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Evaluation of Regular-Based Pore Networks for Simulation of Newtonian Two Phase Flow

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Abstract

Pore network modelling is a well-established and powerful tool for investigating the pore-scale mechanisms in porous media. It can be used for simulation of vast areas involving fluid flow through porous media. The main constraint in this approach is to develop a network structure which is close enough to the real structure of the porous medium. Although recently proposed approaches can capture the real structure of a medium, acquiring a three-dimensional CT image or two-dimensional cross-section of the rock sample is quite expensive for many individual researchers. However, a pore network structure which adequately represents the porous medium and behaves similarly is sufficient for performing the simulations if the aim is to understand flow behaviour in porous media or to perform sensitivity analyses on fluid and rock properties, where. Early results of using regular-based networks in simulations were not promising. However, their inefficiency might be due to the over-simplified equations used in the simulations and ignorance of the presence of wetting layers. This study attempts to understand whether these networks can be a good representation of the porous medium when advanced equations are used in the calculations.

To achieve this aim, a simple regular cubic network was distorted by removing some of the throats randomly. The descriptive parameters of the network were tuned to match the macroscopic properties of the real porous medium. It was then used for simulation of primary drainage and the imbibition process using a quasi-static method. The tuning and validation processes were repeated for water-wet Bentheimer and Berea sandstones and a sample of carbonate rock. This study showed that if advanced equations and concepts (e.g. the shape factor and wetting layers) are used in calculations, the results of using a simple network can be adequately representative of the complexities of a real rock structure. Although the network obtained by tuning is not unique, however, these regular-based networks can be used for sensitivity analysis of pore-scale mechanism, with acceptable results, when a CT image or cross-section image of the rock is not available.

Key words

Pore Network Modelling, Regular network, Distorted network, Macroscopic flow properties.

Introduction

The aim of pore network modelling is to substitute the complex void space of the porous medium with a network of interconnected pores. Fatt (1-3) pioneered the introduction of an idealised two-dimensional pore network model. However, further research in this area was postponed until the late 1970s, when great advances in computer technology occurred and numerical solutions of flow equations became readily available. In 1977, Chatzis and Dullien (4) stated that two-dimensional networks were unable to predict the 3D flow behaviour of fluids in porous media, although they had been widely used before that time. By comparing the results of the regular networks with the experimental data obtained from
sandstones, they showed that the regular networks comprised of circular capillaries could not be a realistic description of the real porous media.

Later, Jerauld and Salter (5) used a cubic lattice of circular capillaries to study the effect of pore structure on relative permeability and capillary pressure. Using this network, they were able to produce relative permeability curves with the same features as those seen in experiments. They reported that some throats could be removed from network in order to reduce the average coordination number of the network and match the coordination number of a given rock sample. Dixit et al (6, 7) used a regular network for analysing relative permeability hysteresis observed in experiments. Although the network did not produce the exact relative permeability in experiments, it explained some trends observed in relative permeability hysteresis. They used the same network for investigating the wettability and its effects on oil recovery; however, in their network, the formation of oil layers during the imbibition stage in mixed-wet rocks was not modelled (8).

Another approach for construction of a more realistic network was the random distribution of some points in the space. The network was constructed by triangulation of these points. However, little difference was observed by Jerauld and Salter when comparing the results of these types of networks to regular ones (5). In 1999, Fischer and Celia (9) tuned the pore space distribution of a regular network to match capillary pressure data and predicted absolute and relative permeability. They used this method for a variety of soils, which worked moderately well. However, in some cases, the water relative permeability prediction was poor. In this study, the cross section of pores was assumed to be circular and also, the presence of wetting layers was ignored in the calculations.

Although these networks did not produce the exact relative permeability measured in the experiments they explained some trends observed in relative permeability hysteresis (5-7, 9). Since these networks were not successful in prediction of experimental results, it became widely believed that regular-based networks cannot provide a good representation of real porous media (10, 11). However, the fundamental equations used in pore networks to calculate the capillary pressure and the conductance of pores and throats were not fully understood at that time. Thus, it is very difficult to understand if the main reason for the inefficiency of these networks was the regular-based structure or the simple equations, along with the many simplifying assumptions used in calculations.

With the development of more advanced technology, new techniques have been proposed for improving the extraction of actual rock void space. One of these methods is the construction of 3D images from a series of 2D cross sections (12, 13). Another method for acquiring a three-dimensional image of pore space is the use of X-ray microtomography. This is a more accurate method but it is not readily available nor cost effective (14-17). The geological reconstruction of a porous medium simulates the sedimentation process in sandstone rocks and captures the developed void space (18-20). However, the pore size distribution of the rock and some information about the connectivity of the rock is also required in this method. Moreover, it can only be applied for simulation of sandstones, since carbonated rocks are formed during a very complex diagenesis procedure which is hard to simulate. Nevertheless, geological reconstruction of rock structure successfully predicted the macroscopic properties of rock for the first time, which was a significant improvement in pore network modelling. However, in addition to using complicated pore
network structures, these models used more complicated and advanced equations for calculation of both capillary pressure and conductance. The presence of wetting layers and their contribution in fluid conductance were some other features that were introduced in these networks.

Currently, the technology for acquiring a 3D image of the rock and extracting the equivalent pore network model is available in certain research groups around the world (21-25). In some cases, the macroscopic flow properties are even calculated directly on 3D images of the rock sample, using Navier–Stokes solvers (26). Therefore, many of the recent papers published in the area of pore network modelling use the real structure of the rock sample to perform pore network simulations (27, 28). However, these facilities are not readily available for many individual researchers in other institutes around the world. These researchers require a pore network modelling tool which can adequately represent the porous media to be used for simulation and gaining a better understanding of pore-scale mechanisms. This tool should be simple enough for performing simulations and inserting new ideas and findings. At the same time, it should be complex enough to mimic the real porous media.

