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An approximate solution for the wave energy shadow in the lee of an array of overtopping type wave energy converters

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Abstract

In this study we investigate how the wave energy deficit in the lee of an array of overtopping type wave energy converting devices (WECs), redistributes with distance from the array due to the natural variability of the wave climate and wave structure interactions. Wave directional spreading has previously been identified as the dominant mechanism that disperses the wave energy deficit, reducing the maximum wave height reduction with increasing distance from the array. In addition to this when waves pass by objects such as an overtopping type WEC device, diffracted waves re-distribute the incident wave energy and create a complex interference pattern. The effect of wave energy redistribution from diffraction on the wave energy shadow in the near and far field is less obvious. In this study, we present an approximate analytical solution that describes the diffracted and transmitted wave field about a single row array of overtopping type WECs, under random wave conditions. This is achieved with multiple superpositions of the analytical solutions for monochromatic unidirectional waves about a semi-infinite breakwater, extended to account for partial reflection and transmission. The solution is used to investigate the sensitivity of the far field wave energy shadow to the array configuration, level of energy extraction, incident wave climate, and diffraction. Our results suggest that diffraction spreads part of the wave energy passing through the array, away from the direct shadow region of the array. This, in part, counteracts the dispersion of the wave energy deficit from directional spreading.

1. Introduction

A Wave Energy Converter (WEC) converts the kinetic and potential energy in the incident wave in one of two distinct ways. Either a volume of water with a positive potential energy is captured from the system (overtopping type device), or wave motion cause the components of the device to move relative to each other so that a destructively interfering wave is generated (point absorber or attenuator). The extraction of wave energy from the system produces a wave energy deficit or shadow down wave. The ability to predict the shadow is of topical interest due to the significant stake holder concerns about potential impacts from wave shadowing arising from wave energy device installations. Waves play a key role in mass transport, assist in mixing and force sediment transport, therefore, quantifying the wave energy reduction and identifying induced wave height gradients would help us to assess the environmental impact of a WEC array on the nearby coastal ocean and shoreline. To minimise environmental impacts, it is desirable that the wave energy deficit redistributes over the widest area in the shortest distance to minimise wave height reductions at any one location, or vice versa if shore protection is a desired goal of the wave energy development. In addition the ability to predict the wave shadow and interference pattern about devices will reveal locations of low energy within the device array due to negative interference and shadowing. A specific spatial arrangement of devices that avoids placing devices in these low energy locations would maximise the collective performance of the array.

For an overtopping type WEC the spatial distribution of the wave energy deficit is affected by wave directional spreading, diffraction and refraction. Wave directional spreading describes the degree of lateral transmission of wave energy in a given sea state. A broader directional spread would disperse the energy deficit over a wider
region and, vice versa, as shown in (Black, 2007). This mechanism is analogous to the dull shadow cast by an object illuminated by diffuse light or the sharp shadow cast by an object illuminated by a point source. The sharp gradient in wave height at the edge of an overtopping type WEC device, from the wave termination at the device, will induce diffraction. This diffraction effect propagates wave energy into the lee region of the WEC initially. Refraction due to bathymetry or ambient current change would deflect the direction of the incident and scattered waves, and therefore alter the wave energy shadows location and distribution at the coast.

Some recent studies have been carried out to investigate the wave height reduction in the lee of WEC arrays. (Millar, et al., 2007) used the third generation phase averaged spectral wave model SWAN to investigate the effects of the scale of energy extraction and incident wave parameters on the far field wave energy deficit. As phase is not resolved in SWAN and the individual WEC devices not delineated the redistribution of energy by diffraction and radiation is not accounted for. The wave height reduction was predicted to reduce monotonically with distance from the array for a final wave maximum reduction 25km behind the array of less than 1% of the incident wave height.

(Venugopal & Smith, 2007) used the Boussinesq wave model, Mike21 BW to assess the resultant wave field about a single row of overtopping type WECs which partially reflect energy to achieve a wave energy deficit in the down wave region rather than by specifically extracting it. Redistribution of wave energy from diffraction is considered but the distance from the array to the shore is 2km which is short compared to some proposed offshore wave array installations.

(Palha, et al., 2010) used the parabolic mild slope wave model REF/DIF to assess the wave shadow in the lee of a series of large energy sinks that represent clusters of devices. Wave structure interactions and the resultant redistribution of wave energy are not considered fully. Individual devices are not delineated so that waves diffract only about the edges of the energy sink regions (WEC clusters) and not the individual devices.

(Beels, et al., 2010) used the time dependent mild slope wave model MILDwave to assess the wave shadow in the lee of a 2D array of Wave dragon overtopping type WEC devices. The devices where implemented within the model as porous layers, the shape of which capture the geometry of the Wave Dragon device. The porous layers specifically reflect and transmit wave energy at the reflecting wings of the structure and extract, reflect, and transmit energy at the main body. The degree of reflection, absorption and transmission are dependent on the draft of the wings and body, the freeboard of the main body and the incident wave height and period.

(Norgaard & Andersen, 2012) assessed the shore protection benefits that might be achieved by placing an array of Wave Dragon overtopping type WEC in the relatively near-shore region using the Boussinesq wave model, Mike21 BW. The devices where implemented in essentially the same way as (Beels, et al., 2010) using frequency dependent sponge layers. They also considered lower resolution approximations of the devices as rectangular porous, permeable, breakwater type structures. For these approximate devices the total variable reflection, absorption and transmission characteristics of the detailed device, were averaged across the device. It was found that in the moderate-field (2 km from the devices) the low resolution approximate representation of the WEC provides excellent accuracy when compared to the more accurate geometrical and variable absorption, transmission and reflection, representation of the Wave Dragon device. In the very near-field there was significant local fluctuations in the wave field for the two device representations but the general wave energy distribution was comparable.

The studies that consider WEC arrays located far offshore (Millar, et al., 2007) and (Palha, et al., 2010) do not properly account for or resolve diffraction. The studies that do resolve diffraction about the individual devices do not consider arrays far offshore (Venugopal & Smith, 2007), (Beels, et al., 2010) and (Norgaard & Andersen, 2012). As such the effect of re-distribution of wave energy over larger distances from scattered waves remains unclear. It is also difficult to cross compare these studies to check for consistency as their models and model implementations are often significantly different from each other.

The motivation for this study was to develop an accessible engineering tool for scaling the far-field wave energy deficit in the lee of an array of overtopping type WEC devices, without the; domain size, resolution and
simulation time, restrictions of a time-stepping phase resolving model and the wave diffraction and interference limitations of a spectral model. Radiated waves associated with point absorber type WEC devices are not considered here in order to exclude the wave energy redistribution due to wave radiation, and to avoid the numerically challenging problem of near trapping of waves associated with an array of point absorbers. This allows us to focus on the effect of wave energy redistribution from diffraction.

A number of analytical solutions and modelling schemes have been proposed for describing the diffracted wave field about solid, porous, dissipating type structures. These include the (Pos & Kilner, 1987) application of the mild slope equations using a finite element method, the eigenvalue expansion approach of (Dalrymple & Martin, 1990). (McIver, 2005) presents the mathematically exact solution for a series of permeable or porous breakwater segments using an application of the Green’s theorem to describe the problem in terms of an integral equation.

In this study we use the computationally efficient classical solution of (Penny & Price, 1952) for the diffracted wave field about a semi-infinite breakwater, as a basic building block for the full solution. By making multiple superposition’s and by applying reflection and transmission coefficients, the Penney and Price solution can be used to describe the wave shadow and interference pattern in the lee of a segmented transmitting and reflecting breakwater series. We use this to approximate a single row array of overtopping type WECs because the degree of absorption and transmission across the breakwater can be set to equal to that of the WEC in a similar manner to the (Norgaard & Andersen, 2012) representation of a simple overtopping WEC device. The approach presented here contain some approximations that affect the accuracy of the solution in the region very close to the array and these will be discussed in more detail later. However, because of the comparatively short calculation time this solution provides a useful alternative for assessing the moderate to far field wave energy shadow for a high resolution spectral / directional wave climate. Also the solution does not require familiarity with the advanced mathematical techniques associated with the mathematically exact boundary element method of (McIver, 2005) and does not have the computational time or domain size / boundary limitations of a time stepping Mildslope or Boussinesq type models.

The objectives of this study were twofold: 1. to construct an approximate analytical solution for the wave field about a single row of overtopping type WEC devices that is sufficiently computationally efficient to scale the very far field wave energy shadow, 2. to investigate the effects of wave directional spreading, incident wave spectrum, diffraction and array configuration on the down-wave, wave energy deficit and interference pattern.

