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Citation for published version:

Digital Object Identifier (DOI):
10.1002/esp.3988

Link:
Link to publication record in Heriot-Watt Research Portal

Document Version:
Peer reviewed version

Published In:
Earth Surface Processes and Landforms

Publisher Rights Statement:
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Numerical modelling of alternate bar formation, development and sediment sorting in straight channels

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ABSTRACT: A two-dimensional shallow water hydro-sediment-morphodynamic model is applied to investigate alternate bar formation, development and sediment sorting in straight channels. The model is coupled, explicitly incorporating the flow-sediment-bed interactions by using the full mass and momentum conservation equations, which are numerically solved by a well-balanced version of the finite volume Slope LImiter Centred (SLIC) scheme. The model is first tested against a flume experiment on alternate bars formed over uniform sediment bed, which clearly exhibits processes of bar formation, migrating and finally approaching an equilibrium state. Then it is applied to another flume experiment on alternate bars due to non-uniform sediment transport. The computational results are evaluated, with a
focus on the longitudinal and vertical sediment sorting. It is argued for the first time that the inconsistent sediment sorting patterns observed in previous studies are determined by different sediment transport conditions, i.e., full versus partial transport. When a condition of full transport is achieved, under which all size fractions are fully mobilized and transported, the longitudinal surface sediment shows a sorting pattern of coarse-on-head and fine-in-pool, and the vertical substrate sediment exhibits an immobile-fine-coarse structure upwards. In contrast, for a partial transport condition, under which only finer fraction participates in the transport process, an opposite longitudinal pattern (i.e., fine-on-head and coarse-in-pool) and a different vertical structure (i.e., immobile-coarse-fine) are observed. Concurrently, numerical experiments with specified conditions show that the critical aspect ratio for the formation of migrating alternate bars is approximately equal to 12. With the increase of the aspect ratio, the bar length grows gradually, while the bar height increases rapidly for moderate values of the aspect ratio and then keeps nearly stable. The bar celerity, however, is weakly sensitive to the variation of this ratio.

KEYWORDS: alternate bar; mathematical model; non-uniform sediment; sediment sorting; aspect ratio

Manuscript description for Twitter publicising (max. 140 characters): Previously observed distinct patterns of sediment sorting over alternate bars can be reconciled as per full versus partial transport regime.
1. Introduction

Alternate bars are typical bed forms in natural rivers, characterized by a sequence of riffles and pools in the downstream direction. They are common in straight and weakly meandering channels and originate from the inherent morphodynamic instability. Typically, the growing bars will result in the change of flow structure, bank erosion as well as the increase of channel width and sinuosity, which induces the initiation of meandering and braided rivers. Thus the formation and development of alternate bars have profound impacts on the evolution of river patterns. Enhanced understanding of the mechanisms underlying alternate bars is of significance for river regulation. Bars can be divided into steady and migrating bars. The migrating celerity of steady bars is small enough (in the order of meters per year) so that the bars can be approximately treated as steady or non-migrating. By contrast, the celerity of migrating bars is usually large, reaching the order of meters per hour, and every bar unit can migrate freely. Bars can also be classified into forced or free bars according to whether a forcing effect (e.g., a permanent obstruction) is imposed. This paper focuses on migrating free bars.

A key factor for the development of alternate bars is the aspect (width-to-depth) ratio of the channel. Generally, migrating free bars can grow in a straight channel when this ratio exceeds a threshold value between 12 and 20. No migrating bars will form at smaller aspect ratios, while for too large values multiple-row bars, meandering or braided rivers are expected to form (Parker, 1976; Struiksma et al., 1985; Colombini et al., 1987).
Theoretically, stability analyses based on the analytical solution of simplified governing equations have contributed a lot to understanding the physical mechanisms in bar formation and development. A linear stability analysis can provide the critical conditions for bar formation and predict the values of bar length, migrating celerity and growth rate, whilst the value of bar amplitude cannot be obtained directly (Blondeaux and Seminara, 1985; Struiksma et al., 1985; Struiksma and Crosato, 1989; Lanzoni and Tubino, 1999; Lanzoni, 2000a). Yet, the linear approximation is invalid once the amplitude of perturbation is not infinitesimal. Therefore, some weakly nonlinear theories were established in the neighborhood of critical conditions (Colombini et al., 1987; Schielen et al., 1993). They account for the nonlinear interactions between different harmonics, thus are valid for slightly larger perturbations and capable of predicting the value of bar amplitude. Unfortunately, both linear and nonlinear stability analyses of migrating alternate bars cannot give well predictions for bar features when flow and sediment conditions are far from the critical status.

Several field observations of alternate bars are available (e.g., Lisle and Madej, 1992; Welford, 1994; Eekhout et al., 2013; Jaballah et al., 2015; Rodrigues et al., 2015), which facilitate a general description of bar development, sediment transport and grain size distribution on the bed surface. Also, there are laboratory experiments on bar development with uniform and non-uniform sediments. Among uniform sediment experiments, Struiksma and Crosato (1989) firstly conducted experiments in a straight flume and a curved flume respectively with constant discharge and a local perturbation imposed near the inflow. Then Tubino (1991) performed experiments to
investigate the development of alternate bars under unsteady flows. Later on, a series of short-duration experiments were conducted by Lanzoni (2000a), which provided detailed measurements of bed profiles and bar features. Crosato et al. (2011, 2012) carried out long-duration experiments with or without a local perturbation near the inflow to reveal the long-term development of alternate bars. With regard to experiments with non-uniform sediment, Lisle et al. (1993) and Venditti et al. (2012) studied the alternate bars’ response to a reduction or termination in sediment supply, while others focused on the characteristics of bar formation and migrating as well as sediment sorting (Lisle et al., 1991; Diplas, 1994; Lanzoni, 2000b; Takebayashi and Egashira, 2001). In particular, Lisle et al. (1991), Lisle and Madej (1992), Diplas (1994) and Lanzoni (2000b) show an obvious trend of bar head coarsening, but in sharp contrast a fining trend on bar heads was observed by Takebayashi and Egashira (2001). To date, observations of sediment sorting over bar heads and pools show inconsistency, for which there have been no reasonable explanations.

Computational river modelling has become a proactive tool for enhancing the understanding of the mechanisms of bar formation and development, as it can consider the fully nonlinear processes without involving any linear approximations. Also it is time-saving and easier to be operated than physical experiments. The last several decades have witnessed the development and application of some mathematical models for alternate bars, but most of them were restricted to cases with uniform sediment transport (Nelson and Smith, 1989; Struiksma and Crosato, 1989; Defina, 2003; Federici and Seminara, 2003; Bernini et al., 2006; Nicholas, 2010;
Crosato et al., 2011, 2012). Only a few models (Takebayashi and Egashira, 2001; Takebayashi and Okabe, 2002; Nelson et al., 2015a, b) were applied for alternate bars formed on bed with non-uniform sediment. Unfortunately, these models are decoupled, whereby neglecting the interactions between the flow, sediment transport and morphological evolution to some extent. Its effect on the results remains unknown. Also, sediment transport is presumed to be always equal to the transport capacity. This assumption, however, may not be generally justified from physical perspectives (Cao et al., 2007; Cao et al., 2016). Further, the numerical results in Takebayashi and Egashira (2001) and Takebayashi and Okabe (2002) show that fine sediment deposits around bar heads whilst coarse sediment is left in pools, which is contrary to most of the experimental findings (Lisle et al., 1991; Lisle and Madej, 1992; Diplas, 1994; Lanzoni, 2000b) and the numerical results in Nelson et al. (2015a, b).

This paper presents a numerical study of alternate bars in straight channels. A 2D shallow water hydro-sediment-morphodynamic model for non-uniform sediment transport is presented and applied to investigate alternate bar formation, development and sediment sorting. Bed load transport is presumed to be dominant over suspended load, and thus the latter is neglected. The model is first tested against a flume experiment with uniform sediment (Lanzoni, 2000a) to show the whole process of alternate bar formation and migrating in a straight channel. Then it is applied to model the processes in a flume experiment with non-uniform sediment (Lanzoni, 2000b).

According to whether all sediment fractions in the bed are set into motion and transport, non-uniform sediment transport can be classified to two conditions, i.e., full
transport and partial transport. The evaluation in this regard is focused on the differential longitudinal and vertical sediment sorting under the conditions of full and partial sediment transport, by which the inconsistent patterns of sediment sorting observed in previous experimental and numerical studies are reconciled. Finally, numerical tests with different aspect ratios are conducted and their influence on alternate bar formation is evaluated.

2. Mathematical model

2.1 Governing equations

The present model is essentially an extension of the recent 1D model (Qian et al., 2015) to two dimensions. Consider two-dimensional (2D) flow in an open channel with rectangular cross-sections of constant width, and over an erodible bed comprising of non-uniform sediment of $N$ size classes. The diameter of the $k$th size sediment is denoted by $d_k$ ($k = 1, 2, ..., N$). The model is based on the widely used three-layer structure (e.g., Hirano, 1971), which consists of bed load layer, active layer and substrate layer. The bed load layer is above the bed surface, in which the sediment particles roll, slide and saltate. The active layer, a conceptual layer on the bed surface, lies between the bed load layer and the substrate layer. The sediment in the active layer is assumed to be distributed uniformly in the vertical and can exchange with the upper and lower layers. The substrate layer, located below the active layer, may have certain vertical structure and the sediment composition can also vary in time.