For many years, researchers used the extracted network of Berea sandstone constructed by Oren et al. (29) for performing simulations and sensitivity analyses when a real 3D image of the rock was not available. However, the permeability of these networks is quite high and their use for simulation of the fluid flow in tight rocks with lower permeability is uncertain. Usually, in order to reduce the permeability of the system, the lengths of pores and throats are reduced by a factor equal to $\sqrt{K^\text{exp}/K^\text{net}}$, in which $K^\text{exp}$ is the desired permeability and $K^\text{net}$ is the permeability of the pore network, based on Berea sandstone. The re-scaled network has the same permeability as the experimental system, although it has the same structure as before. In other words, the average coordination number of the system is still high and the highly connected structure of Berea sandstone is untouched, which might affect the results (11, 30). Other researchers have used the same process to build a sophisticated random pore network quite similar to Berea sandstone and used it for studying the fluid flow in the porous medium (31, 32).

It is widely believed that the rescaled pore network structure of Berea sandstone is a good representation of porous media. However, this triggers the idea that maybe building an artificial pore network with lower connections, lower coordination number and smaller pores and throats could be as efficient as scaling the lengths of pores and throats in a high permeability pore network. Using this network would be easier for researchers who want to test their ideas and formulations using pore network modelling but do not want to get involved in understanding a very complex network. Moreover, they can check the effect of network properties on fluid flow by changing the network parameters, such as porosity, coordination number (degree of connectivity) and pore size distribution. To test this idea, the regular-based network models and their performance was studied to see if they can provide a good representation of real porous media. It has been shown elsewhere that it is easy to use this artificial regular-based pore network structure for calculation of fluid flow properties or to change the governing formulations, for example to simulate non-Newtonian fluid flow (33).
As mentioned above, many of the regular based networks used in the literature, were very simple and comprised of circular capillaries. The cross section of pores and throats was assumed to be circular, so the presence of wetting layers and their conductance was ignored, despite their significant contribution to fluid flow. Furthermore, the capillary pressure and fluid conductance were calculated based on very simple equations available in that time. Therefore, the aim of this paper is to show whether employing recently developed equations and new concepts (e.g. irregular cross sections and wetting layers) can improve the performance of regular-based networks. Although the uncertainty associated with regular-based networks is very large and they cannot be used for predicting the rock properties, they can still be used to perform sensitivity analyses and gain a better understanding of the behaviour of fluid flow in porous media.

To achieve this goal, a regular cubic network was developed and distorted by random elimination of some throats. The average coordination number of the network and the size distribution of pores and throats were tuned until porosity, absolute permeability and capillary pressure curves of the real porous medium were matched. Advanced equations for capillary pressure and fluid conductance were used for simulation of fluid flow in porous media. The results showed that this tuned network was capable of producing satisfactory relative permeability data of the real porous medium. This indicated that using advanced and complex equations in calculations significantly improved the results obtained from regular-based networks. Therefore, it is not always necessary to use complicated network structures to obtain a reliable pore network model which adequately represents a real porous medium. Despite their simplicity, regular-based networks can be used to perform sensitivity analyses with lower costs. Performing such sensitivity analyses helps to understand the physics of porous media and the fluid flow behaviour in porous media.

Pore Network Model

A) Description of Pores and Throats

For performing the present study, a three-dimensional regular cubic lattice pore network model was generated. The maximum coordination number (z) of the regular cubic lattice was six but it can be increased if larger coordination numbers are desired. The pores were located on the corners of cubes and connected to each other by throats (Figure 1). This regular network was then distorted by random removal of some throats. For the distortion process, a set of coordination numbers was generated and assigned to pores, so that the average coordination number of all pores in the network was equal to a user-defined value. Then, each pore was checked and some of the surrounding throats were randomly removed in an attempt to reduce the pore coordination number from six to the assigned value. The porosity and absolute permeability were then calculated and if the results were not satisfactory, this process was repeated by assuming a larger or smaller average coordination number until a good match was obtained.
The cross section of the pores and throats is highly irregular in porous media. Considering this, the wetting fluid can remain in grooves and crevices of pores and throats due to capillary pressure after the primary drainage stage. The presence of these wetting layers was observed in micromodels with a square cross section (34). Hence, the irregular shape of pores and throats is usually described through assigning a dimensionless shape factor to each pore and throat. The dimensionless shape factor is defined as follows (35):

\[ G = \frac{A}{p^2} \tag{1} \]

where \( A \) is the area of the pore or throat cross section and \( p \) is its perimeter. The value of the dimensionless shape factor is somewhere between 0 for a slit-like pore or throat and 0.047 for an equilateral triangle. Next, as a simplification in simulation, the irregular cross sections of pores and throats are replaced with a triangle of the same shape factor and the flow equations are developed based on this equivalent triangle. In this study, the same approach was used to simulate the presence of a wetting layer. For assigning a random shape factor to pores and throats, the Weibull distribution (eq. 2) was used, as follows (36):

\[ G = (G_{\text{max}} - G_{\text{min}})(-\delta \ln (z (1 - e^{-\frac{1}{\delta}}) + e^{-\frac{1}{\delta}}))^{1/\omega} + G_{\text{min}}, \tag{2} \]

The irregular cross sections of pores and throats were replaced with a triangle of the same shape factor and the flow equations were solved for this equivalent triangle. The corner half-angles of these equivalent triangles (\( 0 \leq \beta_1 \leq \beta_2 \leq \beta_3 \leq \pi/2 \)) were calculated using the formulae presented by Patzek (37). Using the shape factor value, the maximum and minimum values for one corner angle (\( \beta_2 \)) were calculated. The exact value of this corner angle can be determined by choosing a random value between the maximum and minimum.