2. Wave diffraction solutions at structures

A WEC array does not remove energy uniformly across the whole wave front passing through the array as modelled in (Millar, et al., 2007), (Black, 2007) and, in part, (Palha, et al., 2010). Instead for an array of overtopping devices wave energy is removed from sections of the wave front by the WEC devices. A breakwater segment with width equal to the overtopping device and with a transmission coefficient equal to the devices will remove the same amount of wave energy from a section of the wave front. As such it has been a common practice to represent an overtopping WEC device as a segmented dissipating or porous breakwater type structure. This method was adopted by (Venugopal & Smith, 2007), (Beels, et al., 2010) and (Norgaard & Andersen, 2012).

By adapting the (Sommerfeld, 1886) solution for the diffraction of polarised light about a screen edge, (Penny & Price, 1952) provide an analytical solution for the diffracted wave field about a semi-infinite breakwater for oblique monochromatic waves. (Silvester & Lim, 1968) suggest that a partially reflecting semi-infinite breakwater can be approximated by applying coefficients to the reflected wave component, and the diffracted wave of the reflected wave component which will be discussed in greater detail in section 2.2. Penney and Price also showed that the diffracted wave field about a gap in an infinite breakwater can be approximated with the superposition of the diffracted wave field about two semi-infinite breakwaters. (Kim & Lee, 2010) extend the breakwater gap solution to account for obliquely incident waves by accounting for the phase shift of waves diffracting about the opposing tips. (Penny & Price, 1952) also propose that the diffracted wave field about a detached breakwater can be approximated with a superposition of two semi-infinite breakwater solutions. (Kim & Lee, 2010) show that approximating a breakwater segment with the superposition of two semi-infinite
solutions gives generally good agreement when compared to the mathematically exact boundary element method with the two methods converging with increasing distance from the breakwater segment and being essentially the same at a distance of 3λ. This is due to the reducing influence of the secondary diffracted waves that are not considered by the superposition method. The solutions diverge by up to approximately 10% at closer distances to the breakwater. (Hotta, 1978) proposes that the approximate solution for a detached breakwater segment can also account for partial transmission by a further superposition of a modification of the solution for a gap in a semi-infinite breakwater, which will be discussed further in section 2.5.

For the purpose of this investigation an approximate analytical solution for the diffracted and transmitted wave field about a series of partially reflecting and partially transmitting breakwater segments was formed using a combination of the aforementioned solutions and methods. The resultant solution presented here is used to consider the wave field about a series of overtopping type wave energy converters approximated as partially transmitting / reflecting, breakwater segments.

The analytical solution for diffraction about a reflecting semi-infinite breakwater can be derived using velocity potential theory and the Laplace equation for incompressible fluid and irrotational flow, and is calculated using the Fresnel integrals. The solution has the boundary conditions of no flow at; the bottom boundary and the upstream and downstream faces of the breakwater. The mathematical derivation is well documented (see for example (Penny & Price, 1952) or (Wiegel, 1964)), and as such the description of the full solution that is used in this is study, will start with the usable final solution of Penney and Price.

2.1. A reflecting semi-infinite breakwater

(Penny & Price, 1952) describe the diffracted wave field about thin fully reflecting breakwater structures with the complex function $F(x, y)$. The modulus of $F$ describes the disturbance or diffraction coefficient $K_d$, which is the ratio of the resultant wave height $H_{xy}$ at the Cartesian coordinate point $x, y$ and the incident undisturbed incident wave height $H_0$, so that;

$$K_d = H_{xy}/H_0 = |F(x, y)| \quad (2.1)$$

The center of the Cartesian coordinate varies with the breakwater configuration being considered but is always on the plane of the breakwater $y = 0$. For incident waves arriving obliquely to the plane of the breakwater it is convenient to work in polar coordinates. $F(x, y)$ becomes $F(r, \theta)$ with $r = (x^2 + y^2)^{1/2}$, $\theta = \tan^{-1}(y/x)$ where $r$ is the distance from the center of coordinate system located on the plane of the breakwater or breakwater system, to the calculation point in the domain, and $\theta$ is the angle between the plane of the breakwater from the lee side and the cord that connects the calculation point in the domain with the centre of the coordinate system.

For the case of a thin semi-infinite fully reflecting breakwater, the complex function describing the diffracted wave field is given in (Penny & Price, 1952) as;

$$F(r, \theta) = f(\sigma)I + f(\sigma')R \quad (2.2)$$

where $I$ and $R$ are the incident and reflected planar wave components respectively and are given by;

$$I = \cos(krcos(\theta - \theta)) - isin(krcos(\theta - \theta)) \quad (2.3)$$

$$R = \cos(krcos(\theta + \theta)) - isin(krcos(\theta + \theta)) \quad (2.4)$$

where the polar coordinate system is centred at the breakwater tip so that $r$ is the distance from the breakwater tip to the domain calculation point, $\theta$ is the angle between the plane of the breakwater from the lee side and the line that connects the calculation point in the domain with the breakwater tip. $\theta$ is the angle between the
incident wave direction and the plan of the breakwater from the lee side as shown in figure 2.1. $k$ is the wave number and is found from the dispersion relationship:

$$\omega^2 = gk \tanh(kh)$$  \hspace{1cm} (2.5)

where $\omega$ is the angular frequency given by $2\pi/T$, where $T$ is the wave period, $g$ is the acceleration due to gravity and $h$ is the water depth.

Also in (2.2):

$$f(\sigma) = 1 - f(-\sigma) = \frac{1 + i}{2} \int_{-\infty}^{\sigma} \exp(-\frac{\pi u^2}{2}) du = \left(\frac{1}{2}\right) \left[ (1 + C(\sigma) + S(\sigma)) - i(S(\sigma) - C(\sigma)) \right]$$  \hspace{1cm} (2.6)

Where the upper limits of the integrals are given by:

$$\sigma = \pm 2\sqrt{\left(\frac{kr}{\pi}\right) \sin \frac{1}{2}(\theta - \Theta)}$$  \hspace{1cm} (2.7)

$$\sigma' = \pm (-2\sqrt{\left(\frac{kr}{\pi}\right) \sin \frac{1}{2}(\theta + \Theta)})$$  \hspace{1cm} (2.8)

The sign of $\sigma$ and $\sigma'$ depends on the region S, O and R (Shadow, Open, Reflection) about the breakwater tip that is being calculated as shown in figure 2.1. The sign of $\sigma$ and $\sigma'$ defines the relative phase of the diffracted wave and the presence or absence of the incident and reflected plane wave for that region due to the relationship $f(\sigma) = 1 - f(-\sigma)$.

In (2.6), $C(\sigma)$ and $S(\sigma)$ are the Fresnel integrals. One might consider using the (McCormick & Kraemer, 2002) polynomial approximations for the Fresnel integrals, as shown in (2.9) and (2.10), which provides excellent accuracy and reduces the computational effort by approximately 90%. This is very important when numerous calculations are required to statistically represent directional and spectral wave conditions which will be introduced later. All results presented in this article where calculated using these polynomial approximations.

$$C(\sigma) = -C(-\sigma) \approx \frac{1}{2} + \frac{(1+0.926\sigma) \sin \left(\frac{\sigma^2}{2}\right)}{2+1.792\sigma+3.103\sigma^2} - \frac{\cos \left(\frac{\sigma^2}{2}\right)}{2+4.142\sigma+3.497\sigma^2+6.670\sigma^3} + \epsilon(\sigma)$$  \hspace{1cm} (2.9)

$$S(\sigma) = -S(-\sigma) \approx \frac{1}{2} - \frac{(1+0.926\sigma) \cos \left(\frac{\sigma^2}{2}\right)}{2+1.792\sigma+3.103\sigma^2} - \frac{\sin \left(\frac{\sigma^2}{2}\right)}{2+4.142\sigma+3.497\sigma^2+6.670\sigma^3} + \epsilon(\sigma)$$  \hspace{1cm} (2.10)

with the remainder for both equations $\epsilon(\sigma) \leq 0.002$.
**Figure 2.1** Definition of coordinate system, incident wave direction and calculation regions for a semi-infinite breakwater. S, O and R represent the Shadow, Open, Reflection region. The sign of $\sigma$ and $\sigma'$ to be used for the different calculations regions is shown.

The solution for the diffracted wave field about a semi-infinite breakwater given in (2.2) is composed of four distinct components. The incident plane parallel wave which is present in regions S and R, the reflected plane parallel wave which is present in region R, and the diffracted wave of the incident wave and diffracted wave of the reflected wave, which are both present in all regions.