In the context of computational river dynamics, it is essential to recognize that water
flow, sediment transport, and riverbed may interact strongly with each other. Specifically, as Figure 1 shows, sediment transport is dictated by water flow while the flow may be modified by a high sediment concentration. The two aspects manifest the two-way flow-sediment interaction. Sediment transport dictates bed deformation (aggradation or degradation) by virtue of sediment deposition and entrainment, while at the same time it is affected by bed conditions. In essence a two-way sediment-bed interaction occurs. Finally, the bed evolution may have a significant effect on the flow especially when the rate of bed evolution is considerable compared to that of the change in flow depth (e.g., in extreme cases such as dam break floods over erodible beds), yet the flow does not directly alter the bed (but indirectly via sediment transport). Thus a one-way flow-bed interaction exists. In general, of particular significance to a mathematical river model is to use the complete governing equations, which fully take into account these physical interactions between the flow, sediment and bed, thereby maximizing model applicability and minimizing its inaccuracy.

Figure 1. Flow-sediment-bed interactions in alluvial rivers
The present depth-averaged 2D governing equations for non-uniform sediment
transport are derived from the complete conservation laws in fluid dynamics, including
the mass and momentum conservation equations for sediment-laden flow, the
size-specific mass conservation equation for bed load transport, the total mass
conservation equation for bed load sediments in the bed (i.e., bed update equation),
and the size-specific mass conservation equation for bed load sediments in the active
layer. In general, the complete governing equations in a conservative form are

\[ \frac{\partial \eta}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = 0 \]  

(1)

\[ \frac{\partial (hu)}{\partial t} + \frac{\partial}{\partial x} \left[ hu^2 + 0.5g(\eta^2 - 2\eta z) \right] + \frac{\partial}{\partial y} (huv) = -gh \frac{\partial \eta}{\partial y} + \frac{\partial \eta}{\partial x} - \frac{(\rho_s - \rho_w)gh^2}{2\rho} \frac{\partial C}{\partial x} \]

(2)

\[ \frac{\partial (hv)}{\partial t} + \frac{\partial}{\partial x} (huv) + \frac{\partial}{\partial y} \left[ hv^2 + 0.5g(\eta^2 - 2\eta z) \right] = -gh \frac{\partial \eta}{\partial x} - \frac{(\rho_s - \rho_w)gh^2}{2\rho} \frac{\partial C}{\partial y} \]

(3)

\[ \rho \frac{\partial (hu)}{\partial t} + \rho \frac{\partial}{\partial x} \left[ \beta_k \frac{\partial u}{\partial x} \right] + \rho \frac{\partial}{\partial y} \left[ \beta_k \frac{\partial v}{\partial y} \right] = E_k - D_k \]

(4)

\[ \frac{\partial \eta}{\partial t} = \frac{D_T - E_T}{1-p} \]

(5)

\[ \frac{\partial (\xi)}{\partial t} + f_k \frac{\partial \xi}{\partial t} = \frac{D_k - E_k}{1-p} \]

(6)

where \( t \) is the time; \( x, y \) are the horizontal Cartesian coordinates; \( \eta \) is the water
level above a datum; \( h \) is the water depth; \( \eta - h \) is the bed elevation; \( u, v \)
are the depth-averaged velocity components in the \( x \) - and \( y \) - directions and
\[ V_t = \sqrt{u^2 + v^2} \] is the total depth-averaged flow velocity; \( g \) is the gravitational acceleration; \( S_{sx}, S_{sy} \) are the friction slopes; \( c_k \) is the size-specific depth-averaged bed load concentration and \( C = \sum c_k \) is the total depth-averaged bed load concentration; \( \rho \) is the bed sediment porosity; \( \rho_w \) and \( \rho_s \) are the densities of water and sediment respectively; \( \rho = \rho_w(1-C) + \rho_s C \) is the density of the water-sediment mixture; \( \rho_0 = \rho_w \rho + \rho_s (1 - \rho) \) is the density of the saturated bed material; \( \gamma'_k \) is the size-specific direction angle of bed load transport (relative to \( x \) axis); \( \beta_k = V_{st}/V_t \) is an empirical coefficient representing the velocity of bed load transport \( V_{st} \) relative to that of flow \( V_t \); \( E_k \) is the size-specific sediment entrainment flux and \( E_T = \sum E_k \) is the total sediment entrainment flux; \( D_k \) is the size-specific sediment deposition flux and \( D_T = \sum D_k \) is the total sediment deposition flux.

It should be noted that Eq. (4) is a depth-averaged equation for bed load transport, which is utilized to resolve the depth-averaged bed load concentration \( c_k \) instead of the mean bed load concentration \( c_{ok} \) within the bed load transport layer with a thickness \( h_{bk} \ll h \). However, the two bed load concentrations can be transferred to each other by the relation \( h_{bk} c_{bk} = h c_k \). For non-uniform sediment transport, the widely used active layer formulation proposed by Hirano (1971), i.e., Eq. (6), is adopted here to resolve the change of bed surface sediment composition. In Eq. (6), \( f_{ak} \) is the fraction of the \( k \) th size sediment in the active layer; \( \delta \) is the thickness of the active layer and \( \delta = 2d_{84} \) (\( d_{84} \) is the particle size at which 84% of the sediment is finer) due to Hoey and Ferguson (1994); \( \xi = z - \delta \) is the elevation of the bottom
boundary of the active layer, and $f_{ik}$ is the fraction of the $k$th size sediment at the bottom boundary of the active layer. The complete set of the governing equations for uniform bed load transport can be easily obtained if $N = 1$ in Eqs. (1-5).

The present model is fully coupled as a few source terms reflecting the feedback effects of bed deformation and sediment transport on the flow are kept in the governing equations (i.e., physically coupled, see Figure 1), and equally importantly the whole set of the governing equations are numerically solved synchronously as briefed in subsection 2.3 (i.e., numerically coupled). Specifically, the third and fourth terms in the right-hand side of momentum Eq. (2) and Eq. (3) represent the effect of spatial variation of sediment concentration, and the last (fifth) term reflects the (apparent) momentum transfer due to sediment exchange between the flow and the underlying erodible bed. These source terms result from a straightforward reorganization of the original momentum conservation equations for water-sediment mixture flow, which eliminates the variable density of the water-sediment mixture from the left-hand sides of the equations using the continuity equations (i.e., Eqs. 1, 4, and 5). The reorganization expedites numerical solution using the established finite volume schemes for shallow clear-water flows (Toro, 2001; Toro, 2009). The practice was first implemented by Cao et al. (2004) and has been widely applied since then (Simpson and Castelltort, 2006; Wu, 2007; Wu and Wang, 2008; Xia et al., 2010; Cao et al., 2012; Qian et al., 2015; Cao et al., 2016). In fact, the equations involving these terms have been presented in the Chinese textbook edited by Xie (1990) for undergraduates and also in Wu (2007). Unfortunately, these source terms are usually
ignored in traditional mathematical river models, which may lead to appreciable errors for fluvial processes and even total collapse of the modelling for extreme cases with very active sediment transport and rapid morphological evolution (e.g., dam break floods over erodible beds). While their effects may be relatively minor for slow morphological evolution, but to date, it remains hard to delimit quantitatively when the effects of these terms are all negligible. Concurrently, keeping these source terms does not incur appreciable computing cost, as compared to that in line with the hyperbolic operator. Therefore, the full, rather than simplified (by neglecting the source terms) set of the governing equations should be generally applied, which can maximize model applicability and minimize its inaccuracy. More broadly, extended versions of these equations have been applied for coastal processes (e.g., Xiao et al., 2010; Kim, 2015; Zhu and Dodd, 2015), watershed erosion processes (e.g., Kim et al., 2013), subaqueous sediment-laden flows (e.g., Hu and Cao, 2009; Hu et al., 2012) as well as sharply stratified processes (e.g., Li et al., 2013; Cao et al., 2015; Zech et al., 2015).

Also, the present model is non-capacity based, which explicitly accounts for the temporal and spatial dimensions required for sediment transport to adapt to its capacity regime by virtue of the mass conservation equation (i.e., Eq. 4). In contrast, Eq. (4) is left out in capacity models where the sediment transport rate or sediment concentration is presumed to be always equal to the transport capacity determined by local flow and sediment conditions (e.g., Nelson and Smith, 1989; Takebayashi and Egashira, 2001; Takebayashi and Okabe, 2002; Defina, 2003; Nelson et al., 2015a, b;
Stecca et al., 2015). Non-capacity transport is widely used for suspended load, whilst bed load transport is intuitively believed to be in capacity or able to adapt to capacity rapidly. Yet, recent years have witnessed the study of non-capacity bed load transport (Wu, 2004; Cao et al., 2012; Pelosi and Parker, 2014; Cao et al., 2016). Succinctly, it should be flagged out that a capacity model is only conditionally appropriate (as opposed to non-capacity models that are generally valid), applicable when the effects of the retarded adaptation of bed load transport are negligible. Equally importantly, it remains hard to date to pinpoint when the effects of the retarded adaptation of bed load transport are negligible. Therefore the present non-capacity model is applied (Cao et al., 2016).

Finally, the present model is well-balanced numerically, which means the model is capable of reproducing the exact solution under stationary flow conditions (Bermúdez and Vázquez, 1994; Zhou et al., 2001). This feature is critical if accurate numerical solution is to be sought. In order to satisfy the well-balanced property, the first step here is to use water level $\eta$ in Eqs. (1-3) instead of water depth $h$. In this way, the fluxes and source terms in the momentum equations, which contain the water level $\eta$, can be mathematically balanced at the discrete level (Liang and Borthwick, 2009). In fact, if the water level $\eta$ in Eqs. (1-3) is replaced by $\eta = z + h$, then the traditional shallow water equations with the variable $h$ can be readily obtained.

