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Frequency diverse array with random logarithmically increasing frequency offset

Authors: Gaojian Huan¹, Yuan Ding², Shan Ouyang¹ and Vincent Fusco³

Institutional Affiliation: ¹School of Information and Communications, Guilin University of Electronic Technology, Guilin 541004, China; ²The Institute of Sensors, Signals and Systems (ISSS), Heriot-Watt University, Edinburgh, EH14 4AS, UK; ³Institute of Electronics, Communications and Information Technology (ECIT), Queen's University of Belfast, Belfast, BT3 9DT, UK; Corresponding author: hmoys@guet.edu.cn

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Abstract: The Frequency diverse arrays (FDAs) have been extensively investigated since it can provide a unique angle-range-dependent beampattern. This range-dependent characteristic can suppress the range ambiguous clutters, leading to many novel potential Radar applications. Two issues, however, exist in conventional FDA (CFDA) beampatterns, i.e., range-angle coupling and range periodicity. Therefore, for a CFDA Radar, target range information cannot be obtained directly from the beampattern peaks and target ambiguity occurs in the range domain. In this work, a new frequency diverse array (FDA) transmitter architecture with random logarithmically increasing frequency offset (log-RFDA) is presented. It is shown that the proposed strategy provides a non-periodic thumbtack-like beampattern, eliminating the ambiguity in range domain, suffered in the CFDA Radar. Furthermore, reduced side lobe and higher detection resolution can be achieved, compared with the previously reported log-FDA.

KEYWORDS: Frequency diverse arrays (FDAs), random logarithmically increasing frequency offset, non-periodic thumbtack-like beampattern, higher detection resolution

1. INTRODUCTION

Frequency diverse array (FDA) has gained increasing attention in recent year in Radar community [1–6] due to its distinct range-dependent beampattern that enables the capability of rejecting interference in range domain. A conventional FDA (CFDA) is constructed by applying a fraction of linear carrier frequency offset across the radiated array elements, named as linear-FDA hereafter. This linear frequency offset creates range-angle-time-dependent radiation patterns [2]. Beampattern peak occurs periodically in each of the three domains, i.e. angle, range and time. Ambiguity exists in range domain as infinite number of field peaks occur when the angle and time are fixed. Moreover, range information cannot be directly obtained from the beampattern peak due to the coupled range-angle relationship. Some recent works have been reported in order to address these issues [3–7]. Specifically, in [3] instead of linear frequency offsets the logarithmic frequency offset was employed to achieve a non-periodic beampattern with compromised resolution performance in the range and angle domains. It was further extended to multi-carrier scenarios in [4]. In [5], a non-monotonically increasing increment used as logarithmic frequency offset was described, achieving improved range resolution at the price of larger bandwidth. Square and cubic frequency offsets were also developed for the decoupling purpose in [6], which, however, can only eliminate the multiple maxima occurring in a specific area. Apart from altering the frequency offset profile, a random frequency diverse array (RFDA) was introduced in [7], providing a decoupled range-angle beampattern. Periodicity, however, still exists. Furthermore, the index modulation (IM) techniques were applied upon antenna elements in a CFDA in [8]. By combing the IM with the CFDA, the range-angle coupling issue can be resolved by way of transmitting two pulses with different number of selected antenna elements, and information bits can also be conveyed by the selected antenna element indices. In [9], a transmit sub-aperturing approach for the FDA was presented, generating a range-angle-dependent beampattern with null depth control for joint Radar and communication applications. Here, a range-angle uncoupled beampattern is produced, meanwhile, for the joint
Radar and communication functionality purpose. The proposed Tx transmitter is highly complex as multi-carriers for each antenna element were employed. In [10], a linear uniformly spaced symmetric FDA with inter-element frequency offsets of Hamming window-based tapering was proposed to generate a so-called range-angle uncoupled beampattern. However, this uncoupled beampattern produced in [10] was erroneously claimed as the time-range relationship was overlooked. This aspect has been recently pointed out and proved in [11]–[13]. In [11], the time-independent beampattern for a given range-angle pair was revisited with the consideration of the time-range relationship. This concept was further studied in [12], suggesting that the time-independent beampatterns cannot be achieved in practical free space applications. Meanwhile, the CFDA and its variants for physical-layer wireless security was carefully examined in [13], proving that the CFDA cannot secure a free-space wireless transmission in the range domain because that the claimed security in range domain is time-dependent and the “secure region” propagates with time.