The diffracted wave of the incident and reflected wave can be isolated by subtracting the incident wave component $I$ and reflected wave component $R$, (if present in that region) from (2.2) giving:

$$
F(r, \theta) = f(\sigma)I + f(\sigma')R - I - R \\
\text{when } 2\pi - \theta < \theta < \theta
$$

**Figure 2.2** Shows the real part of the $F(r, \theta)$ (which gives the water surface elevation $\zeta$) for: (a) the incident and reflected planar wave components, (b) the diffracted wave of the incident wave and the diffracted wave of the reflected wave, and (c) the complete solution which is the sum of (a) and (b) as described by (2.2).

### 2.2. Partially reflecting semi-infinite breakwater

Equation (2.2) satisfies the boundary conditions of zero fluid velocity and describes a fully reflecting breakwater. As shown in figure 2.2 the solution for diffraction at a semi-infinite breakwater can be separated into the components that describe the diffraction of the incident wave and the diffraction of the reflected wave. (Silvester & Lim, 1968) suggest that by treating the reflected and incident wave components independently an approximation of a breakwater that dissipates energy (resulting in partial reflection) can be made by the
application of a reflection coefficient to the components that describe the diffraction of the reflected wave (second term in (2.2)). By doing this the reflected wave and diffracted wave of the reflected wave are reduced proportionally to the desired degree of reflection at the breakwater. With the applied reflection coefficient $c_{\rho}$, the original solution becomes the “simple solution”. This method is also used in (Hotta, 1978), (Ou, et al., 1988) and (Kim & Lee, 2010), and the “simple solution” is given by;

$$F(r, \theta) = f(\sigma)I + c_{\rho}f(\sigma')R$$

with $c_{\rho} = 1$ for total reflection, and $c_{\rho} = 0$ for total absorption.

As (Daemrich & Kohlhase, 1978) show this method provides only an approximate solution as the boundary conditions are no longer fulfilled exactly. (Daemrich & Kohlhase, 1978) found experimentally that the “simple solution” for partial or zero reflection under-predicts the disturbance coefficient close to the lee side of the breakwater. They conclude that although the incident wave is fully absorbed by the breakwater a scattered wave system must still exist which contributes to the resultant wave field in the domain. By taking the mathematically exact solution of diffraction at the end of a guide wall (waves travelling parallel to a breakwater) and the (Mitsui & Murakami, 1967) exact solution of wave diffraction at a wedge, (Daemrich & Kohlhase, 1978) produced a variable weighting factor (dependent on the incident angle and reflection coefficient) that when applied to the second term in (2.2) gives very good accuracy in the down-wave region of the breakwater when compared with experimental data. However, as seen in (Daemrich & Kohlhase, 1978) the two solutions approach each other with increasing distance from the breakwater tip or when $\theta$ gets closer to $\Theta$. As such the simple solution seems a suitable approximation except for the regions very close to the lee side of the breakwater. As this study is primarily concerned with the wave energy shadow in the far down-wave region the approximations associated with the “simple solution” seem acceptable.

2.3. Two semi-infinite partially reflecting breakwater

Penney and Price show that an approximate solution for the diffracted wave field about a gap in an infinite breakwater can be achieved with two superpositions of the semi-infinite solution.

![Figure 2.3](image)

**Figure 2.3** Definition of coordinate system, incident wave direction $\Theta$, tip phase shift $\varepsilon_b$, and regions, for a gap in an infinite breakwater. The subscript $z$ is used as a reference number for the breakwater tip for which the: $I\,R\,r\,\Theta\,\sigma$ and $\sigma^\prime$, terms belong to.

With the partial reflection approximation (described in section 2.2) applied, the Penney and Price solution describing the disturbed wave field about a gap of length $b$ in an infinite breakwater with partial reflection, the
centre of the coordinate system, is now the centre of the gap between the breakwaters (as shown in figure 2.3), is given by:

\[
F(r, \theta) = f(\sigma_1)l_1 + c_\rho f(-\sigma_1')R_1 + f(-\sigma_2)l_2 + c_\rho f(-\sigma_2')R_2 - l_0 \quad \text{region a}
\]

\[
f(-\sigma_1)l_1 + c_\rho f(-\sigma_1')R_1 + f(\sigma_2)l_2 + c_\rho f(\sigma_2')R_2 - l_0 \quad \text{region b}
\]

\[
f(\sigma_1)l_1 + c_\rho f(\sigma_1')R_1 + f(\sigma_2)l_2 + c_\rho f(\sigma_2')R_2 - l_0 \quad \text{region c}
\]

\[
f(-\sigma_1)l_1 + c_\rho f(-\sigma_1')R_1 + f(\sigma_2)l_2 + c_\rho f(-\sigma_2')R_2 - l_0 \quad \text{region d}
\]

\[
f(\sigma_1)l_1 + c_\rho f(-\sigma_1')R_1 + f(\sigma_2)l_2 + c_\rho f(\sigma_2')R_2 - l_0 \quad \text{region e}
\]

\[
f(\sigma_1)l_1 + c_\rho f(-\sigma_1')R_1 + f(\sigma_2)l_2 + c_\rho f(-\sigma_2')R_2 - l_0 \quad \text{region f}
\]

(2.13)

Note that \( l_0 = \cos(kr_0\cos(\theta_0 - \theta_0)) - \sin(kr_0\cos(\theta_0 - \theta_0)) \) has been removed from the solution for all regions. This is because we are superposing two isolated systems which results in the incident planar wave term being added one more time than it should in all regions.

In (2.6) the Fresnel integrals limits for the terms associated breakwater tip \( z \) are now given by:

\[
\sigma_z = 2\sqrt{\frac{kr_2}{\pi}} \sin \frac{1}{2}(\theta_z - \Theta_z) \quad (2.14)
\]

\[
\sigma'_z = -2\sqrt{\frac{kr_2}{\pi}} \sin \frac{1}{2}(\theta_z + \Theta_z) \quad (2.15)
\]

where \( r_2 \) is the distance from the breakwater tip \( z \) to the calculation point in the domain and is given by \( r_1 = ((x + b/2)^2 + y^2)^{0.5} \), \( r_2 = ((x - b/2)^2 + y^2)^{0.5} \).

\( \theta_z \) is the angle between the plane of the breakwater associated with \( z \) (from the lee side) to the line that connects the calculation point in the domain and the breakwater tip \( z \) and is given by; \( \theta_1 = \tan^{-1}(y/(-x - b/2)) \) and \( \theta_2 = \tan^{-1}(y/(x - b/2)) \).

\( \theta_z \) is equal to the incident wave direction \( \Theta \) and \( \theta_1 = \pi - \Theta \), as described in figure 2.3.

(Kim & Lee, 2010) extended the breakwater gap solution to account for oblique waves. This is achieved with a phase shift term \( \epsilon_b \) that is applied to the incident and reflected wave terms. This accounts for the difference in phase of the incident wave driving the system, as it arrives at the opposing breakwater tips relative to the centre of the breakwater gap due to the difference in path length as shown in figure 2.3. The phase shift term \( \epsilon_b \) is given by;

\[
\epsilon_b = \pm k(b/2)\cos(\Theta_0) \quad (2.16)
\]

where \( b \) is gap length and \( \epsilon_b \) is the phase shift as shown in figure 2.3. The sign of \( \epsilon_b \) is given in table 2.1.

**Table 2.1** The sign of \( \epsilon_b \) used with the terms associated with breakwater tip \( z \)

<table>
<thead>
<tr>
<th>Sign of ( \epsilon_b )</th>
<th>( \Theta_0 \leq 90 )</th>
<th>( \Theta_0 &gt; 90 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z = 1 )</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>( z = 2 )</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

For oblique waves with the phase shift term applied, the incident and reflected wave terms associated with the breakwater tip \( z \) become;

\[
l_z = \cos(kr_2\cos(\theta_z - \Theta_z) + \epsilon_b) - \sin(kr_2\cos(\theta_z - \Theta_z) + \epsilon_b) \quad (2.17)
\]

\[
R_z = \cos(kr_2\cos(\theta_z + \Theta_z) + \epsilon_b) - \sin(kr_2\cos(\theta_z + \Theta_z) + \epsilon_b) \quad (2.18)
\]
The incident planar wave which all other parts of the solution are considered to be relative to, has its coordinate
system centred at the middle of the breakwater gap and is given by:

\[ I_0 = \cos(kr_0 \cos(\theta - \Theta)) - is\sin(kr_0 \cos(\theta - \Theta)) \] (2.19)

This solution is an approximation because the boundary conditions are not completely fulfilled due to the
applied reflection coefficient as discussed in section 2.2. An additional approximation is present when more than
one semi-infinite breakwater solution is superposed. This is because the 2nd order effects of the diffracted wave
from one breakwater interacting with the other breakwater are not considered. However, as Penney and Price
show the approximation associated with the superposition is within 2% of the potential maximum of a full
solution provided the gap length is greater than 1\(\lambda\). A comparison of the solution for diffraction about a gap in
an infinite breakwater is made with a comparison against experimental data in section 3.