\[ \beta_{2,\text{min}} = \arctan\left(\frac{2}{\sqrt{3}} \cos\left(\frac{\arccos\left(-\frac{12\sqrt{3}G}{3}\right)}{3} + \frac{4\pi}{3}\right)\right) \tag{3} \]

\[ \beta_{2,\text{max}} = \arctan\left(\frac{2}{\sqrt{3}} \cos\left(\frac{\arccos\left(-\frac{12\sqrt{3}G}{3}\right)}{3}\right)\right) \tag{4} \]

The other two corner angles can then be calculated as follows:

\[ \beta_1 = -\frac{1}{2} \beta_2 + \frac{1}{2} \arcsin\left(\frac{\tan \beta_2 + 4G}{\tan \beta_2 - 4G} \sin \beta_2\right) \tag{5} \]

At this point the third corner-angle can be calculated easily (\( \beta_3 = \frac{\pi}{2} - \beta_1 - \beta_2 \)). The corner angles of all pores and throats were assigned, using this procedure to ensure that the shape
factor of the triangle was the same as the desired shape factor. The cross-sectional area of the triangle was calculated easily, using the inscribed radius of the pore and throat along with the shape factor, as $A = \frac{r^2}{4G}$.

Figure 2 illustrates a small representation of the developed network and the distortion process. Although the cross sections of pores and throats are triangular, they are shown here by spheres and cylinders for the sake of simplicity in illustration. The initial regular network is distorted by removing some throats so that the average coordination number of the network is equal to the determined value.

During the distortion process, it is possible that some of the pores become disconnected from their neighbours. Therefore, by reduction of the coordination number, the possibility of having isolated pores in the network increases. In order to calculate the effective porosity of the network, the volume of isolated pores (which does not contribute in flow) was calculated and subtracted from total porosity. As a result, a pore network with very low coordination number can be used for simulation of vuggy carbonates, since it contains several isolated clusters of pores and throats. For this case, both total porosity and effective porosity can be used for tuning the developed network (Figure 3).
Figure 3: The generation of isolated pores as a result of network distortion.

Furthermore, in this network, both pores and throats had volume and contributed in the calculation of porosity. Therefore, the inscribed radius of both pores and throats had to be determined and assigned. The throat size distribution was an input parameter which could be tuned to match absolute permeability and capillary pressure data of the real porous medium. It is obvious that pore and throat size distribution highly affects the absolute permeability and entry capillary pressure of the network. Both pore and throat size distributions were determined using the Weibull distribution (Eq. 2). However, the maximum and minimum radii used for determination of the pore size distribution were larger than those used for throat size distribution. It should be noted that the pore size distribution is an important factor for matching the porosity, since the main part of the network’s void space (porosity) is generated by the volume of pores. After assigning the pore size distribution, it was checked whether the size of each pore was larger than the size of the connecting throats. If it was smaller, the radius assigned to the pore was replaced by a new value, which was determined using an aspect ratio and the radius of the largest connecting throat, as follows (36):

$$R_p = \max(R_{ti}, i = 1, 2, ..., 6) \cdot \alpha$$

(6)

The pore size distribution and the aspect ratio were changed until a good match with the porosity of the real porous medium was obtained.

**B) Calculation of Absolute and Relative Permeability**

In pore network modelling, in order to calculate the absolute permeability of the network, the flow rate and pressure difference across the network should be computed. The law of conservation of mass is simplified to the conservation of volume for a steady state flow of an incompressible fluid. When the conservation of volume is applied to all pores in the network, a set of linear equations is generated in which the pore pressures are unknown. This set of equations could be solved for both water and oil in the system. For calculation of absolute permeability, it is assumed that all the system is saturated with one fluid. Hence all the calculations are based on water properties and only water conductance is calculated for all of the pores and throats in the system. However, for calculation of relative permeability, both phases of water and oil exist in the pores and throats and the oil and water
conductances are calculated for the system separately. For calculation of water or oil relative permeability in the system, the fluid phase should form a connected path from inlet to outlet. Therefore, the water relative permeability is calculated in every step of primary drainage and imbibition, due to the connectivity of water layers in the corners of the pores and throats. However, the calculation of oil phase relative permeability is started after the oil breakthrough in the primary drainage step and continues until the oil phase becomes disconnected during the imbibition process.

The same procedure was used in this study for calculation of absolute and relative permeability. For each pore, the law of conservation of volume can be written as follows, where $k$ is the coordination number of pore $I$ (37):

$$\sum_{K=1}^{k} Q_{ij,k} = 0, \quad i = w, o$$

(7)

Figure 4 shows the smallest flow element in a network - a duct, which comprises of two pores and a connecting throat. The flow rate of an immiscible fluid $i$ - water or oil- between two adjacent pores was calculated as below, in which $l_{ij}$ is the distance between two pore centres and $P_{i,I}$ is the pressure of phase $i$ in pore $I$ (37):

$$Q_{ij} = \frac{g_{ij}}{l_{ij}} (P_{i,I} - P_{i,J}), \quad i = w, o$$

(8)

![Figure 4: Two adjacent pores with the connecting throat.](image)

The hydraulic conductance of oil or water for the duct shown in Figure 4 was calculated using the harmonic averaging of hydraulic conductance of two pores ($I$, $J$) and their connecting throat ($t$) (37):

$$\frac{l_{ij}}{g_{ij}} = \frac{l_t}{g_{i,t}} + \frac{1}{2} \left( \frac{l_i}{g_{i,t}} + \frac{l_j}{g_{i,t}} \right), \quad i = w, o$$

(9)

