandering has a much stronger influence on the fluctuating component with a maximum difference ($\Delta_{rms}$) of 28%. The relationship of the vortex meandering on the fluctuation as a function of the axial location is still unclear from the current work and further investigation is recommended. It should be noted that at the higher Reynolds number, the velocity components around the vortex core suffer greater uncertainty due to the missing data as discussed previously and hence the current analysis focused on the lower Reynolds number case. From Table 2, the maximum number of holes in the vortex core region increases with downstream distance and can be explained as follows. The fin-tip vortex emanates from the tip of the fin, where initially we can assume it is evenly seeded but as the vortex travels downstream the seeding progressively evacuates from the core due to the centrifugal forces. There is no mechanism for the tracer particles to be re-inserted in the core, hence the decrease in seeding and lower quality PIV data with downstream distance. At the higher speed, this effect is compounded by the larger swirl velocity present.

Table 2. Summary of the key vortex core parameters before and after meandering correction using POD.

<table>
<thead>
<tr>
<th>$U_\infty$ (m/s)</th>
<th>Measurement planes (%L)</th>
<th>Max ‘holes’ (%)</th>
<th>$\langle U_{xyz}\rangle_{max}$ $U_\infty$</th>
<th>Corrected $\langle U_{xyz}\rangle_{max}$ $U_\infty$</th>
<th>$\Delta_{xyz}$ (%)</th>
<th>$\langle U_{rms}\rangle_{max}$ $U_\infty$</th>
<th>Corrected $\langle U_{rms}\rangle_{max}$ $U_\infty$</th>
<th>$\Delta_{rms}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>51.1</td>
<td>0.10</td>
<td>1.15</td>
<td>1.16</td>
<td>0.9%</td>
<td>0.245</td>
<td>0.178</td>
<td>27%</td>
</tr>
<tr>
<td>29</td>
<td>65.0</td>
<td>5.10</td>
<td>1.00</td>
<td>1.10</td>
<td>3.7%</td>
<td>0.305</td>
<td>0.256</td>
<td>16%</td>
</tr>
<tr>
<td>29</td>
<td>81.5</td>
<td>7.00</td>
<td>1.14</td>
<td>1.14</td>
<td>1.2%</td>
<td>0.237</td>
<td>0.185</td>
<td>22%</td>
</tr>
<tr>
<td>61</td>
<td>51.1</td>
<td>0.30</td>
<td>1.21</td>
<td>1.21</td>
<td>0.4%</td>
<td>0.205</td>
<td>0.183</td>
<td>11%</td>
</tr>
<tr>
<td>61</td>
<td>65.0</td>
<td>17.7</td>
<td>1.24</td>
<td>1.24</td>
<td>0.3%</td>
<td>0.437</td>
<td>0.317</td>
<td>28%</td>
</tr>
<tr>
<td>61</td>
<td>81.5</td>
<td>47.3</td>
<td>1.05</td>
<td>1.06</td>
<td>0.9%</td>
<td>0.352</td>
<td>0.332</td>
<td>5.7%</td>
</tr>
</tbody>
</table>

7. Conclusion and recommendations

This paper presented a study of a fin-tip vortex meandering using recently acquired SPIV measurements [24]. The vortex flow is analysed using a snapshot POD technique to establish the dominant large-scale turbulent structure, which grows as the vortex moves downstream. For the vortex core region, the first three POD eigenmodes were the most dominant and represented 55.4% of the total TKE.

Several local vortex identification criteria were then used to identify the vortex centre in the flow field. For the raw data, which generally has small-scale turbulent structure, measurement noise and missing data in the central region, the $\Gamma_1$ criterion is the most robust criteria able to provide an estimation of the meandering corrected flow characteristics. The $\Gamma_1$ criterion resulted in the least scatter and the smallest meandering area in the PDF of the vortex centre location and after correction, resulted in an overall reduction in the turbulence intensity. Whereas, traditional vortex identification criteria, $\omega_x$, Q and $\lambda_2$ showed greater
scatter in the PDFs as well as an overall increase in the turbulence intensity, which was attributed to incorrect vortex centre identification.

Using each of the criteria to identify the vortex centre of the POD reconstructed velocity fields the instantaneous raw velocity vector fields were shifted to remove the meandering effect. The ensemble-averaged velocity and turbulence intensity of the meandering corrected raw data was then determined. For the POD reconstructed velocity vector fields, all criteria used in this study generated a clear view of the vortex core and after meandering correction the flow statistics showed good agreement. The POD method showed better accuracy in the vortex centre identification compared to using $\omega_x$, $Q$ and $\lambda_2$ criteria on the raw data. The vortex meandering correction using POD produced a maximum difference in the resultant velocity of up to 3.7%, whereas the effect on the fluctuating component was much more significant with a reduction in the central region of up to 28%. The meandering correction also produced a significant overall decrease of the peak fluctuating core region, or in other words, the meandering leads to an over prediction of the vortex core fluctuating region.

A comprehensive analysis of the vortex flow at the higher Reynolds number and along the length of the model was hampered due to the absence of seeding in the core, affecting the quality of the PIV data. To obtain better measurements in the core region of gaseous vortex flow, a neutrally buoyant tracer, such as helium-filled soap bubbles is recommended (see [41]). Furthermore, the current study only looked at the statistical results of the fin-tip vortex and its meandering, another possibility is to extend the work to include time-resolved measurements in order to identify the frequency of vortex meandering.

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Reference