2.4 A detached partially reflecting breakwater segment

(Penny & Price, 1952) also propose that an approximate solution for the diffracted wave field about a
detached breakwater segment can be constructed with two superpositions of the semi-infinite solution given in
section 2.1. This is also used in (Hotta, 1978) and (Kim, et al., 2011).

![Figure 2.4 Definition of coordinate system, incident wave direction, tip phase shift and regions for a detached
breakwater segment. The subscript \(z\) is used as a reference number for the breakwater tip](image)

The approximate complex function describing the wave field at a point \(r_0, \theta\) about a detached breakwater
segment with partial transmission, where the centre of the coordinate system is now the geometrical centre of
the breakwater segment, is given by:

\[
F(r, \theta) = \begin{cases} 
    f(-\sigma_1)l_1 + c_p f(-\sigma'_1)R_1 + f(\sigma_2)l_2 + c_p f(-\sigma'_2)R_2 & \text{region } a \\
    f(\sigma_1)l_1 + c_p f(-\sigma'_1)R_1 + f(-\sigma_2)l_2 + c_p f(-\sigma'_2)R_2 & \text{region } b \\
    f(-\sigma_1)l_1 + c_p f(-\sigma'_1)R_1 + f(-\sigma_2)l_2 + c_p f(-\sigma'_2)R_2 & \text{region } c \\
    f(\sigma_1)l_1 + c_p f(\sigma'_1)R_1 + f(\sigma_2)l_2 + c_p f(\sigma'_2)R_2 - l_0 - R_0 & \text{region } d \\
    f(\sigma_1)l_1 + c_p f(\sigma'_1)R_1 + f(\sigma_2)l_2 + c_p f(\sigma'_2)R_2 - l_0 - R_0 & \text{region } e \\
    f(\sigma_1)l_1 + c_p f(\sigma'_1)R_1 + f(\sigma_2)l_2 + c_p f(\sigma'_2)R_2 - l_0 - R_0 & \text{region } f
\end{cases}
\] (2.20)
where \( \sigma_z \) and \( \sigma'_z \) are given by (2.14) and (2.15) respectively, where \( r_z \) is the distance from the breakwater tip \( z \) to the calculation point in the domain given by \( r_1 = (x + B/2)^2 + y^2)^{0.5} \) and \( r_2 = ((B/2 - x)^2 + y^2)^{0.5} \).

\( \theta_z \) is the angle between the plane of the breakwater (from the lee side) to the line that connects the calculation point in the domain and the break water tip \( z \) and is given by; \( \theta_1 = \tan^{-1}(y/(x + B/2)) \) and \( \theta_2 = \tan^{-1}(y/(B/2 - x)) \).

\( \theta_1 \) is equal to the incident wave direction \( \theta_0 \) and \( \theta_2 = \pi - \theta_0 \) as described in figure 2.4.

When incident waves are oblique a phase shift term \( \epsilon_B \) (as shown in figure 2.4) is applied to account for the relative phase shift of the incident wave that drives the system, arriving at the opposing tips of the breakwater. The phase shift is considered relative to the centre of the breakwater segment \( r_0 = 0 \), and is given by;

\[
\epsilon_B = \pm k(B/2)\cos(\theta_0) \tag{2.21}
\]

where \( B \) is breakwater segment length and the sign of \( \epsilon_B \) is given in table 2.2.

**Table. 2.2** The sign of \( \epsilon_B \) for a region relative to breakwater tip \( z \)

<table>
<thead>
<tr>
<th>Sign of ( \epsilon_B )</th>
<th>( \theta_0 \leq 90 )</th>
<th>( \theta_0 &gt; 90 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z = 1 )</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>( z = 2 )</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

For oblique waves with the phase shift term applied the incident and reflected waves associated with the breakwater tip \( z \) become;

\[
I_z = \cos(kr_z \cos(\theta_z - \theta_2) + \epsilon_B) - \sin(kr_z \cos(\theta_z - \theta_2) + \epsilon_B) \tag{2.22}
\]

\[
R_z = \cos(kr_z \cos(\theta_z + \theta_2) + \epsilon_B) - \sin(kr_z \cos(\theta_z + \theta_2) + \epsilon_B) \tag{2.23}
\]

The incident planar wave \( I_0 \) which all other parts of the solution are considered to be relative to, now has its coordinate system centred at the middle of the breakwater segment and is given again by (2.3);

---

**Figure 2.5** Superposition steps to create an approximate analytical solution for a disturbed field about an ideal zero reflection zero transmission overtopping type wave energy converter which we approximate as a detached breakwater segment under unidirectional monochromatic waves conditions. (a) semi-infinite solution with opening at the top of the domain, (b) semi-infinite solution with opening at the bottom of the domain, (c) is the sum of (a) and (b) which gives the approximate single WEC solution.
The solution for the detached partially reflecting breakwater is an approximation because of the non-exact boundary conditions associated with the reflection coefficient using the “simple solution” as discussed in section 2.2. As also discussed in section 2.2, (Daemrich & Kohlhase, 1978) provided a variable weighting factor, dependent on the incident wave angle and reflection coefficient, that accurately describes the down-wave disturbance coefficient when the breakwater is partially / non reflecting. To assess the potential inaccuracy of the approximations associated with the “simple solution”, the disturbed wave field described by Daemrich and Kohlhase’s “weighted solution” was compared to Silvester and Lim’s “simple solution”. The greatest difference between the two solutions was found along the line that bisects the breakwater segment and when the reflection coefficient is zero. As seen in figure 2.6 for regular waves the difference between the two methods is somewhat significant in the immediate lee of the breakwater with a difference of up to 15% of the incident wave height. However this difference decays rapidly with increasing distance from the breakwater and is reduced to 2% of the incident wave height within a distance of 15\( \lambda \). For irregular incident waves (which will be introduced in section 2.7 and 2.8) with a cos power spreading factor of \( m = 50 \) and a JONSWAP peak period of \( T_p = 8 \) the difference is significantly less being 10% of the incident wave height in the immediate lee of the breakwater and decaying very rapidly to 1% at a distance of 9\( \lambda_p \). Past a distance of \( 30\lambda_p \) there is effectively no difference between the “weighted” and “simple” solutions for irregular waves. As the primary focus of this study is to assess the moderate to far field wave energy shadow for irregular waves the simple solution seems a reasonable approximation. In the immediate vicinity of the breakwater the simple solution will underestimate the disturbance coefficient.

Figure 2.6 The maximum difference between the disturbance coefficient for the “weighted” and ”simple” solution, normalised by the incident disturbance coefficient in the lee of a single detached breakwater segment of length 150 m along the line that bisects the device, for regular waves of period \( T = 8 \) and for irregular waves of peak period \( T_p = 8 \) and Cos power spreading factor \( m = 50 \).

Another approximation exists in that the complete system is the superposition of two isolated system. As such the diffracted waves emanating from one breakwater tip will not register that the breakwater section is no longer present past the opposing tip and as such will not perform a secondary diffraction. This results in a small discontinuity along the plane of the breakwater (a step of approximately 2% of the incident wave height for the case of zero reflection) at the open regions because the diffracted wave of the incident wave and the diffracted wave of the reflected wave do not undergo a secondary diffraction which would cause them to overlap and smooth the wave field in this region as they should. This produces a small error in the region close to the breakwater. However, neglecting these secondary diffracted waves should have little effect in the regions away from the immediate vicinity of the breakwater because these waves will be very small initially and due to their radial nature will decreases rapidly with increasing distance from the breakwater. This is confirmed in (Kim & Lee, 2010) who show that when the down-wave distances from the breakwater is greater than 3\( \lambda \) the approximate solution essentially converges with the mathematically exact boundary element method. As the focus of this study is the wave energy shadow in the lee of a WEC array for the moderate too far down-wave region, this approximation seems acceptable.
2.5. A partially reflecting/transmitting breakwater segment