By substitution of equation 9 into equation 8, the following set of linear equations will be formed, which should be solved to determine the fluid pressure in each pore (37):
\begin{equation}
\begin{align*}
P_1 \left( \sum_{K=1}^{k_1} \frac{g_{1,K}}{l_{1,K}} \right) - \sum_{K=1}^{k_1} \left( \frac{g_{1,K}}{l_{1,K}} \right) P_K &= 0 \\
P_N \left( \sum_{K=1}^{k_N} \frac{g_{N,K}}{l_{N,K}} \right) - \sum_{K=1}^{k_N} \left( \frac{g_{N,K}}{l_{N,K}} \right) P_K &= 0
\end{align*}
\end{equation}

Dimensionless hydraulic conductance of a fluid phase in a pore or throat is related to its shape factor, with a constant value. The constant value for a triangular cross section is calculated to be 0.6, which is different from the constant value for a square or circular cross section. In this network, the cross section of all pores and throats was assumed to be triangular, so equation 11 was used for calculation of the fluid’s hydraulic conductance in each pore and throat (38).

\begin{equation}
\tilde{g} = \frac{g_i \mu_i}{A_i^2} = \frac{3}{5} \tilde{G}, \quad i = w, o
\end{equation}

When the fluid pressure was calculated for all pores, the flow rate across the network could be calculated through the summation of fluid flow in the ducts connected to the inlet face (37).

\begin{equation}
Q_{tot} = \sum_{k=1}^{n} g_k \frac{P_k - P_{outlet}}{l_k}
\end{equation}

\(n = \text{number of pores connected to outlet}\)

The Darcy law was then used for calculation of permeability, using the total flow rate and pressure drop across the system at macroscopic scale. \(A_{tot}\) is the area of the inlet/outlet face and \(L_{tot}\) is the length of the network (39).

\begin{equation}
Q_{tot} = \frac{A_{tot} K (P_{in} - P_{out})}{\mu L_{tot}}
\end{equation}

For calculation of absolute permeability, it was assumed that the whole network was filled with only one phase (oil or water) and the above equation was used. In the middle of the primary drainage or imbibition stage, the relative permeability of phase \(i\) can be calculated as (39):

\begin{equation}
k_{rti} = \frac{Q_i}{Q_{tot}} \quad i = w, o
\end{equation}

It should be noted that during primary drainage, oil just occupies the centre of the pore and a layer of water still exists in the corners. The volume of these layers should be calculated and included in the total water saturation. Moreover, these layers have a huge impact on the connectivity of the water phase in the system and their conductance should not be neglected. In the current study, the cross-sectional area of the water layers was calculated
from the following equation, in which the contact angle might be an advancing angle, receding angle or hinging angle, depending on the process (40):

\[ A_w = r_w^2 \sum_{i=1}^{3} \left[ \frac{\cos \theta \cos(\theta + \beta_i) - \pi}{2 + \theta + \beta_i} \right] \quad \theta, \beta_i: \text{rad} \]  

(15)

C) Primary Drainage Stage

For simulation of the primary drainage stage, it was assumed that the network is fully saturated with water (wetting phase). One of the faces of the network was assumed to be an inlet face, from which oil (non-wetting phase) invades the network. This process describes the migration of oil into an initially water-filled porous medium (reservoir rock). For primary drainage, the entry capillary pressure for each pore or throat with a triangular cross section was calculated as follows (40):

\[ P_{c,PD}^e = \frac{r}{r_d} = \frac{r}{r} \cos \theta_r \left( 1 + 2\sqrt{\pi G} \right) F_d(\theta_r, G, C_1), \]  

(16)

where \( F_d \) is a function of contact angle, shape factor and triangle half-angles and \( C_1 \) is only calculated for corners of triangles in which the wetting fluid exists (37):

\[ F_d(\theta_r, G, C_1) = \frac{1+4G \cos^2 \theta_r}{(1+2\pi G)}, \]  

(17)

\[ C_1 = \sum_{i=1}^{3} \left[ \cos \theta_r \frac{\cos(\theta_r + \beta_i)}{\sin(\beta_i)} - \frac{\pi}{2} - \theta_r - \beta_i \right], \]  

(18)

The presence of water layers in the corners of pores and throats was simulated. According to the published literature, the wetting fluid layer can reside in the corner of a triangle if \( \theta_r < \pi/2 - \beta_i \) (40). When \( P_{c,PD}^e \) was calculated for every pore or throat, the inlet pressure was increased in each step and the oil invaded pores and throats whose entry capillary pressure was less than inlet pressure. In each step the volume of the oil-filled pores and throats was calculated and the saturations of the wetting and non-wetting phase were determined. These data were used for plotting the capillary pressure curve as a function of wetting phase saturation. When the oil phase reached the outlet face of the network and the breakthrough occurred, the relative permeability of wetting and non-wetting phases was calculated in each step, as well. In Figure 5, the simulation of the primary drainage process is demonstrated in a small simplified network.
Figure 5: Illustration of primary drainage process in a small network. The fluid invaded the network as the inlet pressure was increased.

D) Imbibition Stage

In this simulation, the imbibition stage began when the network reached the maximum inlet pressure and the network saturation was equal to irreducible water saturation. In this process, the wetting phase invaded the network and the non-wetting phase was pushed towards the outlet. For this stage, the pressure at the inlet face of the network was decreased step by step. In each step, some of the pores and throats were filled by the wetting phase. The non-wetting phase (oil) was displaced by the wetting phase if a connection path existed to the outlet for oil-filled pores and throats, otherwise the non-wetting phase was trapped in the network. The calculations of the saturations and relative permeabilities of the wetting and non-wetting phases were repeated in each step. Figure 6 shows the final stage of the imbibition process and the residual oil saturation due to the trapping, in a small schematic of the developed network.