In light of the insights revealed in the recent papers [11]–[13], we understand that in free space, the beampatterns generated by the FDAs and their variants are always time-variant. Under this background, in this letter we do not intent to synthesize an FDA that produce time-invariant patterns. Instead, we propose a low-complexity log-RFDA transmitter that can produce a range-angle uncoupled and range-non-periodic beampattern at a certain time instant. The uncoupled range-non-periodic beampattern of the proposed log-RFDA can be useful for some important Radar and navigation applications. For instance, a big issue in the forward-looking Radar is how range ambiguous clutteres can be suppressed. This can be addressed by employing this range-non-periodic uncoupled beampattern, since at a specific time instant, this beampattern has the focus of the energy on the target position, while the energy distribution along the non-target positions along the same direction is suppressed. It endows the log-RFDA Radar the capability of rejecting the range ambiguous clutteres. In addition, for Radar detection and ranging, by way of adopting the log-RFDA transmitter, the range information of the detection target can be directly obtained from the beampattern peaks and the range ambiguity can be eliminated due to the fact that the range-angle uncoupled beampattern only has a single maximum in the range domain. Compared with previously reported works, the superiority of the proposed log-RFDA is summarized as follows.

- Low complexity at the transmitter end, since each element employs only a single carrier logarithmic offset and frequency offset optimization is not required;
- The time-range relationship is considered, which has been overlooked in some previous works, e.g. [10];
- The produced thumbtack-like beampattern is much more concentrated, i.e. the energy distribution surrounding the target location is much lower than the peak;
- The range resolution and sidelobe suppression of the produced beampattern are enhanced without consuming additional bandwidth, e.g. when the identical maximum bandwidth 20.794 kHz is adopted in different transmitter schemes, the proposed log-RFDA has a better target resolution compared with the log-FDA.

The paper is organized as follows. In Section 2, the Log-RFDA transmitter architecture is first described, followed by the elaboration of its transmit beampattern analysis. In Section 3, the radiated pattern and range-angle localization simulations of the Log-RFDA are presented and compared with those other previously reported transmitters. Finally, conclusions are drawn in Section 4.

2. PROPOSED LOG-RFDA TRANSMITTER

2.1 Log-RFDA transmitter

Considering a one-dimension (1D) array with uniformly spaced \( N \) identical isotropic radiating elements, a random logarithmic carrier frequency offset \( \Delta f_{\text{cn}} \) is applied onto each antenna element,

\[
f_n = f_0 + \Delta f_{\text{cn}},
\]

\[
\Delta f_{\text{cn}} = \log(C_n) \delta,
\]

where \( f_0 \) denotes the radiated carrier frequency of the \( n^{th} \) element, and \( f_0 \) is the reference frequency satisfying \( f_0 \gg \Delta f_{\text{cn}} \). In (2), ‘log (·)’ denotes the natural logarithm and \( \delta \) represents a frequency offset parameter in the order of kHz. \( C_n \) is a random variable with its probability density function (PDF) \( p(C_n) \). In the study, a discrete uniform PDF is assumed, i.e., \( C_n \in \{1, 2, 3, \ldots, C\} \) and \( p(C_n) = 1/C \). The log-RFDA transmitter architecture is illustrated in Figure. 1.
2.2 Log-RFDA transmit beampattern