(Hotta, 1978) proposes an approximate solution for a detached breakwater segment with partial transmission. This is achieved by superposing a further breakwater system onto the solution of a partially reflecting breakwater segment as given in (2.20). Hotta took the solution for the gap in the infinite breakwater given in (2.13), with a reflection coefficient \( c_p = 0 \) and applied a transmission coefficient \( c_t \) (0 for no transmission and 1 for full transmission) to all terms related to the incident wave and diffracted waves of the incident wave. Also the incident planar wave is removed from the up wave region. In effect this gives a solution for a forward traveling wave generated in a breakwater gap. When this is superposed with the solution of a detached breakwater segment with length equal to the gap, this component represents the wave that has transmitted through the detached breakwater segment. The wave that transmits through the breakwater segment has an initial height of \( c_H_0 \), and a wave front length equal to the length of the breakwater segment. Taking (2.13) with reflection coefficient \( c_p = 0 \), removing \( l_0 \) from the up-wave region and applying \( c_t \) to all terms, we have;

\[
F(r, \theta) = c_t(f(\sigma_1)I_1 + f(\sigma_2)I_2 - l_0) \text{ region a} \\
+ c_t(f(-\sigma_1)I_1 + f(\sigma_2)I_2 - l_0) \text{ region b} \\
+ c_t(f(\sigma_1)I_1 + f(\sigma_2)I_2 - 2l_0) \text{ region c} \\
+ c_t(f(\sigma_1)I_1 + f(\sigma_2)I_2 - 2l_0) \text{ region d} \\
+ c_t(f(\sigma_1)I_1 + f(\sigma_2)I_2 - 2l_0) \text{ region e} \\
+ c_t(f(\sigma_1)I_1 + f(\sigma_2)I_2 - 2l_0) \text{ region f}
\]

Superposing the solution of the wave transmitted through the structure given in (2.24) with the solution for the partially reflecting breakwater segment given in (2.20). We arrive at the approximate solution for the wave field about a detached breakwater segment with partial reflection and transmission;

\[
F(r, \theta) = f(\sigma_1)I_1 + c_pf(-\sigma_1'R_1 + f(\sigma_2)I_2 + c_pf(-\sigma_2'R_2 + c_t(f(\sigma_1)I_1 + f(\sigma_2)I_2 - l_0) \text{ region a} \\
f(\sigma_1)I_1 + c_pf(-\sigma_1'R_1 + f(-\sigma_2)I_2 + c_pf(-\sigma_2'R_2 + c_t(f(-\sigma_1)I_1 + f(-\sigma_2)I_2 - l_0) \text{ region b} \\
f(-\sigma_1)I_1 + c_pf(-\sigma_1'R_1 + f(\sigma_2)I_2 + c_pf(-\sigma_2'R_2 + c_t(f(\sigma_1)I_1 + f(\sigma_2)I_2 - l_0) \text{ region c} \\
f(\sigma_1)I_1 + c_pf(-\sigma_1'R_1 + f(\sigma_2)I_2 + c_pf(-\sigma_2'R_2 + c_t(f(\sigma_1)I_1 + f(\sigma_2)I_2 - l_0) - l_0 - R_0 \text{ region d} \\
f(-\sigma_1)I_1 + c_pf(-\sigma_1'R_1 + f(\sigma_2)I_2 + c_pf(-\sigma_2'R_2 + c_t(f(-\sigma_1)I_1 + f(-\sigma_2)I_2 - l_0) - l_0 - R_0 \text{ region e} \\
f(\sigma_1)I_1 + c_pf(-\sigma_1'R_1 + f(\sigma_2)I_2 + c_pf(-\sigma_2'R_2 + c_t(f(\sigma_1)I_1 + f(\sigma_2)I_2 - l_0) - l_0 - R_0 \text{ region f}
\]

Where the regions and breakwater tip reference \( z \) are the same as in figure 2.4.

(Ou, et al., 1988) obtain wave basin experimental data for the diffracted and refracted wave field about a partially transmitting detached breakwater segment with a mildly varying bed gradient. For the test cases considered generally good agreement was found between the approximate solution of (Hotta, 1978) and experimental data. However, it should be noted that only a single transmission coefficient of \( c_t = 0.3 \) was considered and that the approximate solution being tested has the extension for refraction. As such, further validation would be beneficial when additional data becomes available. In all but one case in the results section we consider WEC devices of perfect energy conversion i.e. zero reflection and zero transmission, to assess the worst case scenario of wave energy shadowing from an overtopping type WEC array. As such the method of partial transmission described above and the associated approximations are only relevant to the one result presented in figure 2.18.

2.6. A series of detached partially transmitting/reflecting breakwater segments

Finally an approximate solution for the diffracted wave field about a segmented breakwater series is constructed using multiple superpositions of the detached breakwater solution given in section 2.5.
Figure 2.7 Definition of coordinate system, incident wave direction, tip phase shift $\varepsilon_B$ and breakwater phase shift $\mu_n$, for a series of detached breakwater segments. The subscript $z$ is used as a reference number for the breakwater tip, $n$ denotes the breakwater segment number.

By removing the incident wave described by (2.3) across the whole domain for the breakwater segment solution given in (2.25) we obtain the “temporary solution” shown in figure 2.8(a) and is defined as;

$$
\tau(r, \theta) = F(r, \theta) - I_0
$$

(2.26)

The temporary solution can now be superposed multiple times without the incident planar wave term stacking up. This is shown in figure 2.8(b) and the solution for this is given by;

$$
\tau_{tot}(r, \theta) = \sum_{n=1}^{n} \tau(r, \theta)_n
$$

(2.27)

where $n$ is the number of breakwater segments and;

$$
I_{z,n} = \cos(kr_{z,n}\cos(\theta_{z,n} - \theta_{x,n}) + \varepsilon_B + \mu_n) - i\sin(kr_{z,n}\cos(\theta_{z,n} - \theta_{x,n}) + \varepsilon_B + \mu_n)
$$

(2.28)

$$
R_{z,n} = \cos(kr_{z,n}\cos(\theta_{z,n} + \theta_{x,n}) + \varepsilon_B + \mu_n) - i\sin(kr_{z,n}\cos(\theta_{z,n} + \theta_{x,n}) + \varepsilon_B + \mu_n)
$$

(2.29)

Using the centre of the first breakwater of the series as the centre of the coordinate system, $r_{z,n}$ is the distance from the breakwater tip $z$ on the breakwater segment $n$ to the calculation point in the domain and is given by;

$r_{1,n} = ((x + B/2 - (n - 1)(B + b))^2 + y^2)^{0.5}$ and $r_{2,n} = (((n - 1)(B + b) - x - B/2)^2 + y^2)^{0.5}$.

$\theta_x$ is the angle between the plane of the breakwater $n$ (from the lee side) to the line that connects the calculation point in the domain and the breakwater tip $z$ and is given by $\theta_{1,n} = \tan^{-1}(y/(x + B/2 - (n - 1)(B + b)))$ and, $\theta_{2,n} = \tan^{-1}(y/((n - 1)(B + b) - x - B/2))$.

$\theta_{1,n}$ is equal to the incident wave direction $\theta_0$ and $\theta_{z,n} = \pi - \theta_0$, as described in figure 2.7.

In (2.28) and (2.29) $\varepsilon_B$ is the again the phase shift term for the tip $z$ relative to the centre of the breakwater segment $n$ and is given by;

$$
\varepsilon_B = \pm k(B/2)\cos(\theta_0)
$$

(2.30)

where $B$ is breakwater segment length and the sign of $\varepsilon_B$ is given in table 2.3

<table>
<thead>
<tr>
<th>Sign of $\varepsilon_B$</th>
<th>$\theta_0 \leq 90$</th>
<th>$\theta_0 &gt; 90$</th>
</tr>
</thead>
</table>
\[
\begin{array}{c|c|c}
  z = 1 & - & + \\
  z = 2 & + & - \\
\end{array}
\]

Also in (2.28) and (2.29), the term \( \mu_n \) is applied in the same fashion as the term \( \varepsilon_0 \). This accounts for the phase shift of the incident and reflected waves at the other breakwater segments of the series (relative to the first breakwater in the series) as shown in figure 2.7, so that the phase of the incident wave arriving at each breakwater tip is considered relative to the phase of the incident wave at the centre of the first breakwater segment of the series.

\[
\mu_n = \pm k(n - 1)(b + B)\cos(\Theta_0) \tag{2.31}
\]

where \( b \) is the length between the centre of each breakwater segment and the sign of \( \mu_n \) is positive when \( \Theta \leq 90 \) and negative when \( \Theta > 90 \). This assumes a series of breakwaters segments with uniform separation distances between each segment and uniform lengths for each breakwater segment.

By re-applying the incident wave given in (2.3) to the solution for the multiple superpositions of the temporary solution given in (2.27), we get the approximate solution for the total wave field about a series of breakwater segments given by:

\[
F(r, \theta)_{tot} = I_0 + \sum_{n=1}^{n} \tau(r, \theta)_n \tag{2.32}
\]

Finally the disturbance coefficient for an incident monochromatic wave of period \( T \), in water depth \( h \), and incident angle relative to the plane of the array \( \Theta_o \), can be obtained by taking the modulus of the complex function \( F \), and the final result is as shown in figure 2.8(c)

Again this solution is an approximation for the same reason discussed in section 2.2, 2.4 and 2.5. Also the influence of neighbouring segments on the waves diffracted by each segment is not accounted for. This approximation is assessed in (McIver, 2005) and is shown to have little significance.