The main displacement mechanisms in the imbibition process are known as piston-type, pore body filling, and snap-off. The entry capillary pressure for each mechanism is different and should be calculated separately. For this study, the formulations presented by Patzek (37) were used for calculation of entry capillary pressure for each mechanism of the imbibition process (Appendix A).
Figure 6: The final stage of the imbibition process and illustration of the residual oil saturation due to trapping.

E) Wettability Alteration

When the oil fills the centre of a pore in the primary drainage process, if the oil contains surface active components, these components can adhere to the surfaces of the pores and throats. Normally, a very thin water layer coats the surface of the pores and throats, which prohibits the adsorption of surface active components and wettability alteration. However, there is a threshold capillary pressure at which this thin water layer collapses and the wettability of the surface is changed. The value of threshold pressure for the collapse of water layers depends on the type of fluid, the mineralogy of the rock and the curvature of the pore or throat. A parametric model is suggested for calculation of this threshold pressure as follows (41):

\[
P_{\text{thresh}} = P_{\text{low}} + \Omega x
\]  

(19)

in which \( x \) is a random number between 0 and 1 and \( P_{\text{low}} \) and \( \Omega \) are input parameters. The first term on the right side of the equation represents disjoining pressure while the second term includes the effect of surface curvature. This process is called wettability alteration and is simulated in the present study. The value of this threshold pressure was calculated for each pore and throat. After calculating the pore pressures across the system, these values were compared with the threshold pressure values. If the pore pressure was more than the assigned \( P_{\text{thresh}} \), the wettability of that pore or throat was changed to mixed-wet and a new value for advancing angle and receding angle was assigned to it. However, the corners of the pores and throats were kept water-wet.

During the imbibition process, if water invades a mixed-wet pore or throat in a piston type mechanism, the centre of the pore/throat is filled with water. If the surface is changed to strongly oil-wet \( \left( \theta_d > \frac{\pi}{2} + \beta_{\text{min}} > \theta_{d,\text{max}} \right) \) there is a possibility that a film of oil is left sandwiched between the water layers which exist in the corners and the water phase in the centre. The threshold pressure at which an oil film is collapsed is given by (40):

\[
\frac{P_c}{P_{c,\text{max}}} = \frac{\sin\beta_i}{\cos(\theta_r + \beta_i)} \left( \frac{1 - d^2}{d \cos\beta_i + \sqrt{1 - d^2 \sin^2\beta_i}} \right)
\]

(20)
in which \( d = 2 + \cos \theta_a / \sin \beta_i \). The oil films increase the connectivity and conductance of the oil phase in the system significantly. The presence of oil layers was also simulated, and the cross-sectional area of the oil films was calculated using the following equation:

\[
A_o = r_w^2 \sum_{i=1}^{3} \left[ \frac{\cos \theta_a \cos(\theta_a + \beta_i)}{\sin \beta_i} - \frac{\pi}{2} \left( \frac{\theta_a + \beta_i}{90} - 1 \right) \right] - A_w \tag{20}
\]

In this formulation, \( A_w \) is the cross-sectional area of the water layers and was calculated using equation (15). The presence of water layers and oil films was not included in any regular-based pore networks previously reported in the literature.

**Results and discussion**

The aim of this study was to investigate if regular-based networks can be used with lower expense by individual researchers for simulations of pore scale mechanisms and performing sensitivity analyses. The main idea was that the inefficiency of regular-based networks was due to the simplified equations which existed that time. Hence, maybe using advanced formulations and concepts could improve their performance. For this purpose, three different sets of experimental data reported in the literature were used to tune the network. The network results obtained in this study were compared to pore network results reported by others in the literature. The results indicated that, although regular-based networks are relatively simple, using advanced equations improves their simulation capabilities significantly.

**A. Bentheimer Rock**

First, the experimental data reported by Øren et al. (40) for Bentheimer Rock were used for tuning the network and comparing the calculated relative permeability results. The results obtained were then compared to the experimental data and network results reported. Øren et al. used the geological reconstruction model to simulate the sedimentation process of sandstone rocks and describe their void space. Using this model, he successfully predicted the measured macroscopic properties of Bentheimer Rock. In this study, porosity, absolute permeability and capillary pressure curve were simultaneously used to tune the pore network model. Our results are in good agreement with the measured values, which means that the simple network was able to produce realistic results.

Table 1 shows the network and fluid properties used for simulation of Bentheimer sandstone after tuning.

Table 2 compares the results obtained from tuning in this study with the measured experimental data and the results of simulations performed by Øren et al.. Our results are in good agreement with the measured values, which means that the simple network was able to produce realistic results.

| Table 1: Network properties used for simulation of Bentheimer sandstone |
|-----------------------------|-------------------|
| **Property**               | **Value**         |
| Pore Number                | 2000              |
Table 2: Comparison of experimental and predicted porosity and permeability values of Bentheimer (B1, B2, B3) and reservoir rock (R1, R2) samples.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Porosity (fraction)</th>
<th>Permeability (md)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Experiment</td>
<td>Øren et al.</td>
</tr>
<tr>
<td>B1(Bentheimer)</td>
<td>0.232</td>
<td>0.210</td>
</tr>
<tr>
<td>B2(Bentheimer)</td>
<td>0.241</td>
<td>0.219</td>
</tr>
<tr>
<td>B3(Bentheimer)</td>
<td>0.237</td>
<td>0.216</td>
</tr>
<tr>
<td>R1(Reservoir rock)</td>
<td>0.297</td>
<td>0.243</td>
</tr>
<tr>
<td>R2(Reservoir rock)</td>
<td>0.293</td>
<td>0.245</td>
</tr>
</tbody>
</table>

As stated before, the network parameters were tuned to match the measured experimental data. Figure 7 shows the calculated capillary pressure curve and the comparison with experimental values. The obtained capillary pressure is in good agreement with the experimental values. After matching the capillary pressure curve, the network was used to calculate the primary drainage relative permeability curve. As shown in Figure 8, the relative permeability data are in good agreement with experimental data. This indicates that when advanced and complicated equations are used in calculations, the performance of regular-based networks improves significantly. It should be noted that the tuning process is limited to obtaining a good match to porosity, absolute permeability and the capillary pressure curve. In other words, no further tuning has been performed to match the relative permeability data shown in Figure 8.