Compared to most of the similar models for non-uniform sediment transport in alluvial rivers, the present model is either physically advanced (e.g., compared to Hoey and Ferguson, 1994; Takebayashi and Egashira, 2001; Takebayashi and Okabe, 2002;
Wu, 2004; Nelson et al., 2015a, b; Stecca et al., 2015), i.e., fully coupled and non-capacity based, or numerically improved (e.g., in comparison with Hoey and Ferguson, 1994; Takebayashi and Egashira, 2001; Takebayashi and Okabe, 2002; Wu, 2004; Wu and Wang, 2008; Nelson et al., 2015a, b; Stecca et al., 2015; Cao et al., 2016) due to the employment of well-balanced scheme. Thus the present model is generally valid and applicable, even for cases with strong interactions between flow, sediment transport and bed evolution.

2.2 Model closure

Empirical relationships are required to close the governing equations. Firstly, the Manning formula is used for the friction slopes with the Manning roughness $n$

$$S_x = \frac{n^2 u \sqrt{u^2 + v^2}}{h^{4/3}}, \quad S_y = \frac{n^2 v \sqrt{u^2 + v^2}}{h^{4/3}}$$

(7)

The velocity of bed load transport $V_{st}$ is usually appreciably lower than the flow velocity $V_t$ (Einstein, 1950; Chien and Wan, 1999; Greimann et al., 2008), so the sediment-to-flow velocity ratio $\beta_k$ is introduced and estimated by the relation due to Greimann et al. (2008)

$$\beta_k = \frac{V_{st}}{V_t} = \frac{u \cdot 1.1(\frac{\theta_k}{0.047})^{0.17}[1 - \exp(-5\theta_k/0.047)]}{\sqrt{0.047}}$$

(8)

where $u_*$ is the bed shear velocity and $\theta_k = \frac{u_*^2}{(\rho_s/\rho_w - 1)gd_k}$ is the size-specific Shields parameter. Also, bed load transport often deviates from the direction of flow velocity or bed shear stress due to the influence of transverse bed slope and helical
Herein, the influence of helical flow is ignored because of the small curvature of flow pattern induced by bar topography in a straight channel (e.g., Lanzoni, 2000a, b). Therefore, an approach given by Sekine and Parker (1992) is applied to determine the direction angle \( \gamma_k \) of bed load transport

\[
\tan \gamma_k = \frac{\sin \varepsilon - \chi_k \frac{\partial Z}{\partial y}}{\cos \varepsilon - \chi_k \frac{\partial Z}{\partial x}}
\]

(9)

where \( \varepsilon = \arctan(v/u) \) is the angle of flow velocity; \( \chi_k = 0.75(\theta_c/\theta_k)^{1/4} \) is the weighing coefficient suggested by Sekine and Parker (1992), where \( \theta_c \) is the critical Shields parameter for sediment incipient motion and \( \theta_c = 0.047 \) is used for consistency with that in the following formula of Meyer-Peter and Müller (1948). In Eq. (9), if the bed slope effects are ignored, the direction of bed load transport coincides with that of flow velocity, i.e., \( \gamma_k = \varepsilon \).

Sediment entrainment due to turbulence and sediment deposition due to gravitational action are two distinct mechanisms in the sediment exchange between the bed load carrying flow and the underlying bed. Empirically, the sediment entrainment and deposition fluxes are estimated by

\[
E_k = \alpha_k \omega_k c_{bk}
\]

(10)

\[
D_k = \alpha_k \omega_k c_k
\]

(11)

where \( \omega_k \) is the size-specific settling velocity calculated by the formula of Zhang and Xie (1993); \( \alpha_k = c_{bk}/c_k \) is an empirical coefficient physically representing the difference between the mean bed load concentration \( c_{bk} \) in the bed load transport layer and depth-averaged bed load concentration \( c_k \). According to mass
conservation, \( c_k = c_{bk}/c_k = h/h_{bk} \) can be readily derived where the thickness of bed load layer \( h_{bk} \) is determined as \( h_{bk} = \max\left(9\theta d_k, 2d_k\right) \) (Hu et al., 2014). The size-specific sediment concentration at capacity \( c_{ek} \) is calculated by

\[
c_{ek} = \frac{q_{bk}}{h\beta \sqrt{u'^2 + v'^2}}
\]

where \( q_{bk} \) is the unit-width size-specific bed load transport rate at capacity. In the processes of the incipient motion and movement of non-uniform sediment, an important mechanism exists, i.e., coarse grains are easier to be entrained than their counterparts in uniform case because they have higher exposure chance to flow; On the contrary, fine grains are more difficult to be entrained as they are more likely sheltered by coarse grains. This is the so-called hiding and exposure effect. To date, most of the studies on non-uniform sediment transport rate are based on introducing a correction factor, which accounts for the hiding and exposure effect, to modify the existing formulas for uniform sediment transport. In the present work, the sediment is presumed to be only transported as bed load, and the original formula of Meyer-Peter and Müller (1948) (MPM) for uniform bed load transport is modified and extended to be applicable for non-uniform bed load transport by considering the hiding and exposure effect, just similar to Bui and Rutschmann (2010) and Tritthart et al. (2011). The original MPM formula is given as

\[
\frac{q_b}{\sqrt{(\rho_s/\rho_w - 1)gd^3}} = 8(\mu \theta - \theta_c)^{1.5}
\]

where \( \theta = u'^2/[(\rho_s/\rho_w - 1)gd] \) is the Shields parameter; \( \theta_c = 0.047 \) is the critical Shields parameter for sediment incipient motion; \( \mu \leq 1 \) is the ripple factor which...
accounts for the effect of the bedforms on the bed load transport ($\mu = 1$ if no bedform exists). An extension of the MPM formula for non-uniform bed load transport can be obtained by expressing Eq. (13) for several size fractions and incorporating a hiding and exposure factor $\xi_k$:

$$\frac{q_{bd}}{\sqrt{(\rho_s/\rho_w - 1)g\sigma_k^3}} = 8f_{ek}(\mu \theta_k - \xi_k \theta_c)^{1.5}$$

(14)

The hiding and exposure factor $\xi_k$ is usually expressed as a function of $d_k/d_{50}$ or $d_k/d_m$, where $d_{50}$ and $d_m$ are the 50% sieve and arithmetic mean diameters of bed material (e.g., Parker, 1990; Buffington and Montgomery, 1997; Wilcock and Crowe, 2003). However, this method does not account for the influence of bed-material gradation (Wu et al., 2000). Therefore, the factor $\xi_k$ in the present work is determined following Wu et al. (2000) as a function of the hidden and exposed probabilities $p_{hk}$ and $p_{ek}$, i.e., $\xi_k = (p_{ek}/p_{hk})^m$, where the exponent $m$ is theoretically within -1 and 0 and calibrated to be -0.6 in Wu et al. (2000). In the computation of the hidden and exposed probabilities, not only the influence of sediment particle size but also that of bed-material gradation (i.e., the percentage of each size sediment in the active layer) are considered. Specifically, the hidden and exposed probabilities are defined as

$$p_{hk} = \sum_{i=1}^{N} f_{ai} \frac{d_i}{d_k + d_i}$$

(15a)

$$p_{ek} = 1 - p_{hk} = \sum_{i=1}^{N} f_{ai} \frac{d_k}{d_k + d_i}$$

(15b)

The key parameter $f_{ik}$ in the evolution of substrate sediment composition is determined by a commonly used relationship (Hoey and Ferguson, 1994;
Toro-Escobar et al., 1996), which reads

\[
f_{ik} = \begin{cases} 
    f_{sk} & \frac{\partial \xi}{\partial t} \leq 0 \\
    \phi C_k / C + (1 - \phi) f_{sk} & \frac{\partial \xi}{\partial t} > 0 
\end{cases} \tag{16a, b}
\]

where \( f_{sk} \) is the fraction of the \( k \)th size sediment in the substrate layer and \( \phi \) is an empirical weighting parameter. There have been two options about the value of the parameter \( \phi \). The first one is \( \phi \neq 0 \). For example, the parameter \( \phi \) was suggested to range between 0.61 and 0.86 as a function of sediment size in Toro-Escobar et al. (1996). However, the other option is \( \phi = 0 \), i.e., \( f_{ik} = f_{sk} \) when \( \frac{\partial \xi}{\partial t} > 0 \) (Wu, 2004; Wu and Wang, 2008). In this paper, \( \phi = 0 \) is firstly applied for simulation and then discussion is conducted on these two options.