In the proposed log-RFDA, the radiated signal by the $n^{th}$ element can be written as

$$s_n(t) = W_n e^{-j2\pi f_0 t} 0 \leq t \leq T,$$

where $W_n = a_n \exp(j\varphi_n)$. $a_n$ and $\varphi_n$ respectively represent the excitation amplitude and phase at the time instant $t = 0$. Here uniform amplitude is considered, namely $a_n = 1$. $T$ refers to one single pulse duration. As seen in Figure 1, a far-field target $A$ is assumed at $(R_1, \theta)$ with the first $(n = 1)$ antenna element taken as the range reference and the array boresight set as the angle reference. The waveform $S$ reaching the target $A$ can be expressed as

$$S(t; R_1, \theta) = \sum_{n=1}^{N} \frac{1}{R_n} S_n \left( t - \frac{R_n}{c} \right),$$

where $R_n = R_1 - (n-1)d\sin\theta$ and the weighting coefficient $1/R_n \approx 1/R_1$ due to far-field operation. $c$ is the speed of light in free space. Substituting (1), (2) and (3) into (4), we get

$$S(t; R_1, \theta) = \frac{1}{R_1} e^{-j2\pi f_0 (t-R_1/c)} \sum_{n=1}^{N} e^{j\varphi_n} e^{-j2\pi \left( \log(C_n)\delta \left( t - \frac{R_n}{c} \right) + \frac{f_0 + \log(C_n)\delta(n-1)d\sin\theta}{c} \right)},$$

In (5), $\delta << f_0$, and $\log(C_n)$ is a number smaller than 3 when $C = 16$, thus $\log(C_n)\delta << f_0$. Then (5) can be approximated as

$$S(t; R_1, \theta) = \frac{1}{R_1} e^{-j2\pi f_0 (t-R_1/c)} \sum_{n=1}^{N} e^{j\varphi_n} e^{-j2\pi \left( \log(C_n)\delta \left( t - \frac{R_n}{c} \right) + \frac{f_0(n-1)d\sin\theta}{c} \right)}.$$

In (6), the term $e^{j\varphi_n}$ is exploited to steer the direction of the radiation peak. Different with those in previous works, e.g. [10], here, the $e^{j\varphi_n}$ is designed as

$$e^{j\varphi_n} = e^{j2\pi \left( \frac{f_0(n-1)d\sin\theta}{c} \right)} $$

FIGURE 1 Structure of proposed FDA with random logarithmically increasing frequency offset.

2.2 Log-RFDA transmit beampattern

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$$e^{j\varphi_n} = e^{j2\pi \left( \frac{f_0(n-1)d\sin\theta}{c} \right)} $$

FIGURE 1 Structure of proposed FDA with random logarithmically increasing frequency offset.
to project the radiation pattern peak to a target direction at $\theta_X$. Based on (6) and (7), the magnitude of transmit beampattern of the proposed log-RFDA at the target direction $\theta_X$ can be expressed as

$$B(t; R, \theta) = \left| \sum_{n=1}^{N} e^{-j2\pi \left( \log(C_n) \delta \left( t - \frac{R}{c} \right) + \theta_{X} - \frac{\sin \theta_{X}}{c} \right)} \right|^2.$$  

In (8), when the maxima of the beampattern arrived at the target position $(R_1, \theta_X)$, we have $R = R_1$ and $t = R_1/c$.

3. SIMULATION RESULTS AND COMPARISON ANALYSIS

To validate the efficacy of the proposed log-RFDA scheme, the transmit beampattern is simulated and compared with the CFDA, log-FDA, RFDA, square and cubic frequency offset FDA schemes. Specifically, for the log-RFDA and RFDA schemes, the averaged transmit beampatterns with 10,000 Monte Carlo trials are shown as their frequency offsets are random variables. It is noteworthy that the designed excitation weightings are employed by all comparison schemes. In addition, two examples of range-angle localization are presented by using multiple signal classification (MUSIC) algorithm for these transmitter schemes. The system parameters used in the simulations are listed in Table 1. For fair comparison, the signal bandwidth in the log-RFDA, log-FDA, RFDA and CFDA schemes are set to be approximately identical, and the other parameters are kept the same as those for the proposed log-RFDA.