The imbibition relative permeability curve for Bentheimer sandstone is shown in Figure 9. As displayed in this figure, the comparison between experimental data and simulation results indicate that the regular-based networks can be used for simulation of the imbibition process as well, and their application is not limited to primary drainage calculations. Although the relative permeability results in Figure 8 and Figure 9 do not perfectly match the experimental data, nevertheless, these results show that the fluid flow behaviour in this
network is similar enough to that in the real porous medium. Hence, this method can be used for sensitivity analyses by individual researchers, with lower costs. It should be noted that we are not using these simple networks for prediction purposes, so we were not looking for a perfect predictive match. In this study, the verifications and simulations have been performed on water/oil systems, but the application to gas flow is similar.

Figure 7: Calculated capillary pressure curve for Bentheimer sandstone.
Figure 8: Calculated primary drainage relative permeability curve for Bentheimer sandstone.

Figure 9: Calculated Imbibition relative permeability curve for Bentheimer sandstone.

B. Water-wet Berea sandstone (Oak 1990)

In 1990, Oak (42) reported the experimental data of two and three phase relative permeability measured for Berea sandstone. In 2003, Valvatne and Blunt (8) used the network model created by Øren et al. (40) to predict the relative permeability of Berea
sandstone. They used the same network without any changes, because the network was originally developed for sandstone rock. They used only the primary drainage contact angle for matching relative permeability. This process seems controversial, as the absolute permeabilities reported by Oak (42) for the samples were 200, 800 and 1000 md, whereas Valvatne and Blunt (11) used the experimental data for 200 md sandstone, which was considerably lower than the permeability of the Bentheimer rock structure developed by Øren et al., which is around 2900 md (40). However, the results of their simulation (shown in Figure 10) were quite satisfactory, despite the large difference in absolute permeability values.

Table 3 compares the experimental and calculated values of porosity and permeability for Berea sandstone using a regular-based network model. The comparison of experimental and calculated relative permeability is shown in Figure 10. The relative permeability data indicate that a regular-based network can be used for developing pore networks with the same macroscopic properties as the real porous media. It should be mentioned that the capillary pressure data was not available in this case for tuning the network, which could improve the results obtained for relative permeability significantly.

Table 3: Comparison of experimental and predicted porosity and permeability values of Berea sample.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Porosity (fraction)</th>
<th>Permeability (md)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experiment (Oak 1990)</td>
<td>Valvatne &amp; Blunt (2003)</td>
</tr>
<tr>
<td>Berea Sandstone</td>
<td>N/A</td>
<td>0.24</td>
</tr>
</tbody>
</table>
C. Carbonate rock

The final experimental data used in this study for verification were the measured data for a carbonate sample. Even though the absolute permeability of this sample is quite low, the predicted relative permeability by the tuned network is in good agreement with the experimental data. This example illustrates that the regular-based networks described in this study can also be used for studies of carbonate rocks with low absolute permeabilities.

The network used by Valvatne and Blunt was the Berea network (originally developed by Oren et al.). The network was rescaled by reducing the length and radius of the pores and throats, such that the target porosity was achieved. However, the ratio of the radius to the length for each pore or throat was kept constant, equal to that in the original network. The comparison of the results of the rescaled network with the experimental values shows that the rescaled network can be a representative of the porous medium. Although the Berea network used in these calculations is a very complex structure and a good representation of the rock sample, it should be noted that the rescaled network has the same coordination number and structure as a sandstone with very small pores and throats. The average coordination number of the network is reported to be 4.45 and the absolute permeability was about 1100 md (43). The maximum coordination number was reported as 19, which is very high for a tight rock. Therefore, the use of rescaled networks for performing sensitivity analysis should be very careful, especially in the subjects where the flow properties depend on the radius of pores and throats or the connectivity of the network. The flow of polymers in porous media is a very good example, since the viscosity of the non-Newtonian fluid depends on the size of pore or throat (30). Hence, using a network with low coordination number and reasonable pore size distribution yields quite different results compared to a
network with higher coordination number and very narrow pore size, although both networks might have the same absolute permeability and porosity.

Figure 12 demonstrates that the results of the regular-based network were comparable with the rescaled network. This is another indication that the network adequately represents the structure of the porous medium. The average coordination number of the system is 4, which is almost equal to that in the network used by Valvatne & Blunt. However, the maximum coordination number is six, which is more reasonable for tight samples. It should be noted that both methods are associated with a high degree of uncertainty and neither of them can be selected as superior. Therefore, selecting a method depends on the researcher’s opinion, the facilities and the subject of the study. For example, if a researcher wants to investigate the effect of network structure on the result, it may be more logical to develop a network and change the structure according to the subject of study. However, the aim of this study was just to show that regular-based networks can also be good representatives of rock structure, and thus, and re-scaling the available networks is not the only solution for individual researchers in pore network modelling simulations.