2.3 Numerical algorithm

The numerical algorithm employed in the present 2D model is, in principle, an extension of that in the 1D well-balanced model proposed by Qian et al. (2015). Eqs. (1-4) constitute a hyperbolic system, which can be solved together by a finite volume SLIC scheme (Toro, 2009). Particularly, special treatments of wet-dry interfaces together with a surface gradient method (SGM), which uses the water level \( \eta \) instead of water depth \( h \) in the governing equations and data reconstructions, are incorporated to seek well-balanced solutions. Detailed description and demonstration of the well-balanced property of the present model can be referred to Qian et al. (2015). Here, the numerical algorithm is just introduced simply. Firstly, Eqs. (1-4) are written in a matrix form as
\[
\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{S}_b + \mathbf{S}_f \tag{17}
\]

\[
\mathbf{U} = \begin{bmatrix} \eta \\ \text{hu} \\ \text{hv} \\ \text{hc}_k \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \text{hu} \\ \text{hu}^2 + 0.5g(\eta^2 - 2\eta z) \\ \text{hu}v \\ \text{cos}(\gamma_k)\eta \beta \text{V}_i c_k \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} \text{hv} \\ \text{hv}u \\ \text{sin}(\gamma_k)\eta \beta \text{V}_i c_k \end{bmatrix} \tag{18a, b, c}
\]

\[
\mathbf{S}_b = \begin{bmatrix} 0 \\ -g\eta \frac{\partial z}{\partial x} \\ -g\eta \frac{\partial z}{\partial y} \\ 0 \end{bmatrix} \tag{18d}
\]

\[
\mathbf{S}_f = \begin{bmatrix} -g\text{h}S_{ix} - \frac{(\rho_s - \rho_w)\text{h}^2}{2\rho} \frac{\partial C}{\partial x} + u \frac{\rho_s - \rho_w}{\rho} \sum_{i} \left( \frac{\partial (\beta_i - 1)huc_k}{\partial x} + \frac{\partial (\beta_i - 1)hvc_k}{\partial y} \right) - u(\rho_s - \rho) \frac{E_i - D_i}{\rho} \right] \\ -g\text{h}S_{iy} - \frac{(\rho_s - \rho_w)\text{h}^2}{2\rho} \frac{\partial C}{\partial y} + v \frac{\rho_s - \rho_w}{\rho} \sum_{i} \left( \frac{\partial (\beta_i - 1)huc_k}{\partial x} + \frac{\partial (\beta_i - 1)hvc_k}{\partial y} \right) - v(\rho_s - \rho) \frac{E_i - D_i}{\rho} \right] \tag{18e}
\]

where \( \mathbf{U} \) is a vector of conserved variables; \( \mathbf{F}, \mathbf{G} \) are the convective flux vectors of the flow in the \( x \) - and \( y \) - directions respectively; \( \mathbf{S}_b \) is the source term related to the bed slopes and \( \mathbf{S}_f \) is the source term including the friction terms and other terms representing the effects of sediment transport and bed evolution. Then an explicit finite volume discretization of Eq. (17) gives:

\[
\mathbf{U}_{i,j}^n = \mathbf{U}_{i,j}^{n-1} - \frac{\Delta t}{\Delta x} \left( \mathbf{F}_{i+1/2,j} - \mathbf{F}_{i-1/2,j} \right) - \frac{\Delta t}{\Delta y} \left( \mathbf{G}_{i,j+1/2} - \mathbf{G}_{i,j-1/2} \right) + \Delta t \mathbf{S}_{b,i,j} \tag{19}
\]

\[
\mathbf{U}_{i,j}^{n+1} = \mathbf{U}_{i,j}^n + \Delta t \mathbf{S}_{f,i,j} \tag{20}
\]

where \( \Delta t \) is the time step; \( \Delta x, \Delta y \) are the spatial steps; the subscripts \( i, j \) denote the spatial node indexes in the \( x \) - and \( y \) - directions respectively; the superscript \( m \) denotes the time step index; \( \mathbf{F}_{i+1/2,j}, \mathbf{F}_{i-1/2,j}, \mathbf{G}_{i,j+1/2}, \mathbf{G}_{i,j-1/2} \) are the inter-cell numerical fluxes computed by a well-balanced version of second-order
SLIC scheme (Qian et al., 2015); the bed slope source term $\mathbf{S}_{bi,j}$ is discretized with a centered difference scheme as

$$
\mathbf{S}_{bi} = 
\begin{pmatrix}
0 \\
- g \frac{\eta_{i+1/2,j}^L + \eta_{i-1/2,j}^R}{2} \frac{z_{i+1,j} - z_{i-1,j}}{2\Delta x} \\
- g \frac{\eta_{i,j+1/2}^L + \eta_{i,j-1/2}^R}{2} \frac{z_{i,j+1} - z_{i,j-1}}{2\Delta y} \\
0
\end{pmatrix}
$$

(21)

where $\eta_{i+1/2,j}^L$, $\eta_{i-1/2,j}^R$, $\eta_{i,j+1/2}^L$, and $\eta_{i,j-1/2}^R$ are the reconstructed water levels at the cell interfaces (Qian et al., 2015); the source term $\mathbf{S}_{f}^{RK}$ is computed by a second-order Runge-Kutta (R-K) method:

$$
\mathbf{S}_{f}^{RK} = \frac{1}{2} \left[ \mathbf{S}_i(\mathbf{U}_{i,j}^1) + \mathbf{S}_i(\mathbf{U}_{i,j}^2) \right]
$$

(22)

$$
\mathbf{U}_{i,j}^1 = \mathbf{U}_{i,j}^* 
$$

(23)

$$
\mathbf{U}_{i,j}^2 = \mathbf{U}_{i,j}^1 + \Delta t \mathbf{S}_i(\mathbf{U}_{i,j}^1) 
$$

(24)

Generally, an explicit finite volume method for the hyperbolic system should satisfy the Courant–Friedrichs–Lewy (CFL) condition to ensure its stability, i.e., the Courant number $Cr \leq 1$ (Toro, 2009). However, the stability limit for the SLIC scheme, which uses the FORCE method to calculate numerical fluxes, is a decreasing function of the dimension parameter $\lambda$ ($\lambda = 1, 2, 3$), i.e., $Cr \leq \sqrt{2\lambda - 1}/\lambda$ (Toro, 2009). For the present 2D modelling,

$$
Cr = \Delta t \max \left( \frac{|u_{i,j}| + \sqrt{gh_{i,j}}}{\Delta x}, \frac{|v_{i,j}| + \sqrt{gh_{i,j}}}{\Delta y} \right) \leq \frac{\sqrt{3}}{2} = 0.87
$$

(25)

Two types of boundaries, i.e., open and closed boundaries, are involved in this work.

At an open boundary, such as the inlet or outlet of a channel, the method of characteristics is used for subcritical flow conditions to obtain the updated values of
flow variables, which however should be directly prescribed at the inlet and set to be zero gradients at the outlet for supercritical flows. The depth-averaged sediment concentration $c_k$ at an open boundary, however, needs to be specified. At a closed boundary, such as the side walls of a channel, a free-slip and non-permeable condition is employed, i.e., $\eta_B = \eta_L$, $h_B = h_L$, $u_{Bn} = 0$, $u_{Br} = u_{Lr}$ and $c_{Bk} = c_{Lk}$, where the subscripts $B$ and $L$ denote the positions of the closed boundary and the adjacent inner cell, and the subscripts $n$ and $\tau$ denote the direction normal and tangential to the boundary (Liang and Borthwick, 2009).

The bed deformation and bed surface sediment composition are updated by the discretizations of Eq. (5) and Eq. (6) respectively

$$z_{i,j}^{m+1} = z_{i,j}^m + \Delta t \frac{\sum (D_k - E_k)_{i,j}^{RK}}{1 - \rho}$$

Finally, the sediment composition in the substrate layer is also updated following the updating of the sediment composition in the active layer. In fact, the entire substrate layer is further divided into several storage layers, with the thickness of each storage layer being represented by $L_s$. However, the thickness of the top storage layer is variable in the range of $L \leq L_s$ due to bed aggradation or degradation. In each storage layer, the sediment is assumed to be well mixed. The updating procedure of the substrate sediment composition can be classified into two cases (i.e., bed aggradation and degradation), with a detailed description of the procedure given in Qian et al. (2015).
The treatments of wet-dry interfaces are of great importance in the numerical modelling, and a special treatment is applied in the present model in order to satisfy the well-balanced property. That is, if the water level in a wet cell \((i, j)\) is lower than the bed elevation of its adjacent dry cell \((i + 1, j)\), then the bed elevation and water level of this dry cell are both set equal to the water level of the wet cell temporarily only in the flux calculation section, i.e., \(\eta_{i+1,j} = \eta_{i,j} = \eta_{i,j}\). As a consequence, the depth in the dry cell \((i + 1, j)\) is still zero. Besides, to avoid the occurrence of very small water depth, which can lead to instabilities, a small threshold depth \((1.0 \times 10^{-5} \text{ m})\) is introduced. If the computed water depth is lower than this small threshold depth, then the computed depth, velocity and sediment concentration are all set to be zero.

3. Case study

To evaluate the capability of the present model in modelling the alternate bar formation, development and sediment sorting, several cases are numerically revisited, including a flume experiment with uniform sediment transport (Lanzoni, 2000a), another experiment with non-uniform sediment (Lanzoni, 2000b) and its varied case. Particularly, special emphasis is attached on the longitudinal and vertical sediment sorting over bar topography. The weighting parameter \(\phi\) in Eq. (16) is set to be zero in the modelling except for subsection 3.4, in which \(\phi = 0\) and \(\phi = 0.65\) are both applied and compared. Finally, numerical tests are conducted to study the influence of the aspect ratio on bar formation and development. For all cases, a uniform mesh with \(\Delta x = 0.15 \text{ m}\) and \(\Delta y = 0.05 \text{ m}\) is used, and the time step is specified by \(Cr = 0.6\).
according to Eq. (25) along with empiricism, which always ensures stability. The values of other common parameters are $\rho_w = 1000$ kg m$^{-3}$, $\rho_s = 2650$ kg m$^{-3}$ and $g = 9.8$ m$^2$ s$^{-1}$.

3.1 Alternate bars with uniform sediment transport

Lanzoni (2000a) conducted several experiments about alternate bar development using uniform sediment in a straight rectangular channel, which was 50 m long, 1.5 m wide and 1 m deep. The bed consisted of a single sediment with a geometric mean diameter of 0.48 mm and a density of 2650 kg m$^{-3}$. The initial sloping bed was almost but not exactly plane, with some small bottom disturbances distributed randomly on the bed, which triggered the formation of alternate bars. Steady and constant inflow was maintained at the inlet of the flume and the water level at the downstream end was controlled by an automatic tailgate. Sediment was recirculated during the experiments, that is, the sediment exiting from the flume was transported to the inlet for supplying. Each experiment was stopped when equilibrium conditions were obtained, that is, the bed slope was equal to water surface slope and, on average, the solid discharge and bar features reached nearly constant values.