The solution for a segmented breakwater series appears to be complicated with multiple values for \( r_z \) and \( \theta_z \) and many different “regions” to define (with increasing complexity for additional segments). However practically the solution is not complicated to implement and the end result can be achieved in the following steps:

1.) Calculate the domain about a single breakwater segment with (2.25)
2.) Remove the incident wave field \( I_0 \), in (2.3), uniformly across the domain calculated in step 1 to create the “temporary solution” given by (2.26)
3.) Repeat steps 1 and 2 for each segment of the series accounting for the phase shift of the incident wave arriving at each segment due to its location relative to the first segment of the series.
4.) Superpose the “temporary solution” for each breakwater segments in the location in the domain defined by the relative phase shift that is applied.
5.) Re-apply the incident wave field \( I_0 \), in (2.3), relative to the first breakwater segment of the series uniformly across the domain calculated in step 4, to get the final output
Figure 2.8 Superposition steps to for the approximate solution for the field about an array of overtopping type WEC converters approximated as breakwater segments, under unidirectional monochromatic wave conditions. (a) gives the temporary solution for single WEC, (b) gives multiple superpositions of the temporary solution shown in (a) to account for the diffracted waves (with relative phase shift) about each WEC in the array, (c) gives the solution shown in (b) with the incident wave field re-applied to give the complete approximate solution for the array.

From this point forward we will refer to the approximate analytical solution for a segmented series of partially transmitting, partially reflecting breakwaters as the “approximate analytical solution for a WEC array” because we are approximating an array of overtopping WEC device as a series of thin dissipative breakwater segments of length equal to the WEC device with the same transmission and reflection coefficients as the WEC devices that we are considering.

2.7. Unidirectional spectral wave

At this stage the incident wave field is both unidirectional and monochromatic. The wavelength is discreet creating a strong periodic interference pattern, with maximas when the difference in path length between two breakwater tips and a point in the domain is an integer number of wave lengths. Similarly a minimum is formed when the difference in path length is a half integer number of wave lengths. A realistic wave climate is not monochromatic and the JONSWAP period spectrum $S(T)$ is one method of statistically describing a spectral wave climate. The JONSWAP spectrum can be expressed as:

$$S(T) = \frac{\alpha g^2 y^5}{(2\pi)^4} \exp\left[ -\frac{5}{4} \left( \frac{T_p}{T} \right)^{-4} \right] y^q$$  \hspace{1cm} (2.33)

where $T$ is the period, $T_p$ is the peak period of the spectrum, $\alpha$ is the Phillip’s constant ($\alpha = 0.0081$), $y$ is the peak enhancement factor and $q = \exp[-((1/T) - (1/T_p))^2 T_p^{-2} / 2\sigma^2]$, with $\sigma = 0.07$ when $T \geq T_p$ and $\sigma = 0.09$ when $T < T_p$.

The period spectrum function $S(T)$ can be scaled to a probability density function $P(T)$, by:

$$P(T) = S(T)/\int_0^{\infty} S(T)dT$$  \hspace{1cm} (2.34)

where $\frac{1}{\int_0^{\infty} S(T)dT}$ is the normalising coefficient, and $\int_0^{\infty} P(T)dT = 1$.

The JONSWAP spectrum can be used to find the statistical mean diffracted wave field about the array under spectral wave conditions in the following manner. The diffracted wave energy field for each integer period component $T[s]$ of the period spectrum is multiplied by the respective $P(T)$ value for that period to give the scaled diffracted wave energy field for that particular value of $T[s]$. As $\int_0^{\infty} P(T)dT = 1$, each scaled diffracted
wave energy field for each period component of the spectrum can be linearly superposed to form the mean diffracted wave field for the full JONSWAP period spectrum. A spectral resolution of one second was chosen because sensitivity studies showed that using a higher period resolution only provides a very small amount of additional smoothing and has no appreciable effect on the trend of energy redistribution.

By simulating a spectrum of wave frequencies, waves will have a range of discreet wavelengths. The locations and area of the maxima and minima interference pockets will change for each period component of the imposed spectrum. This causes the interference pattern to smooth so that the maxima and minima are closer to the local mean disturbance coefficient.

Figure 2.9 (a) Disturbance coefficient $K_d$ in the lee of five WEC devices with width 150m, a separation distance of 600m and no reflection or transmission (total absorption). Incident waves are unidirectional and spectral with a JONSWAP probability density described by (2.32) with $\gamma = 3.3$ and $T_p = 8[s]$.

2.8 Multi-directional spectral wave

A complete representation of the mean wave diffraction patterns under field conditions is achieved when the directional spread is convolved with the frequency distribution given in section 2.7. The statistical distribution of wave directions for a random wave climate can be described, for example, with the Mitsuyasu type (Goda, 2010), directional spread, which is given in a form independent of frequency as;

$$G(\theta) = \cos^2 s \left( \frac{\theta - \theta_0}{\pi} \right)$$ (2.35)

where $\theta_0$ is the mean wave direction and $s$ is the spreading parameter.

Alternatively, in the SWAN wave model the directional spread distribution is described by;

$$G(\theta) = \cos^m(\theta - \theta_0)$$ (2.36)

where $\theta_0$ is the mean wave direction and $m$ is the spreading parameter.

Implementation of either of these or similar schemes is essentially the same. In section 3 the spreading parameter $s$ is used to match experimental parameters of (Yu, et al., 2000). At all other times spreading parameter $m$ is used for easy comparison against the wave shadow predictions in (Millar, et al., 2007) and (Black, 2007).

The spread function $G(\theta)$ can be scaled to a probability density function, by;

$$D(\theta) = G(\theta) / \int_{-\pi}^{\pi} G(\theta) d\theta$$ (2.37)

where; $1/\int_{-\pi}^{\pi} G(\theta) \theta$ is the normalising coefficient.

The analytical solution was used to calculate the diffracted wave energy field for each integer degree of the direction spread. Multiplying this by the directional spread probability function $D(\theta)$ for that angle gives the
scaled diffracted wave energy field for that angle component. As \( \int_{\pi}^{\pi} D(\Theta)d\Theta = 1 \), each scaled diffracted wave energy field for each angle can be linearly superposed to give the mean diffracted wave field for the full directional spread. A directional resolution of one second was chosen because sensitivity studies showed that no appreciable difference is achieved using a higher resolution.

Small regular fluctuations in the disturbance coefficient are seen in figure 2.10. These are the result of the regular layout of the array and discreet representation of the random wave field. Both of these factors will act to produce small pockets of enhanced constructive / destructive interference as a number of diffracted waves come into or out of phase at specific locations. These fluctuations can be minimised further by increasing the directional or spectral resolution but this requires more computational effort and does not affect the trend of the disturbance coefficient. Conversely the computational effort can be reduced by lowering the spectral and directional resolution but this will result in stronger fluctuations. This is also seen in the studies of (Millar, et al., 2007) and (Beels, et al., 2010) where restrictions of computational effort requires a course representations of directional spread and frequency spectrum. The general trend of the disturbance coefficient with distance from the array is maintained at a lower directional and spectral resolution but as the solution is relatively quick to compute we have chosen a relatively fine resolution. A single wave height is used for all calculations as diffraction is not a function of wave height for linear theory. For a realistic device the reflection and transmission coefficients would be a function of wave height. As such a wave height distribution would need to be considered in the similar manor as the frequency spectrum and directional spread when the wave height dependence of the reflection and transmission coefficient for a device is determined.

3. Comparison of approximate analytical solution to experimental data

(Putnam & Arthur, 1948) found good agreement between wave basin experimental data and the (Penny & Price, 1952) solution for oblique wave diffraction about a semi-infinite breakwater so no further validation is given here. As discussed in section 2.5, (Ou, et al., 1988) compare the approximate solution for the detached partially transmitting breakwater segment with experimental wave basin data and found generally good agreement. The approximate analytical solution can be further validated with a comparison against experimental data for the diffracted wave field about a gap in an infinite breakwater for directional spectral waves as presented in (Yu, et al., 2000). This in part tests the method of superposing independent diffracted wave fields to form the total diffracted wave field between two neighbouring devices and also the superposition method for statistically representing directional and spectral wave states. The experimental layout and the transects for which the experimental and analytical values of \( K_d \) are compared along is shown in figure 2.11. The approximate analytical solution for the breakwater gap as given in section 2.3, with the extensions for spectral and directional waves as given in sections 2.7 and 2.8 was calculated with full reflection (\( c_r = 1 \)) and zero transmission (\( c_t = 0 \)), with a period spectrum and directional spread to match the experimental parameters as described in table 2.4.
Figure 2.11 Layout of wave basin for the breakwater gap experiments after (Yu, et al., 2000). Transects of wave height measurements used for comparison at \( x = 0, y = 7.8 \) & \( y = 16 \). The still water depth was 0.4m throughout the basin.