Table 4: Comparison of experimental and predicted porosity and permeability values for a carbonate sample.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Porosity (fraction)</th>
<th>Permeability (md)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbonate Rock</td>
<td>N/A</td>
<td>0.26</td>
</tr>
</tbody>
</table>
Conclusion

The aim of this paper was to show whether the inefficiency of regular-based networks reported in the literature had been due to the inadequate network structure or the simple equations used for calculations. For this purpose, a regular-based network was generated and distorted by removing some throats. The cross section of pores and throats was
assumed to be irregular, therefore, the presence of wetting layers and their contribution in the fluid flow was modelled. Advanced equations were used for capillary pressure calculation in the primary drainage and imbibition processes and the calculation of fluid conductance in pores and throats.

The macroscopic properties of three rock samples were used for tuning the regular-based network. The comparison of calculated relative permeabilities with experimental values revealed that the inefficiency of regular-based networks reported in the literature might be due to the simple equations used in calculations and important phenomena such as wetting-layer flow being ignored. However, it should be noted that the regular-based networks cannot be used for prediction of macroscopic properties of a real porous medium, because the tuning procedure is not unique and many different network structures have the same properties. However, they can be used widely for investigation of pore-scale mechanisms and sensitivity analysis of network parameters with low costs by individual researchers. The results of this paper suggest that the capabilities of regular-based networks should not be underestimated simply because they are based on a regular structure. These networks can be used in many fields of petroleum engineering as rapid and simple simulation tools to investigate the pore-scale mechanisms active in porous media.

Appendix A: simulation of imbibition process

During the imbibition process, the oil pressure is decreased step by step, which results in lowering the capillary pressure and the system experiences a free imbibition process. The water pressure is then increased step by step, which is equivalent to decreasing the capillary pressure to values lower than zero and the imbibition process is forced. It should be noticed that simulation of the imbibition process is more complicated, as the water phase can displace oil from a pore or throat only if it is connected to the outlet. Hence, in every step the connectivity of oil-filled pores and throats should be checked and the oil-filled pores and throats which are connected to the outlet labelled. Sometimes displacement of oil from only one pore or throat affects the connectivity of other pores and throats. Therefore, the connectivity of the oil phase should be checked just before every single displacement during the imbibition process in the system. During the primary drainage stage, the water phase was connected throughout the network through water layers existing in the corners, so there was no need to check the connectivity of the water phase.

As the capillary pressure drops during the imbibition process, the water phase invades the smallest available water-wet pore or throat. The displacement mechanisms during the imbibition stage are piston-type displacement, pore body filling and snap-off, which has been discussed in detail by Lenormand et al. (34). In this section we only mention the formulations which have been used in this study for simulation of the imbibition stage. In every step of reducing the capillary pressure, the threshold capillary pressure for each displacement is calculated.

For a snap-off displacement there is no need for the fluid interface to reach the entrance of the pore or throat. Therefore, at each capillary pressure, the entire network was searched and the pressure difference was compared to the snap-off threshold capillary pressure. If the water phase had the ability to displace the oil phase and the oil phase was connected to the outlet, the water phase is imbibed by the pore or throat. This pore or throat was
labelled as filled with water. When this search was finished the rest of the imbibition process was continued at the interface of fluids. When the water interface reached a pore or throat, the threshold capillary pressure for piston-type displacement and pore body filling was compared to the pressure difference. If the water was able to displace the oil phase and the oil phase was connected to the outlet, the pore or throat was labelled as a water-filled pore or throat. The water-filled pores and throats are added to the front function, to be checked for the possibility of being imbibed by connected pores or throats. Even any pore or throat which is filled by snap-off should be added to this function, as it forms an interface which can be imbibed by the nearest pores and throats.

**Piston type displacement:**

In this type of displacement, depending on the contact angle hysteresis, two cases are considered (40):

1. If the there is no contact angle hysteresis ($\theta_r = \theta_a$):

The threshold capillary pressure for piston type displacement in this case equals the value calculated for primary drainage.

2. If the there is no contact angle hysteresis ($\theta_r \neq \theta_a$):

Every pore or throat in the network has three defining corner half angles ($0 \leq \beta_1 \leq \beta_2 \leq \beta_3 \leq \pi/2$). The maximum advancing angle should be calculated as follows for each pore and throat, since this value is an indication of whether the imbibition process is forced or spontaneous (40):

$$\cos \theta_{a,\text{max}} = \frac{-4G \sum_{i=1}^{3} \cos(\theta_r + \beta_i)}{P_{c,\text{max}} r / \gamma - \cos \theta_r + 12G \sin \theta_r} \quad (A.1)$$

The value of $P_{c,\text{max}}$ in the above formulation is the maximum capillary pressure reached during primary drainage stage.

2.1 If $\theta_a > \theta_{a,\text{max}}$: forced imbibition (40)

   2.1.1 If $\theta_a \geq \frac{\pi}{2} \text{ + max}(\beta_i)$: the threshold capillary pressure is calculated from the primary drainage formulation. However, the receding angle in the formulation is replaced by the advancing angle.

   2.1.2 If $\theta_a < \frac{\pi}{2} \text{ + max}(\beta_i)$: the threshold pressure is calculated as:

$$P_{c,PT}^e = \frac{2 \gamma \cos \theta_a}{r} \quad (A.2)$$

2.2 If $\theta_a \leq \theta_{a,\text{max}}$: Spontaneous imbibition (37)

The threshold pressure is calculated by solving a set of non-linear equations and determination of seven unknowns ($\theta_{h,i}, \alpha_i, i = 1,2,3, r_p$) as follows (37):
\[
\theta_{h, i} = \min \left\{ \arccos \left[ \frac{r_{pd}}{r_p} \cos(\theta_r + \beta_i) \right] - \beta_i, \theta_a \right\} \quad i = 1, 2, 3 \tag{A.3}
\]