One of the experiments, Run P1505, is reproduced here firstly and the related experimental conditions are reported in Table 1. In the modelling, a single small bump (0.6 m long, 0.75 m wide and 5 mm high) instead of random disturbances was set up near the inlet, next to the left side wall and perpendicular to the flow direction. Similar to Defina (2003), the length of the computational domain was extended from the initial
50 m to 120 m so that a few formed bars were possible to attain an equilibrium state before reaching the downstream end of the flume. Also, the sediment transport rates at the inlet and outlet of the flume were assumed to be in equilibrium. The ripple factor in Eq. (14) was estimated to be 0.62 according to the total shear stress and its skin friction portion given in Lanzoni (2000a). Meanwhile, the bed load transport rate computed by the extended MPM formula (i.e., Eq. 14) was multiplied by a coefficient of 0.85 in order to fit the measured value (i.e., 94.5 L h⁻¹).

Table 1. Summary of experimental conditions

<table>
<thead>
<tr>
<th>Run</th>
<th>Duration (h)</th>
<th>Discharge (L s⁻¹)</th>
<th>Slope (%)</th>
<th>Depth (cm)</th>
<th>Velocity (m s⁻¹)</th>
<th>Transport rate (L h⁻¹)</th>
<th>Roughness (s m⁻¹/3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1505</td>
<td>28</td>
<td>30</td>
<td>0.452</td>
<td>4.4</td>
<td>0.45</td>
<td>94.5</td>
<td>0.018</td>
</tr>
<tr>
<td>P2009</td>
<td>3</td>
<td>45</td>
<td>0.526</td>
<td>5.0</td>
<td>0.6</td>
<td>391.7</td>
<td>0.0157</td>
</tr>
</tbody>
</table>

The time evolution of bed configuration is shown in Figure 2, in which the bed elevation in each point is plotted with respect to its initial value. The bar formation and development mechanisms can be described as follows. First of all, the initial bump is eroded quickly but it distorts the otherwise uniform flow pattern and gives rise to the first bar downstream the bump. Then the first bar triggers a train of new bars downstream. The formed bars migrate downstream gradually, increasing their lengths and heights. Finally they approach an equilibrium state and the bar features (length, height, celerity) tend to be stable or vary slowly. As shown in Figure 2, the channel
bed exhibits a clear alternate pattern during the bar development, with recursive scour/deposition sequences and diagonal fronts. This process is in accordance with the experimental observations of Fujita and Muramoto (1985) and the numerical findings of Defina (2003).

Figure 3 shows (a) the longitudinal bed profiles and (b) the difference between RHS (right-hand-side) and LHS (left-hand-side) bed elevations during the equilibrium phase of Run P1505 ($t = 28$ h). Herein, the LHS and RHS denote the longitudinal sections located 20 cm from the left and right side walls respectively. In Figure 3(b), to allow for a straightforward comparison with the computed bar sequences, an extra line is added, representing the measured signal shifted downstream for a distance of 42 m. The computed results in Figure 3 show a favorable comparison with the experimental findings, though the numerical bars exhibit a more ordered pattern. For quantitative comparison, the computed and measured equilibrium bar features are listed in Table 2, where bar length denotes the wavelength of a bar unit; bar height is defined as the difference between the maximum and minimum bed elevations within a bar unit; and bar celerity is estimated by comparing the plots of bed profiles at different times. The first three bars in Figure 3 approximately reach the equilibrium state and thus are selected to estimate averaged bar features. It can be seen from Table 2 and Figure 3 that the computed bar height is very similar to the measured, whilst the bar length is notably larger and the celerity is smaller than the observed. Yet, the computed bar length and celerity of Run P1505 are close to those simulated by Defina (2003), in which the computed bar length is 14 m, the height is 8 cm and the
Several reasons might be responsible for the discrepancies between the experimental and numerical results. First, the initial conditions are difficult to be set as the same as in the experiments, especially the initial bed disturbance. Second, the boundary conditions in the modelling are somewhat different from the experimental set-up. For example, the sediment feeding at the inlet is assumed to be uniformly distributed across the channel width in the modelling, but this is difficult to be satisfied during the experiments. The otherwise non-uniform distribution of sediment at the inlet, however, may act as an extra disturbance affecting the formation of alternate bars. Meanwhile, the presumed equilibrium conditions at the upstream and downstream ends are just approximations to the experimental conditions. Finally, the empirical relationships and parameters for model closure may also inevitably bring about some discrepancies.
**Figure 2.** The time evolution of bed configuration from plan views for Run P1505. Transverse scale is enlarged in order to better visualize alternate patterns.

![Figure 2](image)

**Figure 3.** The longitudinal bed profiles \( z_b \) and the difference between RHS and LHS bed elevations \( \Delta z_b \) during the equilibrium phase of Run P1505 \((t = 28 \text{ h})\)

![Figure 3](image)

**Table 2.** The computed and measured bar features

<table>
<thead>
<tr>
<th>Run</th>
<th>Length (m)</th>
<th>Height (cm)</th>
<th>Celerity (m h(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1505</td>
<td>10.0 / 15.0</td>
<td>6.0 / 5.7</td>
<td>2.8 / 1.8</td>
</tr>
<tr>
<td>P2009</td>
<td>10.2 / 12.8</td>
<td>3.4 / 3.2</td>
<td>11.0 / 10.2</td>
</tr>
</tbody>
</table>

The values before and after "/" are the measured and computed bar features respectively; * the value of bar height in Run P1505 is given as 7.0 in Lanzoni (2000a)
but modified here to be 6.0 according to the detailed data of bed profiles reported by Lanzoni (2000a).

3.2 Alternate bars with non-uniform sediment transport

Similar to the experiments using uniform sediment in Lanzoni (2000a), a series of experiments about the alternate bar development due to non-uniform sediment transport were also carried out by Lanzoni (2000b). The flume, control devices and experimental procedures are all the same as those in Lanzoni (2000a). The sediment mixture, composed of 67% fine sand and 33% coarse gravel, was strongly bimodal and each fraction can be approximately treated as uniform sediment. The geometric mean diameters of the sand and gravel were 0.2 mm and 2.0 mm respectively, and thus the weighted mean diameter of the initial sediment mixture was calculated to be nearly 0.8 mm. In the modelling, an initial single bump (0.6 m long, 0.75 m wide and 5 mm high) was introduced and the computational domain was extended to 120 m. Equilibrium conditions for sediment transport were imposed at the upstream and downstream ends of the flume. Run P2009 is chosen herein to study the bar development mechanisms and characteristics over non-uniform sediment bed, and its experimental conditions are listed in Table 1. The ripple factor was estimated to be 0.8 and the bed load transport rate computed by the extended MPM formula (i.e., Eq. 14) was multiplied by 1.3 to fit the measured value (i.e., 391.7 L h⁻¹).

Figure 4 shows (a) the longitudinal bed profiles and (b) the difference between RHS and LHS bed elevations (20 cm from each wall) at the initial phase of Run P2009 (t =
In accordance with the experimental observations, a regular sequence of well-formed alternate bars are numerically reproduced quickly after the beginning of the run ($t = 1$ h). And both the computed bar shapes and amplitudes are in fairly satisfactory agreement with the observed ones (Figure 4). Quantitatively, the first two well-formed bars in Figure 4 are selected to estimate the bar features, and it is found that the computed bar length is a little larger whilst the bar height and celerity are somewhat smaller in comparison with the measured, but the differences are rather limited (Table 2). On the whole, the present model reproduces the bar development and features reasonably well.

**Figure 4.** The longitudinal bed profiles ($Z_b$) and the difference between RHS and LHS bed elevations ($\Delta Z_b$) at the initial phase of Run P2009 ($t = 1$ h)
In modelling the alternate bar formation and development, the effect of transverse bed slope (i.e., gravity effect) on the bed load transport direction has been accounted for in previous studies (e.g., Struiksma and Crosato, 1989; Defina, 2003; Federici and Seminara, 2003; Bernini et al., 2006; Crosato et al., 2011, 2012). In order to shed light on this effect, the computed bar sequences with considering the effect of transverse bed slope ($\gamma_k \neq \varepsilon$) or not ($\gamma_k = \varepsilon$) are compared. It is seen in Figure 5 that the height of alternate bars is dampened if the bed slope effect on bed load transport is included, and this damping effect is more pronounced for Run P1505 with larger bed deformation and steeper bed slopes. However, the bar wavelength and celerity seem to be weakly affected. The damping effect, due to the transverse down-slope pull of sediment particles by gravity from bar heads, was also numerically resolved by Defina (2003) and Bernini et al. (2006).
Figure 5. The measured and computed bar topography (i.e., the difference between RHS and LHS bed elevations) with considering the effect of transverse bed slope 
\((\gamma_k \neq \varepsilon)\) or not \((\gamma_k = \varepsilon)\) for (a) Run P1505 and (b) Run P2009