Table 2.4 Wave parameters for tests in (Yu, et al., 2000) experiments on wave diffraction through a breakwater gap. \( H_s \) is significant wave height, \( T_p \) is peak period, \( s \) is directional spreading parameter describe by (2.35), \( \lambda_p \) is peak wavelength, and \( b \) is gap width, the significant wave height was 0.05 [m], mean direction was \( \theta_0 = 90 \) which is the angle relative to the plane of the breakwater gap (x-axis) and JONSWAP peak enhancement parameter was 4 for all cases.

<table>
<thead>
<tr>
<th>Reference</th>
<th>( T_p (s) )</th>
<th>( S )</th>
<th>( \lambda_p (m) )</th>
<th>( b (m) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a-s6</td>
<td>1.2</td>
<td>6</td>
<td>1.96</td>
<td>3.9</td>
</tr>
<tr>
<td>a-s19</td>
<td>1.2</td>
<td>19</td>
<td>1.96</td>
<td>3.9</td>
</tr>
<tr>
<td>b-s6</td>
<td>0.92</td>
<td>6</td>
<td>1.31</td>
<td>3.9</td>
</tr>
<tr>
<td>b-s19</td>
<td>0.92</td>
<td>19</td>
<td>1.31</td>
<td>3.9</td>
</tr>
<tr>
<td>c-s6</td>
<td>1.2</td>
<td>6</td>
<td>1.96</td>
<td>7.8</td>
</tr>
<tr>
<td>c-s19</td>
<td>1.2</td>
<td>19</td>
<td>1.96</td>
<td>7.8</td>
</tr>
<tr>
<td>d-s6</td>
<td>0.92</td>
<td>6</td>
<td>1.31</td>
<td>7.8</td>
</tr>
<tr>
<td>d-s19</td>
<td>0.92</td>
<td>19</td>
<td>1.31</td>
<td>7.8</td>
</tr>
</tbody>
</table>
Figure 2.12 Comparison of the experimental data (markers) and analytical solution (line) for incident waves with a mean direction of 90° to the breakwater plane. The incident wave parameters for each case are given in table 2.1. Individual analytical/experimental runs are grouped by the directional spread parameter s. (a-b) gives $K_d$ along the transect $x = 0$, (c-d) transect $y = 7.8 \text{(m)}$ and (e-f) transect $y = 16 \text{(m)}$.

The standard deviation of the difference between the measured and predicted $K_d$ (at all wave gauge measured locations for the central, 7.8 [m], and 16 [m], transects combined) was; $\sigma = 0.025$ when $\theta_0 = 90^\circ$. The same validation series was done for waves with mean incident angle $\theta_0 = 45^\circ$ which gave a higher standard deviation of $\sigma = 0.097$. The graphical comparison for analytical versus experimental for $\theta_0 = 45^\circ$ has been omitted for conciseness. There is a moderate deviation between the measured and predicted diffraction coefficient at a small number of measurement locations but overall the approximate analytical solution describes the diffracted wave field well.

4. Results

A series of tests were performed to investigate how: diffraction, wave directional spreading, frequency distribution, device separation length and transmission coefficient, affect the re-distribution of wave energy with distance down wave of an array. This was performed by independently varying the parameters affecting diffraction and wave directional spreading about a set of base parameters and the wave field re-calculated for each variation made. The base incident wave parameters were defined as: peak period - $T_p = 8 \text{[s]}$, spread parameter - $m = 50$, JONSWAP peak enhancement - $\gamma = 3.3$. The base parameters for the WEC array and layout were: WEC length $B = 150 \text{[m]}$, tip to tip device separation distance $b = 600 \text{[m]}$, 0% transmission and 0% reflection (perfect absorption for worst case scenario). The length of the domain was set at 25[km] as this is a relevant down wave distance from a WEC array to the coast. The depth was set at a uniform 50[m] as refraction is not yet considered in this initial study. This produces a wavelength of 100m at the peak of the base peak period value $T_p = 8 \text{[s]}$. In figures 13(b), 15(b), 16 and 18, the disturbed wave height predicted by the analytical solution for the different parameter permutations, is given along the central transect, which is defined as a transect perpendicular to the array that bisects the central WEC device of the series. This is because in the absences of refraction and for an incident wave field with a mean wave direction perpendicular to the plane of
the array, the central transect will generally show the maximum reduction in wave height and therefore the greatest potential impact.

4.1. Directional spreading

As shown in previous studies the broadness of the directional spread has a strong effect on the wave energy shadow because it controls the rate of dispersion of the wave energy deficit as seen in figures 13(b) and 14(a-d). For a very broad directional spread (i.e. $m = 2$), the energy deficit is spread over a very wide area so that the disturbance coefficient is almost unity at a distance of 25km. In contrast, for a very narrow directional spread (i.e. $m = 800$) a significant maximum reduction in wave height of almost 20% is predicted 25km away from the array. The disturbance coefficient profile at the central transect generally shows a steady recovery with increasing distance from the array with the exception of a very narrow directional spread, which will be discussed future in section 5. However, significant fluctuations exist in the moderate to near field due to constructive and destructive interference from the diffracted waves. The strength of these fluctuations depends primarily on the breadth of the directional spread. A single discreet incident direction will result in a complex maxima and minima pattern as shown in figure 2.9. As directional spreading is applied the locations of these maxima and minima will shift with each discreet direction component of the spread distribution. This results in a smoothing of the interference pattern as the maxima/minima for one discrete direction overlaps the minima/maxima for another discrete direction, thus partially cancelling each other. The narrower the spread the smaller the range of spatial shift of the maxima and minima locations resulting in less smoothing of the interference pattern, and vice versa. In figures 13 (b) and 14 (a-d) there exist some regular fluctuations in the moderate to far field, the reason for this is discussed in section 2.7 and 2.8. These fluctuations can be reduced with a higher directional and spectral resolution if required.

![Figure 2.13](image)

**Figure 2.13** (a) Wave directional spread probability density (b) $K_d$ profile along the central transect for an array for the same directional spread parameters for directional spread parameter $m = 2,10,100 & 800$. 


4.2. Effect of peak period

As seen in figure 2.15(b), varying the peak period $T_p$ has very little effect on the far field wave energy shadow, with the disturbance coefficients at 25km being within 1% of each other for the different peak period values considered. It should be noted that the length of each plot in figure 2.15(b) differs because a distance of 25km is reached in fewer wavelengths when the peak period is longer and vice versa. In the near field there is an appreciable difference in $K_d$, due to the interference of diffracted waves. It is seen that a spectrum with a longer peak period produces maxima and minima interference with greater magnitude in the near field. This is because the area of the domain in which two diffracted waves come into phase to produce a maxima (or vice versa for a minima), is larger when the wavelength is longer. When the wave direction changes (for each discreet component of the directional spread) the location of the interference pockets shift. Broader interference pockets for longer wavelengths are more likely to overlap for each discreet directional superposition than for narrower interference pockets for shorter wavelengths. The greater the overlap of interference pockets with the same polarity the stronger the interference pocket will be when all directions of the spread and frequencies of the spectrum are superposed. Similarly, significant maxima and minima pockets are only present in the near field because as the distance from the array increases so does the length of the shift in the location of the interference pocket between each component of the directional spread. As the distance from the array increases, the amount of overlapping of interference pockets decreases for each superposition. This results in a smoothing of the overall interference across all directions of the spread and frequencies of the spectrum, with increasing distance from the array and vice versa. For a realistic device the power take-off will be strongly dependent on the incident wave period and so the peak period of the device will likely have a significant influence on the wave energy shadow.
4.3. Effect of gap length

Varying the gap length between the individual WEC devices whilst maintaining the same energy absorption at each device and the same total energy absorption across the array, requires changing the length of the total WEC array. This changes the transmission ratio of the array as a whole with a shorter gap length resulting in a lower array transmission to absorption ratio and vice versa as seen in figure 2.16. Consequently there is a greater reduction in the disturbance coefficient initially for an array with shorter gap lengths as the wave energy deficit is more concentrated. However, this initial enhanced reduction is countered with increasing distance from the array as the spread of the un-attenuated waves from the open regions (to the sides of the array) requires a shorter distance to reach the central transect and therefore acts to reduce the disturbance coefficient closer to the array (compared to a wider array).

Figure 2.15 (a) JONSWAP period spectrum probability density (b) $K_d$ profile along the central transect for an array for peak period values of $T_p = 8,10,12 \& 14 \,[s]$.