\[
b_i = \begin{cases} 
\frac{r_{pd}}{\sin \beta_i} & \text{if } \theta_{h,i} \leq \theta_a \\
\frac{r_p \cos(\theta_a + \beta_i)}{\sin \beta_i} & \text{if } \theta_{h,i} > \theta_a
\end{cases} \quad i = 1, 2, 3 \tag{A.4}
\]

\[
\alpha_i = \begin{cases} 
\arcsin \left( \frac{b_i}{r_p} \sin \beta_i \right) & \text{if } \theta_{h,i} \leq \theta_a \\
\frac{\pi}{2} - \theta_a - \beta_i & \text{if } \theta_{h,i} > \theta_a
\end{cases} \quad i = 1, 2, 3 \tag{A.5}
\]

\[
r_p = \frac{\frac{r^2}{4G} - r_p \sum_{i=1}^{3} b_i \cos \theta_{h,i} + r_p^2 \sum_{i=1}^{3} \left( \frac{\pi}{2} - \theta_{h,i} - \beta_i \right)}{2r_p \sum_{i=1}^{3} \alpha_i + \left( \frac{r}{2G} - 2 \sum_{i=1}^{3} b_i \right) \cos \theta_a} \tag{A.6}
\]

For solving this set of equations an initial value for \(r_p\) is guessed. Then the other parameters are calculated which are used for calculation of \(r_p\), which is calculated again and updated in every step, until the difference between the initial and calculated value is less than a pre-defined value of \(\epsilon\). This value of \(r_p\) is used for calculation of threshold capillary pressure for spontaneous piston-type displacement (37).

\[
P_{c,PT}^e = \frac{Y}{r_p} \tag{A.7}
\]

**Snap-off displacement:**

The threshold capillary pressure in this case is a function of the curvature of wetting layers in the corner. The maximum capillary pressure reached during the primary drainage stage determines the curvature of wetting layers. The capillary pressure required for this type of displacement is calculated as follows (37):

\[
\theta_a < \pi - \beta_{\text{min}} : \quad p_c^e = \frac{\gamma_{\text{ow}}}{r_{so}} = \frac{P_{c,\text{max}} \cos(\theta_r + \beta_{\text{min}})}{\cos(\theta_a + \beta_{\text{min}})} \tag{A.8}
\]

\[
\theta_a \geq \pi - \beta_{\text{min}} : \quad p_c^e = \frac{\gamma_{\text{ow}}}{r_{so}} = \frac{P_{c,\text{max}}}{\cos(\theta_r + \beta_{\text{min}})} - 1 \tag{A.9}
\]

**Pore-body filling displacement:**

This mechanism is more complicated and the associated threshold capillary pressure is calculated depending on how many of the connected throats are filled with the water. For a pore with coordination number of \(z\), there are \(z-1\) pore-body filling mechanisms. If only one of the connecting throats is filled with water, the mechanism is called \(I_1\) and the displacement is similar to the piston-type mechanism mentioned above. Hence, the threshold capillary pressure is also the same. For the \(I_2\) to \(I_{z-1}\) mechanisms, we use the following formula, in which \(W_{j,k,...,n}\) is the uniform random weight between zero and one (37).

\[
R_n = \frac{1}{\cos \theta_a} \left( R_p + \omega^{(n)} \sum_{n-\text{couple}s} W_{j,k,...,n} (r_j + r_k + \cdots + r_n) \right) \tag{A.10}
\]
\( w^{(2)} = 0.72, \ w^{(3)} = 0.45, \ w^{(4)} = 1.2, \ w^{(5)} = 1.5, \ w^{(n\geq 5)} = 5 \) \hspace{1cm} (A.11)

\[ p_{c,n}^e = \frac{2\gamma}{R_n} \] \hspace{1cm} (A.12)

**Nomenclature**

A: Cross-sectional area of pores or throats \( (m^2) \).

G: Shape factor \(-\).

g: Hydraulical conductance \( (m^2/Pa.s) \).

k: Coordination number \(-\).

K: Rock absolute permeability \( (md) \).

l: The length of pores and throats \( (m) \).

p: Perimeter of pores or throats \( (m) \).

P: Pressure \( (Pa) \).

Q: Volumetric flow rate \( (m^3/s) \).

r: Inscribed radius of pores and throats \( (m) \).

z: Random numbers between 0 and 1 \(-\).

δ: Weibull distribution parameter \(-\).

ω: Weibull distribution parameter \(-\).

\( \beta_1, \beta_2, \beta_3 \): Corner half-angles of triangular cross-sections of pores and throats \( (rad) \).

\( \alpha \): Aspect ratio \(-\).

Ø: Rock porosity \(-\).

μ: Fluid viscosity \( (Pa.s) \).

\( \theta \): Contact angle \( (rad) \).

\( \gamma \): Interfacial tension, IFT \( (mN/m) \).

Ω: Arbitrary pressure for assigning threshold capillary pressure \( (Pa) \).

**Subscripts/Superscripts**

exp: Experimental.

net: Network.

thresh: Threshold capillary pressure for wettability alteration.

a: Advancing.

c: Capillary.

c,max: Maximum capillary pressure reached during primary drainage.

d: Drainage.

e: Entry.

h: Hinging.

in: Inlet.

min: Minimum.

max: Maximum.

o: Oil.

out: Outlet.

p: Pore body

PD: Primary drainage.

PT: Piston type

r: Receding (contact angle).
r: Relative (Permeability).

t: Pore throat.

tot: Total.

w: Water.

References

Highlights

- A simple regular cubic network is distorted by removing some of throats randomly.
- Advanced equations has been used for simulation of fluid flow in porous media.
- The tuned network produces reasonable relative permeability data of porous media.
- Using advanced equations in simulations is more vital than using complex networks.
- Regular-based networks can be used for sensitivity analysis with lower costs.