3.3 Sediment sorting

Sediment sorting is one of the important issues on the study of alternate bars, which includes longitudinal and transverse sorting due to selective transport of sediment fractions over bar topography as well as vertical sorting resulting from bar migration through scouring and filling. The last several decades have seen many investigations on sediment sorting over bar topography. Lisle and Madej (1992) conducted field observations and analysis on the Redwood Creek with a large in-channel supply of
bed load. The results show that alternate bar topography induces strong variations in bed shear stress and thus fine sediment is winnowed from zones of high stress such as riffles or bar heads, whilst deposits as thin sheets in zones of low stress such as pools. In addition to field observations, there have been some laboratory experiments focusing on the sediment sorting during bar development. The experiments conducted in a steep channel with mixed sediment by Lisle *et al.* (1991) show that coarse particles accumulate on bar heads, which prevents downstream migration of bars by inhibiting bar-head erosion and bed load transport over bars. Diplas (1994) sampled the bed surface sediment at the head, middle, tail and pool of a bar unit, and found that their diameters satisfy the relationship $d_{\text{head}} > d_{\text{middle}} \approx d_{\text{pool}} > d_{\text{tail}}$. The phenomenon of bar head coarsening was also observed by Lanzoni (2000b), which revealed that the longitudinal sorting of the bed surface sediment is dictated by two factors, i.e., bed shear stress and gravity. The spatial variation of bed shear stress leads to selective deposition of coarse particles toward bar heads whilst the gravity tends to pull coarse particles downward to pools. However, the former mechanism appears to prevail because of the relatively gentle slope of the bar topography. On the contrary, the laboratory experiments performed by Takebayshi and Egashira (2001) suggest that fine sediment deposits around bar heads whilst coarse sediment is left in pools. Similarly, Blom *et al.* (2003) examined the sediment sorting on dune-covered beds through flume experiments, and found that when a relatively large amount of coarse sediment does not participate in the transport process (i.e., a condition of partial transport), the vertical sediment shows a downward coarsening trend within the
dunes and the surface sediment in pools is coarser than that on bar heads. Apart from field observations (Lisle and Madej, 1992) and laboratory experiments (Lisle et al., 1991; Diplas, 1994; Lanzoni, 2000b; Takebayashi and Egashira, 2001; Takebayashi and Okabe, 2002) as just briefed, a few numerical studies of sediment sorting over bar topography are also available (Takebayashi and Egashira, 2001; Takebayashi and Okabe, 2002; Nelson et al., 2015a, b). Unfortunately, previous experimental and numerical studies to date have shown two inconsistent sediment sorting patterns, i.e., a pattern of coarse-on-head and fine-in-pool (Lisle et al., 1991; Lisle and Madej, 1992; Diplas, 1994; Lanzoni, 2000b; Nelson et al., 2015a, b) and the opposite, i.e., a pattern of fine-on-head and coarse-in-pool (Takebayashi and Egashira, 2001; Takebayashi and Okabe, 2002; Blom et al., 2003). The mechanisms behind the inconsistent sorting patterns, however, have so far remained poorly understood. According to whether all sediment fractions in the bed are set into motion and transport, non-uniform sediment transport can be classified to two conditions, i.e., full transport and partial transport. In this subsection, sediment sorting over bar topography is numerically studied as per full and partial transport conditions.

3.3.1 Full transport

For Run P2009 with high bed shear stress and a bimodal sediment mixture, a condition of full transport is attained, under which both the two size fractions of the bed sediment can take part in the transport process completely (Lanzoni, 2000b). As Figure 6 shows, the computed surface sediment distribution from plan view exhibits a
similar "bar-pool" sequence as the bed topography (Takebayashi and Egashira, 2001).

Particularly, the sediment in the deposition areas (e.g., bar heads) is usually coarse, whereas fine sediment prevails in the scour areas (e.g., pools). This is the case for both longitudinal and transverse sections. Typically, only longitudinal sediment sorting is addressed further in detail. Figure 7 quantitatively shows the computed bed profiles and surface mean sediment diameter on the LHS (20 cm from the left wall) for Run P2009, where the bed elevation in each point is plotted with respect to its initial value and the mean sediment diameter $d_m$ is defined as a weighted average of the diameters of all sediment fractions by their percentages. It can be seen from Figure 7 that the longitudinal diameter profiles closely resemble the bed profiles in the shape, and concurrently the mean sediment diameter on the bed surface increases (decreases) with the increasing (decreasing) of the bed elevation roughly. Though the first couple of bars are the largest in amplitude and length, the most appreciable sediment sorting occurs around the second or the third couple of bars (Figures 6 & 7) probably because the flow and sediment conditions near the first couple of bars are still in adjusting and transition. Anyway, the sediment around the bar head is the coarsest whilst it is the finest close to the pool within a bar unit (Figure 7). This longitudinal sorting pattern agrees with the experimental findings reported by Lisle et al. (1991), Lisle and Madej (1992), Diplas (1994) and Lanzoni (2000b) as well as the numerical results in Nelson et al. (2015a, b). In fact, nearly all sediment fractions can join in the transport just as Lisle et al. (1991), Lisle and Madej (1992), Diplas (1994) and Nelson et al. (2015a, b) indicated. In Lanzoni (2000b), though there are also a few
experimental runs under conditions of partial transport, only the sediment sorting for Run P2009 with regard to full transport was measured and addressed. Thus it can be inferred that the sorting pattern of coarse-on-head and fine-in-pool observed by Lanzoni (2000b) represents the full transport condition.

In essence, the sediment incipient motion, transport and sorting are mainly dictated by two factors, i.e., bed shear stress and gravity (Lanzoni and Tubino, 1999; Lanzoni, 2000b). When the shear stress is large enough to be dominant such as in Run P2009, all sediment fractions are set into motion and fully transported, then the sediment selective transport and the sorting process are mainly governed by bed shear stress. As Figure 8 shows, the shear stress along the bar upstream face increases rapidly, so a large amount of fine sediment and a relative small part of coarse sediment are winnowed from the bed, consequently the bed surface sediment gets coarsening gradually. In the region around the bar head, the shear stress starts to decrease and then the coarse sediment in the flow deposits firstly whilst the fine sediment on the bed surface keeps being winnowed, so the surface sediment near the bar head continues coarsening. On the lee face of bar, the shear stress drops very rapidly and thus a lot of fine sediment begins to deposit, which leads to bed sediment fining. Finally, fine sediment keeps depositing in the pool area where the shear stress is still at low values.
Figure 6. The plan views of (a) bed topography and (b) surface mean sediment diameter during the initial and final phases of Run P2009.
Figure 7. The bed profiles and surface mean sediment diameter on the LHS (20 cm from the left wall) during the initial and final phases of Run P2009 (bed elevation is plotted with respect to the initial value).
Figure 8. The bed profiles and bed shear stress on the LHS (20 cm from the left wall) during the initial and final phases of Run P2009 (bed elevation is plotted with respect to the initial value).

Figure 9 shows the distribution of the mean sediment diameter $d_m$ in the substrate layer at three longitudinal sections (i.e., 20 cm from the left wall, 20 cm from the right wall and the axis of the channel) for Run P2009 at $t = 1$ h. Vertically, a typical bar unit can be divided into three layers from the bottom to top, i.e., immobile layer, fining layer and coarsening layer (Figure 9a, b). The immobile layer is located in the bottom of substrate, where the sediment cannot be disturbed and does not exchange with the upper sediment. The fining layer lies between the immobile layer and coarsening layer,
and is usually underneath the averaged bed level. When the fining layer comes to
form, the local bed elevation is low and the shear stress is small, so that fine sediment
is transported towards and then deposits in this layer, therefore the sediment is finer
than the initial sediment mixture. The uppermost layer is the coarsening layer, which
is usually above the averaged bed level. Because of the high bed elevation and large
shear stress during the formation of this layer, fine sediment is winnowed whilst
course sediment deposits on the bed, so the sediment in this layer keeps coarsening.
However, for the axis of the channel, only immobile layer and coarsening layer can be
observed clearly due to the small bed deformation (Figure 9c).

For a quantitative description of the sediment sorting, a non-dimensional parameter
\( \frac{d_m}{d_0} - 1 \) is introduced, with \( d_m \) and \( d_0 \) denoting the final and initial mean
sediment diameter in the substrate layer, respectively. Apparently, \( \frac{d_m}{d_0} - 1 > 0 \)
indicates sediment coarsening whilst \( \frac{d_m}{d_0} - 1 < 0 \) means sediment fining. The
computed and measured vertical distributions of the parameter \( \frac{d_m}{d_0} - 1 \) are plotted
in Figure 10 for four different locations (i.e., pool, head, middle places of upstream
face and downstream face), which is sampled along the second bar unit on the LHS
for Run P2009. It can be seen from Figure 10 that the computed results generally
agree well with the measured data except for some local deviations. As shown in
Figure 10(a), the vertical sediment diameter in the pool keeps roughly equal to the
initial value except that the sediment approaching the bed surface exhibits a little
fining. For the remaining three locations, however, the substrate sediment shows a
trend of firstly fining and then coarsening from the bed bottom to top (Figure 10b, c, d).
Generally, the present model reproduces the sediment sorting process in the experiments of Lanzoni (2000b) fairly well.

Figure 9. The distribution of the mean sediment diameter in the substrate layer for Run P2009 \((t = 1 \text{ h})\)
Figure 10. The comparison of the computed (present model) and measured (Lanzoni, 2000b) vertical distributions of the quantity $d_m/d_0 - 1$ at four different locations sampled along the second bar unit on the LHS at $t = 1$ h for Run P2009.
3.3.2 Partial transport

Besides experimental runs under full transport conditions like Run P2009, there are also some runs under partial transport in line with low bed shear stress in Lanzoni (2000b). However, in some runs where only fine fraction was transported while coarse fraction kept still, no bars were observed to form except for ripples. In other runs under conditions of partial transport, a great proportion of coarse sediment took part in the transport with the increase of bed shear stress, while only the very big coarse particles remained immobile. The bimodal sediment mixture in these runs, however, can be approximately treated as full transport if two representative diameters are used in the modelling. Moreover, no measurements about the sediment sorting in these runs were conducted or addressed by Lanzoni (2000b). Therefore, to examine the sediment sorting under the condition of partial transport, a varied case from Run P2009 is designed in this subsection with a new sediment mixture, i.e., \( d_1 = 2 \text{ mm} \) (67%) and \( d_2 = 6 \text{ mm} \) (33%), and accordingly the initial mean sediment diameter was about 3.32 mm. Other hydraulic conditions and parameters were set to be the same as Run P2009. In this case, the first sediment fraction \( (d_1 = 2 \text{ mm}) \) can be fully mobilized, whereas the second one \( (d_2 = 6 \text{ mm}) \) is too coarse to participate into the transport.