3.1 Beampattern simulations and comparisons

Seen from Figures. 2(a), (g), the proposed log-RFDA exhibits a thumbtack-like beampattern with most of energy concentrated at the location (225 km, 30°) and only a single maximum energy point occurs at a certain time instant. Compared with Figures. 2(d), (h), a periodicity of thumbtack-like beampattern peaks results when RFDA is used, indicating that ambiguity exists in unlimited areas of the range domain when the travel time is fixed. In contrast, Figure. 2(b), the log-FDA shows a non-periodic beampattern peak. However, the energy distribution at the expected location is not as much concentrated as log-RFDA or RFDA, e.g., the sidelobe energy levels at two locations, (211.9 km, 25°) and (236.4 km, 42°), are about the same, 50% and 46%, respectively, when compared with the energy level at (225 km, 30°). By contrast, the two location energy levels are much lower in the log-RFDA scheme. Thus, the proposed log-RFDA achieves a better sidelobe suppression behavior. While in Figure. 2(c), CFDA a periodic transmit beampattern with range-angle coupling is shown, suggesting that the targets’ range information cannot be directly obtained from the beampattern peak, meanwhile, multiple maxima beampattern characteristics exacerbate the capability of rejecting interference. In Figures. 2(e), (f), both the square and cubic frequency offset FDAs show a periodic beampattern with multiple maxima peaks. Especially, higher side lobe levels are achieved by the cubic frequency offset FDA.

In Figure. 3, the half power bandwidth (HPBW) beampattern shows the superiority of the proposed log-RFDA in respect of sidelobe suppression compared with log-FDA. Also, comparing Figure. 3(a) with Figure. 3(c), we can conclude that the log-RFDA can achieve the same array resolution as the RFDA scheme when their radiated signal bandwidths are identical. Meanwhile, Comparing Figures. 3(d), (e) with the others, the square and cubic frequency offset FDAs show higher range resolutions because of the larger bandwidth.

<table>
<thead>
<tr>
<th>Array name</th>
<th>Frequency offset of the $n^{th}$ element</th>
<th>Number of elements $N$</th>
<th>Reference frequency $f_0$ (GHz)</th>
<th>Element spacing $d$ (m)</th>
<th>$\Delta f$ (kHz)</th>
<th>$\delta$ (kHz)</th>
<th>$C$</th>
<th>Maximum bandwidth (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log-RFDA</td>
<td>$\log(C_n)\delta$</td>
<td>16</td>
<td>8</td>
<td>0.015</td>
<td>$\times$</td>
<td>7.5</td>
<td>16</td>
<td>20.794</td>
</tr>
<tr>
<td>Log-FDA</td>
<td>$\log(n)\delta$</td>
<td>16</td>
<td>8</td>
<td>0.015</td>
<td>$\times$</td>
<td>7.5</td>
<td>$\times$</td>
<td>20.794</td>
</tr>
<tr>
<td>RFDA</td>
<td>$C_n\Delta f$</td>
<td>16</td>
<td>8</td>
<td>0.015</td>
<td>$\times$</td>
<td>1.5</td>
<td>$\times$</td>
<td>16</td>
</tr>
<tr>
<td>CFDA</td>
<td>$n\Delta f$</td>
<td>16</td>
<td>8</td>
<td>0.015</td>
<td>$\times$</td>
<td>1.5</td>
<td>$\times$</td>
<td>24</td>
</tr>
<tr>
<td>Square frequency offset FDA</td>
<td>$n^2\Delta f$</td>
<td>16</td>
<td>8</td>
<td>0.015</td>
<td>$\times$</td>
<td>1.5</td>
<td>$\times$</td>
<td>384</td>
</tr>
<tr>
<td>Cubic frequency offset FDA</td>
<td>$n^3\Delta f$</td>
<td>16</td>
<td>8</td>
<td>0.015</td>
<td>$\times$</td>
<td>1.5</td>
<td>$\times$</td>
<td>6144</td>
</tr>
</tbody>
</table>
FIGURE 3  HPBW beampattern comparisons of range and angle at the location (225 km, 30°) at time instant $t = 0.75$ ms, $\theta_k = 30^\circ$ for (a) the proposed log-RFDA; (b) the log-FDA; (c) the RFDA; (d) the square frequency offset FDA; and (e) the cubic frequency offset FDA.