Figure 2.16 $K_d$ profile along the central transect for an array with WEC tip to tip separation lengths of $b = 200,400,600 \& 800 \,[m]$. WEC device length is fixed at the base value of 150[m].
Figure 2.17 Horizontal spatial distribution of $K_d$ in the lee of an array with WEC separation length of: (a) $b = 200$, (b) $b = 400$, (c) $b = 600$ and (d) $b = 800$ [m].

4.4. Effect of transmission coefficient

The transmission and reflection coefficient of the device defines the energy extraction by the WEC device. A higher transmission coefficient permits more wave energy to pass below the device, resulting in a smaller wave energy deficit in the lee. When wave energy is transmitted through the device the diffracted wave reduces in magnitude as there is a smaller wave gradient along the geometrical shadow of the device and therefore less for the diffraction mechanism to compensate for. When the diffracted wave is smaller its modification of the incident wave field will be reduced and the interference maxima and minima will be closer to the local mean as shown in figure 2.18.

Figure 2.18 $K_d$ profile along the central transect for an array for WEC devices with transmission coefficients of 0, 0.25, 0.5, 0.75% with zero reflection.

4.5. Effect of JONSWAP peak enhancement

The peak enhancement factor changes the shape of the period spectrum whilst maintaining the peak period value. A lower peak enhancement flattens the shape of the spectrum and vice versa. The peak enhancement factor was found to have negligible effect on the disturbance coefficient profile, possibly because any change in
the strength of interference from the change in proportion of waves with longer period than the peak wave is equal and opposite to the change in the strength of interference by the proportion of wave with shorter period than the peak wave.

5. Discussion

As the approximate analytical solution does not account for wave breaking and friction, one might expect that, even in the absence of directional spreading, the wave energy in the direct lee of the array would increase with increasing distance from the array due to diffraction spreading energy into the shadow region thus dispersing the wave energy deficit. To assess this we sum the total energy in each grid cell of a one grid cell wide band, with length equal to the “direct lee region”, which we define as grid points from b/2 before the first device in the series to b/2 past the last device in the series, as shown in figure 2.19.

This was done for each band of the domain from the first to the last and was normalised by the incident wave energy, to give a profile of the energy flux in the “direct lee region” with increasing distance from the array as shown in figure 2.20 (a). Contrary to expectations the energy in the direct lee region actually reduces with increasing distance from the array. To account for this the diffracted wave component was isolated by removing the incident/transmitted wave component from the total wave in the same manner as seen in figure 2.8(b), and the same summation of grid points in each column band of the domain for the “direct lee region” was performed. The diffracted wave component of the total wave field is also seen to lose energy with increasing distance from the array (figure 2.20b). The loss of energy for the diffracted wave component (figure 2.20b) is approximately equal to the loss of energy for the total wave field (figure 2.20a). This suggests that the reduction in energy with distance from the array is from the diffracted wave component. The diffracted waves that emanate from each lateral tip of each WEC device are circular in form (as seen in figure 2.2b). Initially the diffracted wave spreads energy from the higher energy open region to the lower energy “direct lee region” of the array. With increasing distance from the array, a portion of the diffracted wave overshoots the lower energy “direct lee region” propagating energy back into the open region, resulting in a steady reduction of energy in the direct lee region.

Figure 2.19 Schematic of “direct lee region” and energy summation bands, used to assess the net flux of wave energy.
Figure 2.20 Profile of $E_d$, which is defined as the total wave energy in strip of width 25m and length 3.75km (1 x 150 grid cells) parallel to the array normalised by the un-disturbed total wave energy in a band of the same size (i.e. the energy of the incident wave if no array was present) for (a) the complete wave field, and (b) the diffracted wave component only.

For the specific case presented in figure 2.20, which represents an array of five WEC devices of length 150m, tip to tip separation distance of 600m, total absorption, under unidirectional spectral waves conditions, this mechanism causes a significant further reduction in energy of 16% (after the initial absorption by the array) in the “direct lee region”. This equates to an average wave height reduction of 8% at a distance of 25km. This effect will only be of significance when the directional spread is very narrow because the geometrical shadow for all direction components will be in a similar region so that the defocusing of energy will be away from that shared region. This is seen in figure 2.13 (b) for the spreading parameter $m = 800$ (very narrow spread) where the negative redistribution of wave energy from the defocusing of wave energy by diffraction is at a similar rate and opposite to positive redistribution from the directional spread so that the wave shadow does not significantly recover with distance. It is not fully clear whether this mechanism is physical or an artificial shortcoming of the analytical solution. However, in a similar case (Monk et al., 2011) predictions from the analytical solution, the numerical Boussinesq model FUNWAVE (Wei, et al., 1995) (Kirby, et al., 2005) and the parabolic mild slope model REF/DIF (Kirby & Dalrymple, 1994) (Kirby & Ozkan, 1994), all exhibited a similar energy profile trend under unidirectional monochromatic wave conditions.

Further validation of the analytical solution is given in (Monk, et al., 2011) which compares the analytical solution to the modelled wave field about a breakwater series using the Boussinesq wave model FUNWAVE, and the parabolic mild slope model REF/DIF. The magnitude and location of the interference maxima and minima were found to be in good agreement between the analytical solution for a fully reflective breakwater and the FUNWAVE model outputs at distance greater than $2\lambda$ from the array which is expected due to the approximations of the analytical solution. Similarly good agreement is seen between the analytical solution for a fully absorbent breakwater (as a parabolic model does not account for reflected waves) and the REF/DIF model outputs for distance greater than $2\lambda$ from the array.

The analysis presented here, on the most part (except figure 2.18), assesses the worst case scenario for an ideal overtopping type device with full absorption of the incident wave energy across the beam of the device. In reality an overtopping type WEC device would reflect, absorb and transmit a portion of the incident wave energy. These ratios would be dependent on the dimensions of the device, namely the draft of the device, the height of the freeboard of the reservoir, and the slope of the ramp. The ratios would also be dependent on the incident wavelength, height and direction. For a fixed device (Beels, et al., 2010) calculate the transmitted wave energy below the device for the specific device dimensions and incident wave state and this is found by integrating the wave energy from the sea floor to the draft of the device. Similarly the absorbed wave energy is calculate by the overtopping rate based on the device draft, freeboard height, reservoir ramp width and the incident wave height and wavelength. The remaining wave energy is reflected.
(Norgaard & Andersen, 2012) provide wave basin based experimental results for the wave transmission, reflection and absorption ratios for a floating and fixed 1:51.8 scale Wave Dragon device. They found that for the sea states investigated the wave height in the lee of the floating device was approximately 23% greater than for a non-oscillating fixed device. These methods and results can be used to estimate the reflected, absorbed and transmitted wave energy at the device based on the device dimensions and incident wave parameters. These ratios could then be applied to the approximate analytical using the reflection and transmission coefficients. This can then be used to scale the far field wave energy shadow prior to an actual WEC array deployment.

6. Conclusions

An approximate analytical solution was developed to assess the disturbed wave field about a single row array of overtopping WEC devices, represented as a series of partially absorbing and transmitting breakwater segments. This provides a tool for quickly scaling the wave field about a single row of overtopping WEC devices of a segmented breakwater, for very large areas and for a wide range of wave conditions. This would not be possible / practical with a time-stepping phase resolving numerical model and has been shown to achieve very similar results to phase resolving wave models for smaller domains except for the region very close to the array due to the inherent approximations of the solution.

For arrays placed far offshore, it was confirmed that directional spreading has the greatest influence on the maximum wave height reduction at the coast behind the WEC array. For example, specifically, an 18% drop of wave height for a very narrow directional spread compared to only 2% for a very broad directional spread were found at a distance $x = 250\lambda_p$ behind the WEC array configurations and properties considered.

The separation length between WEC devices has a strong influence on the wave energy shadow in the near field with a short separation length producing greater wave height reductions initially. This is because the wave energy deficit is concentrated when the gap length is shorter. This however is counteracted with increasing distance from the array because of the more rapid influence of directional spreading when the array length is shorter. There is no appreciable difference in the wave energy shadow for different gap lengths in the far field.

The peak period of the spectrum did not affect the maximum wave height reduction in the far field but did have a small influence in the near field with a longer peak period producing more amplified minima and maxima pockets. This is because waves of longer period have maxima minima interference pockets that span greater areas. This results in greater overlap of interference pockets from the diffracted waves from different devices. The peak enhancement factor of the spectrum did not affect the wave shadow.

The approximate analytical solution suggests that diffraction deepens the wave energy shadow with increasing distance due to an over shoot of the energy re-distribution from diffraction effect. This propagates a portion of the diffracted wave energy away from the geometrical shadow region of the array, thus defocusing the wave energy passing through the array. This is only significant for wave states of narrow directional spread.

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