Figure 11 shows the bar-pool sequences and sediment sorting pattern from plan view for this varied case. In this case, the sediment is usually fine in the deposition areas but coarse in the scour areas (Figure 11). Typically, as per the longitudinal section 20 cm from the left wall, the bed surface sediment in the pool area of a bar unit is coarse
whilst it is fine around the bar head (Figure 12), which is opposite to the longitudinal surface sediment sorting under the condition of full transport (i.e., Figure 7). But this sorting pattern of fine-on-head and coarse-in-pool is consistent with the observations by Takebayshi and Egashira (2001) and Takebayashi and Okabe (2002), where the shear stresses for coarse fractions were estimated to be below the threshold for incipient motion, and also by Blom et al. (2003) on dunes, where partial transport conditions were present in all experiments as the proportions of coarse fractions in the bed load transport were appreciably smaller than those in the substrate. In this varied case from Run P2009, both gravity and bed shear stress play important roles in the processes of sediment transport and sorting. The coarse particles keep nearly immobile throughout the case as gravity prevails, whilst the fine particles are entrained from the pool surface due to bed shear stress and then deposit around the bar heads.

The vertical sediment sorting in the substrate layer for the varied case from Run P2009 is illustrated in Figure 13, which shows that the substrate sediment of a typical bar unit is characterized by an immobile-coarse-fine structure from the bed bottom to surface, which is also different from Figure 9. The thin coarsening layer, which is above the immobile layer, is generated due to the accumulation of coarse particles by gravity and also the winnowing of fine particles. Fine particles conversely begin to deposit because of the variation of shear stress, and as a result, a fining sediment layer develops and covers the coarsening layer. This kind of firstly coarsening and then fining trend for alternate bars under partial transport conditions is very similar to
that for dunes in Blom et al. (2003).

In summary, the present numerical study shows that different transport conditions of non-uniform sediment (i.e., full versus partial transport) give rise to different or even inconsistent sediment sorting patterns, which have been observed separately in previous experimental and numerical studies. Therefore the long-standing inconsistency on non-uniform sediment sorting over bar topography is reconciled.

Figure 11. The plan views of (a) bed topography and (b) surface mean sediment diameter for the varied case from Run P2009.
Figure 12. The bed profiles and surface mean sediment diameter on the LHS (20 cm from the left wall) for the varied case from Run P2009 (bed elevation is plotted with respect to the initial value).
Figure 13. The distribution of the mean sediment diameter in the substrate layer for the varied case from Run P2009 ($t = 3$ h)
3.4 Discussion on weighting parameter $\phi$

In the simulation of alternate bar development due to non-uniform sediment transport, it was found that the value of the empirical weighting parameter $\phi$ in Eq. (16) has great effects on the computed results especially the sediment sorting process. According to Eq. (16), $\phi = 0$ indicates that each sediment fraction at the bottom boundary of the active layer is just equal to that in the active layer (Wu, 2004; Wu and Wang, 2008), while $\phi \neq 0$ represents a linear weighted average of sediment fractions in the flow and active layer (Toro-Escobar et al., 1996). To examine which option is more applicable for the bar modelling, comparisons are made between results computed with $\phi = 0$ and $\phi \neq 0$ in the following. For simplicity, $\phi = 0.65$ is used as suggested by Toro-Escobar et al. (1996).

3.4.1 Full transport

Firstly, the computed results with $\phi = 0$ and $\phi = 0.65$ are compared for Run P2009, in which the sediment is under the condition of full transport. As shown in Figure 14, the bed profiles computed with $\phi = 0$ and $\phi = 0.65$ are very similar in shape and amplitude except for a small phase difference, and both the two longitudinal distributions of sediment diameter follow the rule of coarse-on-head and fine-in-pool. However, the mean sediment diameter in each pool computed with $\phi = 0$ is always finer than the initial one, but it is not the case for $\phi = 0.65$ (Figure 14). With regard to the vertical sediment sorting in the substrate layer shown in Figure 15, both the results from $\phi = 0$ and $\phi = 0.65$ show a clear three-layer structure (i.e., immobile layer,
fining layer and coarsening layer), but the sediment in the top coarsening layer computed with $\phi = 0$ tends to be coarser than that for $\phi = 0.65$.

**Figure 14.** Computed bed profiles and surface mean sediment diameter with $\phi = 0$ and $\phi = 0.65$ on the LHS (20 cm from the left wall) for Run P2009 at $t = 1$ h.
**Figure 15.** Computed distribution of the mean sediment diameter in the substrate layer with $\phi = 0$ and $\phi = 0.65$ for Run P2009 at $t = 1$ h

3.4.2 Partial transport

For the condition of partial transport, the differences between computed results with $\phi = 0$ and $\phi = 0.65$ are also studied based on the varied case from Run P2009. Figure 16 shows that the bar shapes, amplitudes and phases with $\phi = 0$ and $\phi = 0.65$ are all different with each other. Also, the sediment around bar heads with $\phi = 0$ is always finer than the initial sediment mixture whilst it is the opposite for $\phi = 0.65$. Theoretically, in this case fine sediment is winnowed from the pool area of a
bar unit and then deposits around the bar head whilst coarse sediment keeps nearly immobile, thus the mean sediment diameter on the head should be finer than the initial diameter. That is to say, the longitudinal sediment sorting with $\phi = 0.65$ is unreasonable whilst that with $\phi = 0$ is satisfactory (Figure 16). Vertically, a thin coarsening sediment layer forms in the bed substrate with $\phi = 0$ (Figure 17a), as observed by Blom et al. (2003), which however does not exist if bars are simulated with $\phi = 0.65$ (Figure 17b).

In general, whether the weighting parameter $\phi$ is set to be zero makes a great difference to the modelling of alternate bar development due to non-uniform sediment transport. Particularly, the simulated sediment sorting process with $\phi = 0.65$ does not agree with the experimental findings if partial transport condition prevails. In fact, the coarse sediment rarely joins in the transport under the condition of partial transport, thus the coarse fraction in the flow is nearly nonexistent (i.e., $C_k/C = 0$) and thus $f_{ik} = (1-\phi)f_{ak}$ can be derived according to Eq. (16). Theoretically, only $f_{ik} = f_{ak}$ (i.e., $\phi = 0$) can guarantee the mass conservation for bed sediments. That explains why the computed results with $\phi = 0.65$ are unreasonable under the condition of partial transport.
Figure 16. Computed bed profiles and surface mean sediment diameter with $\phi = 0$ and $\phi = 0.65$ on the LHS (20 cm from the left wall) for the varied case from Run P2009 at $t = 3$ h

Figure 17. Computed distribution of the mean sediment diameter in the substrate layer with $\phi = 0$ and $\phi = 0.65$ for the varied case from Run P2009 at $t = 3$ h
3.5 Bar development with different aspect ratios

Theoretical studies (e.g., Colombini et al., 1987; Tubino, 1991; Lanzoni and Tubino, 1999; Siviglia et al., 2013) suggest that bar features depend on three non-dimensional parameters, i.e., aspect ratio $\beta = B/\bar{h}$ ($B$ is the channel width and $\bar{h}$ is the averaged flow depth), Shields parameter $\theta$, and relative grain roughness height $d_s = d/\bar{h}$. Qualitatively, no migrating bars can form in a narrow and deep channel with a small aspect ratio. When the ratio exceeds a critical value $\beta_c$, migrating alternate bars start to grow. For a too large aspect ratio, however, multiple-row bars or linguoid bars instead of alternate bars appear. Chang et al. (1971) demonstrated experimentally that migrating bars can form when aspect ratio exceeds 10, i.e., $\beta_c \approx 10$. Moreover, linear stability theories show that this critical ratio is related to Shields parameter $\theta$ and relative grain roughness height $d_s$, and some critical values given in previous literatures are listed in Table 3. Generally, the critical aspect ratio varies between 10 and 20 according to existing studies.

To give a further insight into the effect of this ratio using the present 2D model, a series of numerical tests (Table 4) is conducted based on the flow and sediment conditions of Run P2009 by only varying channel widths. Specifically, the flow depth, velocity, bed slope, sediment composition and sediment transport rate in the numerical cases are all set to be the same as Run P2009 whilst the flow discharge varies in proportion to channel width (the unit-width discharge is kept unchanged). In these tests, the Shields parameter $\theta$ and relative grain roughness height $d_s$ keep the same, i.e., $\theta = 0.18$, $d_s = 0.017$, while the aspect ratio differs with each other.
The present numerical tests (Table 4) show that under the condition of $\theta = 0.18$ and $d_s = 0.017$, no migrating bars can occur in a straight channel if the aspect ratio $\beta < 12$. For the channel with the ratio $12 \leq \beta \leq 50$, alternate bars form and develop. However, if the channel is too wide and the water is too shallow ($\beta > 50$), multiple-row bars, instead of alternate bars, are spotted from the numerical solutions. Therefore, the critical aspect ratio for the formation of migrating alternate bars here is approximately 12, which is consistent with the results of Siviglia et al. (2013) and close to the estimations of Chang et al. (1971) and Federici and Seminara (2003). It should be noted that the critical aspect ratio 12 may not be generally valid, as it will vary with different values of Shields parameter $\theta$ and relative grain roughness height $d_s$ (Table 3). Particularly, in Colombini et al. (1987), the critical ratio $\beta_c$ is found to be greater for smaller values of $d_s$ and closer values of $\theta$ to 0.18. Additionally, a natural river usually tends to be meandering if the aspect ratio is relatively large, which however does not occur in the present numerical tests because the channel banks are presumed to be straight, parallel and non-erodible.