FIGURE 2  Normalized transmit beampatterns versus range and angle at the location (225 km, 30°) at time instant $t = 0.75$ ms, $\theta_k = 30^\circ$ for (a) the proposed log-RFDA; (b) the log-FDA; (c) the CFDA; (d) the RFDA; (e) the square frequency offset FDA; (f) the cubic frequency offset FDA; (g) the averaged results of the proposed log-RFDA using 10,000 trials; and (h) the averaged results of RFDA using 10,000 trials.
In Figures 4(a)–(f), the time-range beampatterns of the six transmit schemes are depicted when the azimuth angle $\theta$ is fixed. It can be seen that the beampattern peaks in all schemes are propagated along the distance as time elapses, confirming the findings in [11–13]. Distinctively, only a single maximum at a time instant for the proposed log-RFDA and the log-FDA can be observed, i.e. non-periodicity beampattern in the range domain. In addition, in Figure 4(a), it can be observed that the time-range relationship of the beampattern propagation can be met in the proposed log-RFDA scheme. In Figures 5(a)–(f), the beampatterns of time versus angle for the compared...
schemes are shown at a fixed range. It can be seen that the CFDA provides a varying beampattern peak in the angle domain and the multiple maximums present in the angle domain in the beampatterns of the RFDA, the square and cubic frequency offset FDAs as time passes by, see Figures. 5(c)–(f). Hence, the range ambiguity issue exists in these schemes. In contrast, only a single maximum in the angle domain as time elapses is observed in the log-RFDA beampattern, suggesting that the proposed log-RFDA scheme has a better performance with regard to preventing range ambiguity than do other compared schemes.

3.2 Range-angle localization

Example 1: Single Target

Here, only one transmission pulse is used to obtain the joint range and angle estimation of targets by way of employing the MUSIC algorithm. In this example, it is assumed that a single target A is positioned at (120 km, 35°). Figure. 6 shows the simulation results of the target localization for the six transmitter array schemes. Seen from Figures. 6 (a), (b), it is shown that the target information can be successfully estimated in the range-angle dimension when the log-RFDA and log-FDA schemes are adopted, respectively. By contrast, for RFDA, square and cubic frequency offset FDA schemes fake targets detection occurs, suggesting that target ambiguity exists in these transmit schemes, see Figures. 6 (c), (e), (f). In Figure. 6 (c), for the RFDA, ambiguity exists only in the

![FIGURE 6 MUSIC spectra of example 1. (a) The proposed log-RFDA; (b) The log-FDA; (c) The RFDA; (d) The CFDA; (e) The square frequency offset FDA; (f) The cubic frequency offset FDA.](image)

![FIGURE 7 MUSIC spectra of example 2. (a) The proposed log-RFDA; (b) The log-FDA.](image)
range domain. Hence, we can get correct angle information of the target A. While, ambiguity exists in the range and angle domains for square and cubic frequency offset FDA schemes, resulting in errors in the measured target position. Figure. 6(d) shows that the range-angle two-dimensional information cannot be obtained by the CFDA scheme.

Example 2: Two Targets

In this example, to compare the detection resolutions between the log-FDA and the proposed log-RFDA, the two targets A and B are assumed to be positioned at (120 km, 35°) and (125 km, 37°), respectively. In Figure. 7(a), it is shown that target A and B can be detected and separated by the proposed log-RFDA scheme. In contrast, in Figure. 7(b), these two targets are entangled, leading to error occurs in locating the targets. Therefore, the proposed log-RFDA scheme can achieve a higher detection resolution than the log-FDA scheme.

4. CONCLUSIONS

Frequency diverse random frequency offset combined with log-FDA was proposed in this letter. It has been shown that in the proposed scheme the characteristics of non-periodic beampattern achieved by log-FDA is preserved and that the sidelobe levels in the log-FDA scheme are suppressed. Meanwhile array resolution is improved. Therefore, the proposed log-RFDA using single carrier for each element integrates the benefits of both log-FDA and RFDA without consuming additional bandwidth.

REFERENCES