Figure 18 shows the difference between LHS and RHS bed elevations (B/10 from each wall) at $t = 1$ h for different aspect ratios, and the corresponding bar features are listed in Table 4. It can be seen that there is a progressive growth in bar length with the increase of the aspect ratio $\beta$, whilst the bar celerity is weakly sensitive to this ratio as it only increases slightly. The bar height exhibits a rapid growth when the aspect ratio increases but still has moderate values ($\beta = 12 \sim 30$), because the restriction of sidewalls to the lateral spread of turbulence is weakened and the wider
channel allows the full development of bars. If this ratio continues increasing ($\beta = 30 \sim 50$), the bar height keeps nearly stable, probably because the effect of the sidewalls on the bar development can be ignored. Similarly, Crickmore (1970) and Friedrich et al. (2007) conducted experiments on the effect of flume width on the characteristics of small bed forms such as dunes, and found that the height of the bed forms in a wide flume is larger than that in a narrow flume. Thus a conclusion was drawn, i.e., the bed form height will grow with the increase of channel width, which is similar to our findings about bar development with moderate values of the aspect ratio ($\beta = 12 \sim 30$).

Unfortunately, to the authors' knowledge, there have been few experimental or numerical studies on the variation of alternate bar features with different aspect ratios. In this sense, the present findings by numerical tests remain to be confirmed by observations.
Table 3. Summary of critical values $\beta_c$ in previous studies

<table>
<thead>
<tr>
<th>Literatures</th>
<th>Shields parameter $\theta$</th>
<th>roughness height $d_s$</th>
<th>critical value $\beta_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chang et al. (1971)</td>
<td>/</td>
<td>/</td>
<td>10</td>
</tr>
<tr>
<td>Colombini et al. (1987)</td>
<td>0.3</td>
<td>0.01</td>
<td>21</td>
</tr>
<tr>
<td>Lanzoni and Tubino (1999)</td>
<td>0.07</td>
<td>0.01</td>
<td>17</td>
</tr>
<tr>
<td>Lanzoni and Tubino (1999)</td>
<td>0.1</td>
<td>0.01</td>
<td>20</td>
</tr>
<tr>
<td>Federici and Seminara (2003)</td>
<td>0.057</td>
<td>0.053</td>
<td>11.2</td>
</tr>
<tr>
<td>Siviglia et al. (2013)</td>
<td>0.1</td>
<td>0.061</td>
<td>12</td>
</tr>
</tbody>
</table>

Table 4. Bar features in numerical cases with different width-to depth ratios

<table>
<thead>
<tr>
<th>Case</th>
<th>Aspect ratio</th>
<th>Length (m)</th>
<th>Height (cm)</th>
<th>Celerity (m/h)</th>
<th>Remarks</th>
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<tbody>
<tr>
<td>P11</td>
<td>11</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>no bars</td>
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<tr>
<td>P12</td>
<td>12</td>
<td>10.0</td>
<td>0.5</td>
<td>9.5</td>
<td>alternate bars</td>
</tr>
<tr>
<td>P15</td>
<td>15</td>
<td>10.2</td>
<td>2.0</td>
<td>9.9</td>
<td>alternate bars</td>
</tr>
<tr>
<td>P20</td>
<td>20</td>
<td>11.3</td>
<td>2.8</td>
<td>10.0</td>
<td>alternate bars</td>
</tr>
<tr>
<td>P30 (P2009)</td>
<td>30</td>
<td>12.8</td>
<td>3.2</td>
<td>10.2</td>
<td>alternate bars</td>
</tr>
<tr>
<td>P40</td>
<td>40</td>
<td>14.2</td>
<td>3.2</td>
<td>10.5</td>
<td>alternate bars</td>
</tr>
<tr>
<td>P50</td>
<td>50</td>
<td>15.0</td>
<td>3.1</td>
<td>10.6</td>
<td>alternate bars</td>
</tr>
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<td>P55</td>
<td>55</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>multiple-row bars</td>
</tr>
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</table>
Figure 18. The difference between LHS and RHS bed elevations (B/10 from each wall) at $t = 1 \text{ h}$ for different aspect ratios

4. Conclusion

Alternate bar formation, development and sediment sorting in straight channels are numerically investigated using a coupled, non-capacity and well-balanced 2D shallow water hydro-sediment-morphodynamic model. The following findings can be drawn from the numerical solutions:

(1) The whole process of alternate bar formation and development is well reproduced, as indicated by the fairly satisfactory agreement for the bar features (i.e., bar length, height and celerity) with currently available experimental data and numerical solutions.

(2) The present numerical study facilitates new understanding of the longitudinal and
vertical sediment sorting. Specifically, under the condition of full transport of
non-uniform sediment, the surface sediment on the bar head is coarser than in the
pool of a bar unit and the vertical sediment in the substrate layer shows an
immobile-fine-coarse structure from bottom to top. On the contrary, under the
condition of partial transport, the surface sediment is finer on the bar head but coarser
in the pool, and an immobile-coarse-fine structure is observed for the vertical
distribution of substrate sediment. This finding reconciles the long-standing
inconsistency between previous studies on non-uniform sediment sorting over
alternate bar topography.

(3) The value of the empirical weighting parameter $\phi$ in Eq. (16), which estimates
the sediment composition at the bottom boundary of the active layer, has a
considerable effect on the sediment sorting. Only $\phi = 0$ can guarantee the mass
conservation for bed sediments whilst $\phi \neq 0$ (e.g., $\phi = 0.65$) gives rise to unrealistic
results, especially when partial transport condition prevails.

(4) Numerical experiments with specified conditions (i.e., Shields parameter $\theta = 0.18$
and roughness height $d_s = 0.017$) show that migrating alternate bars can form and
develop in a straight channel with non-erodible banks when the aspect ratio ranges
between 12 and 50. In particular, with the increase of the aspect ratio, the bar length
grows gradually, while the bar height increases rapidly for moderate values of the
aspect ratio and then keeps nearly stable. The bar celerity, however, is weakly
sensitive to the variation of this ratio.

The present findings are qualitatively encouraging and physically meaningful. Yet,
uncertainties are inevitable quantitatively, which may arise from the empirical parameters in the model and computational conditions, such as the boundary conditions and initial disturbances in space and time. Also, the Reynolds stresses representing turbulent effects are ignored in the model, which however should be included when the turbulent effects are considerable such as in acutely curved and meandering channels. Importantly, more experimental investigations are warranted to enhance the understanding of the alternate bars with heterogeneous sediment compositions and to further confirm the present findings.

Acknowledgements

This work is funded by National Natural Science Foundation of China (Grants No. 11172217, 51279144 and 11432015).
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Lisle TE, Iseya F, Ikeda H. 1993. Response of a channel with alternate bars to a


Parker G. 1976. On the cause and characteristic scales of meandering and braiding in


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Table 1. Summary of experimental conditions

<table>
<thead>
<tr>
<th>Run</th>
<th>Duration (h)</th>
<th>Discharge (L s(^{-1}))</th>
<th>Slope (%)</th>
<th>Depth (cm)</th>
<th>Velocity (m s(^{-1}))</th>
<th>Transport rate (L h(^{-1}))</th>
<th>Roughness (s m(^{-1/3}))</th>
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</thead>
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<td>30</td>
<td>0.452</td>
<td>4.4</td>
<td>0.45</td>
<td>94.5</td>
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<td>P2009</td>
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<td>45</td>
<td>0.526</td>
<td>5.0</td>
<td>0.6</td>
<td>391.7</td>
<td>0.0157</td>
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Table 2. The computed and measured bar features

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<th>Height (cm)</th>
<th>Celerity (m h⁻¹)</th>
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<tr>
<td>P1505</td>
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<td>6.0 / 5.7</td>
<td>2.8 / 1.8</td>
</tr>
<tr>
<td>P2009</td>
<td>10.2 / 12.8</td>
<td>3.4 / 3.2</td>
<td>11.0 / 10.2</td>
</tr>
</tbody>
</table>

The values before and after " / " are the measured and computed bar features respectively; * the value of bar height in Run P1505 is given as 7.0 in Lanzoni (2000a) but modified here to be 6.0 according to the detailed data of bed profiles reported by Lanzoni (2000a).
<table>
<thead>
<tr>
<th>Literatures</th>
<th>Shields parameter $\theta$</th>
<th>roughness height $d_s$</th>
<th>critical value $\beta_c$</th>
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<td>0.061</td>
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<td>Case</td>
<td>Aspect ratio</td>
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<td>Height (cm)</td>
</tr>
<tr>
<td>------</td>
<td>--------------</td>
<td>------------</td>
<td>-------------</td>
</tr>
<tr>
<td>P11</td>
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</table>
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**Figure 1.** Flow-sediment-bed interactions in alluvial rivers

**Figure 2.** The time evolution of bed configuration from plan views for Run P1505. Transverse scale is enlarged in order to better visualize alternate patterns.

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Figure 16. Computed bed profiles and surface mean sediment diameter with $\phi = 0$ and $\phi = 0.65$ on the LHS (20 cm from the left wall) for the varied case from Run P2009 at $t = 3$ h.

Figure 17. Computed distribution of the mean sediment diameter in the substrate layer with $\phi = 0$ and $\phi = 0.65$ for the varied case from Run P2009 at $t = 3$ h.

Figure 18. The difference between LHS and RHS bed elevations (B/10 from each wall) at $t = 1$ h for different aspect ratios.